



Complex frequency modeling of inverter resources

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Panel Session: Modeling of Inverter-Based
Resources for Large System Stability Studies



Research Question 1

What is the “internal” frequency of a converter?

- We know well that the internal frequency of a synchronous machine is the rotor angular speed.
- The rotor speed is the frequency of the internal emf of the machine
- Can we define a similar “internal” frequency for converters and, more in general, for devices that do not have a rotor?

Research Question 2

What is the link between frequency and power injection?

This is relevant because, if we know this link, then:

- We can understand better what to expect from existing controllers
- It is easier to design new and effective controllers

Complex Frequency

Definition

$$\bar{u} = u_d + j u_q = u \exp(j\varphi)$$



$$\bar{u} = \exp(\ln(u) + j\varphi)$$



$$\dot{\bar{u}} = (\dot{u}/u + j\dot{\varphi}) \bar{u} = (\rho_u + j\omega_u) \bar{u} = \bar{\eta}_u \bar{u}.$$

Complex Frequency (CF) of Voltage and Current

$$\dot{\bar{v}}_h = \bar{\eta}_v \bar{v}_h$$

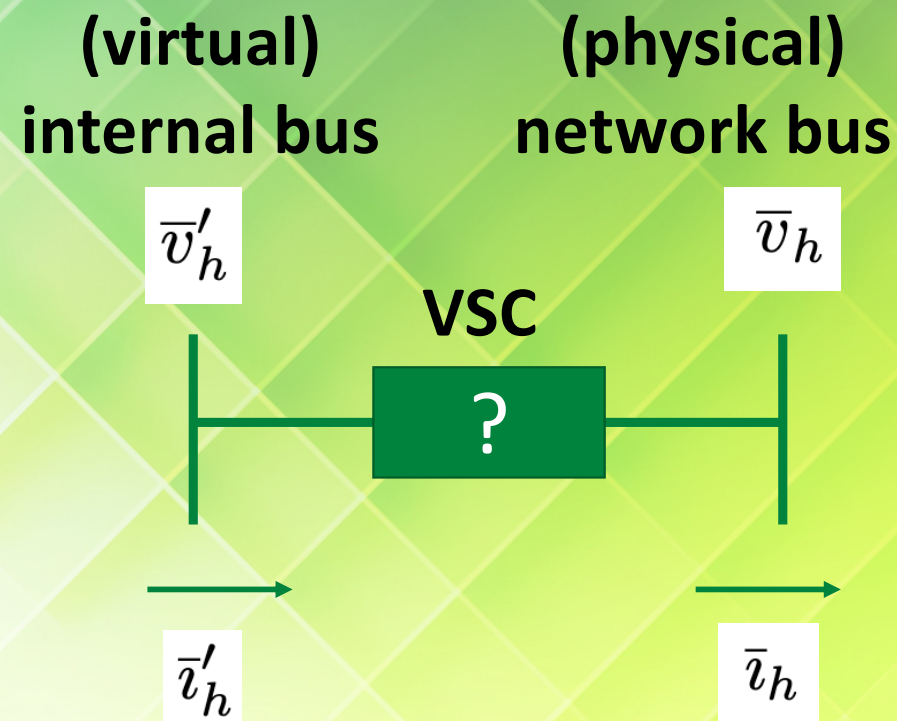
$$\dot{\bar{i}}_h = \bar{\eta}_i \bar{i}_h$$

The CF includes:

- a real part, which represents a *translation* and depends only on the magnitude of the Park vector; and
- an imaginary part, which represents a *rotation* and depends only on the phase angle of the Park vector.

Terminal bus vs “internal” bus

The goal is to use the CF to find the equations that describe what happens in the box.



Complex Frequency and Complex Power

$$\bar{s}_h = \bar{v}_h \bar{i}_h^*$$

The complex power is an invariant (it is the same, independently from the reference frame):

$$\begin{aligned} \dot{\bar{s}}_h &\rightarrow = (\bar{\eta}_v + \bar{\eta}_i^*) \bar{s}_h \\ &\rightarrow = [\bar{\eta}'_v + (\bar{\eta}'_i)^*] \bar{s}_h \end{aligned}$$

Ideal Controllers

**Constant current – constant power –
constant impedance**

Ideal DER Controllers - 1

- Ideal constant current control:

$$\dot{i}_h = 0, \quad \bar{\eta}_i = 0$$

$$\dot{s}_h = \bar{\eta}_v \bar{s}_h.$$

- Constant current source and constant power factor

$$\rho_i = 0, \quad \omega_v = \omega_i.$$

$$\dot{s}_h = \rho_v \bar{s}_h.$$

Ideal DER Controllers - 2

- Constant Power:

$$\dot{\bar{s}}_h = 0, \quad \bar{\eta}_v = -\bar{\eta}_i^*$$

- Constant admittance

$$\rho_v = \rho_i, \quad \omega_v = \omega_i$$

$$\dot{\bar{s}}_h = 2\rho_v \bar{s}_h.$$

Ideal DER Controllers - 3

- Constant active power and constant voltage:

$$\rho_v = 0, \quad \dot{p} = 0$$

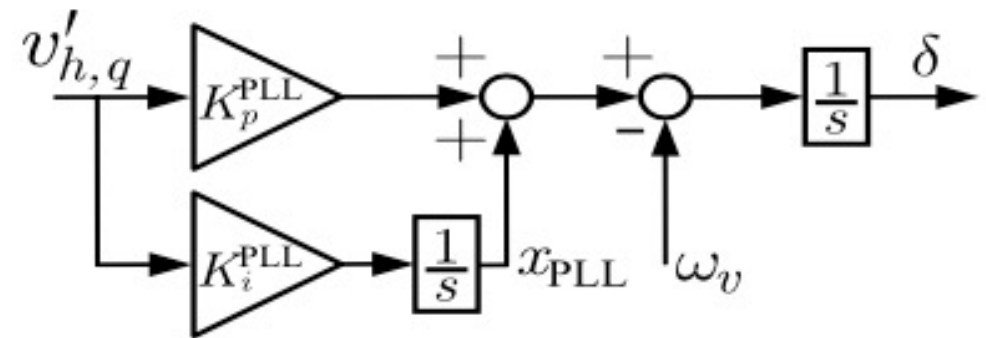
$$\frac{q}{p} = \frac{\rho_i}{(\omega_v - \omega_i)}$$

Current Controllers

**Effect of PLL and current
controllers of converters**

Effect of PLLs

The very first device that we can consider within the DER is the PLL which introduces a (transient) shift between the grid and the internal reference frame of the voltage and current of the DER:



$$\bar{v}_h = \exp(j\delta) \bar{v}'_h, \quad \bar{i}_h = \exp(j\delta) \bar{i}'_h \quad \longrightarrow \quad \begin{aligned} \bar{\eta}'_v &= \bar{\eta}_v - j\dot{\delta} \\ \bar{\eta}'_i &= \bar{\eta}_i - j\dot{\delta} \end{aligned}$$

Current Control

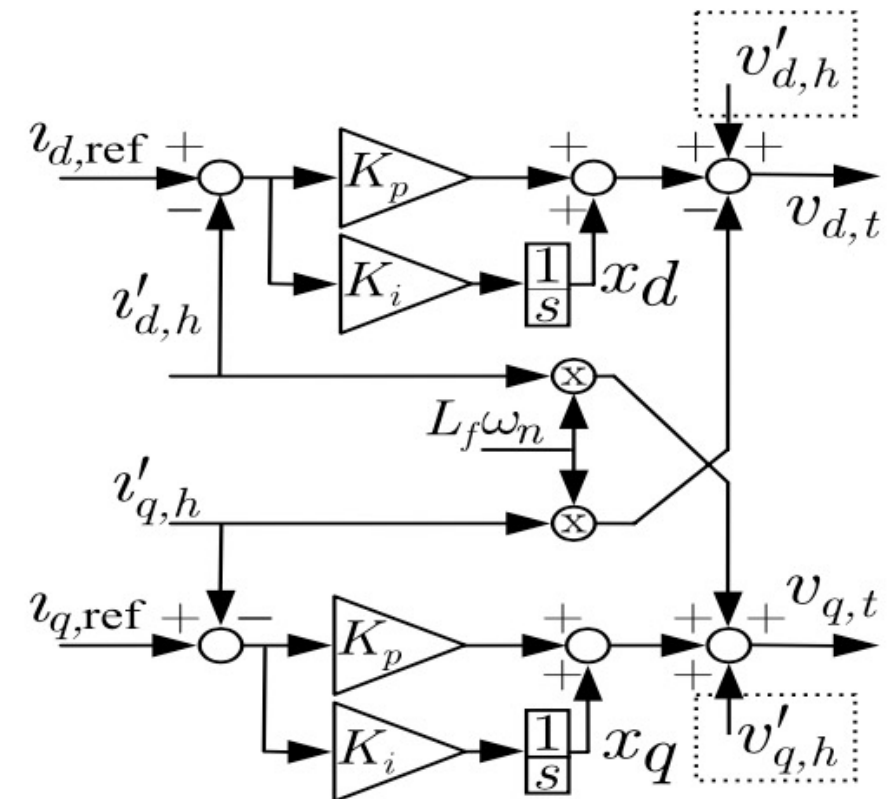
The effect of the current control is only on the real part of the CF:

$$\dot{\bar{s}}_h = (\bar{\eta}_{i_{\text{ref}}}^* + \kappa_{\text{PI}}) \bar{v}'_h \bar{i}_{\text{ref}}^* + (\bar{\eta}'_v - \kappa_{\text{PI}}) \bar{s}_h$$

where: $\kappa_{\text{PI}} = K_i/K_p$

$$(\bar{\eta}'_i)^* + \kappa_{\text{PI}} = \bar{\eta}_i^* + (\kappa_{\text{PI}} + j\dot{\delta})$$

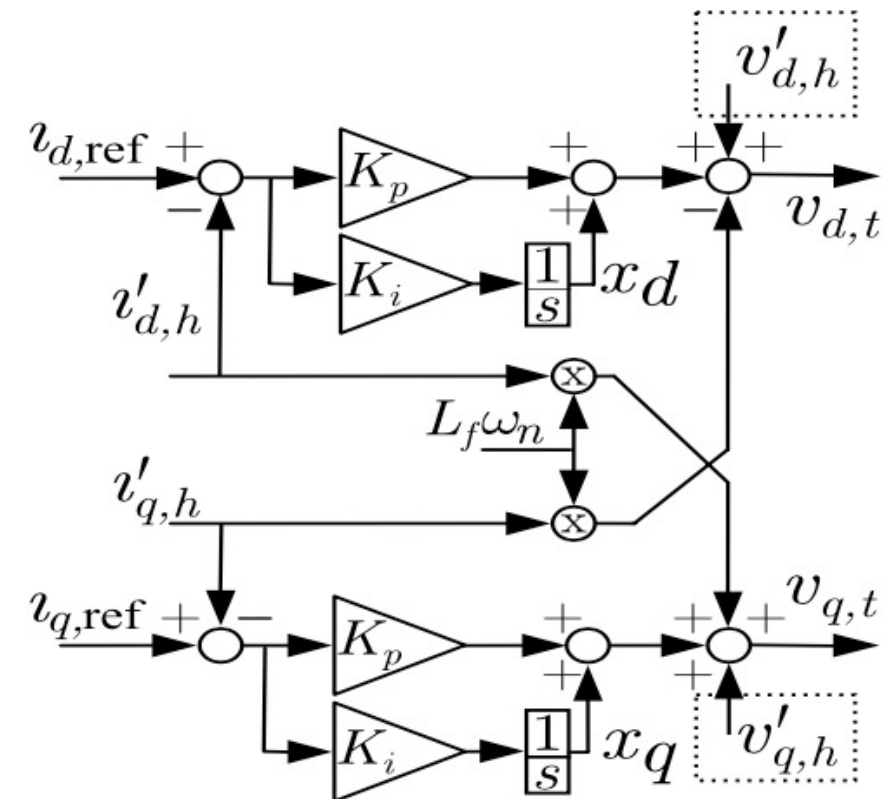
$$\bar{\eta}'_v - \kappa_{\text{PI}} = \bar{\eta}_v - (\kappa_{\text{PI}} + j\dot{\delta})$$



Voltage Feed Forward (VFF)

The effect of the VFF (dotted boxes in the figure) is inversely proportional to the proportional gain of the current controller:

$$\dot{\bar{s}}_h = (\bar{\eta}_{i_{\text{ref}}}^* + \kappa_{\text{PI}}) \bar{v}'_h \bar{i}_{\text{ref}}^* + (\bar{\eta}'_v - \kappa_{\text{PI}}) \bar{s}_h - \frac{1}{K_p} (\bar{\eta}'_v)^* v_h^2$$



GFL Converters

Constant current reference
Constant power reference
Virtual admittance loop

GFL Controllers

Current control with constant current reference:

$$\dot{\bar{s}}_h = \kappa_{\text{PI}} \bar{v}'_h \bar{i}_{\text{ref}}^* + (\bar{\eta}'_v - \kappa_{\text{PI}}) \bar{s}_h$$

Current control with constant power reference:

$$\dot{\bar{s}}_h = (\bar{\eta}'_v - \kappa_{\text{PI}}) (\bar{s}_h - \bar{s}_{\text{ref}})$$

Current control with virtual admittance loop:

$$\dot{\bar{s}}_h = -v_h^2 \bar{Y}_v^* 2\rho_v + \bar{Y}_v^* \bar{\eta}'_v \bar{v}'_h \bar{v}_{\text{ref}}$$

GFM Converters

**Voltage controller – Synchronization –
Outer voltage loop**

GFL controllers

Voltage control:

$$\dot{\bar{s}}_h = (K_p^v \bar{\eta}_{v_{\text{ref}}}^* + K_i^v) \bar{v}_h' \bar{v}_{\text{ref}}^* - (K_p^v (\bar{\eta}_v')^* + K_i^v) v_h^2 + (\bar{\eta}_v' - \kappa_{\text{PI}}) \bar{s}_h$$

Synchronization:

$$\dot{\delta} = \omega_{\text{VSM}} - \omega_v ,$$

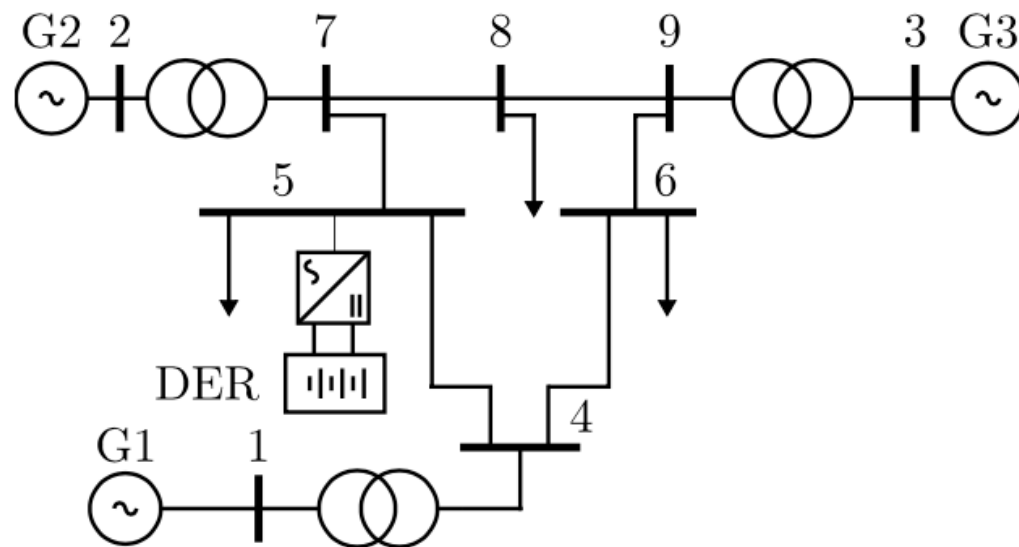
$$\dot{\omega}_{\text{VSM}} = \frac{1}{J_v} \left(\frac{p_{\text{ref}}}{\omega_n} - \frac{p_h}{\omega_{\text{VSM}}} + D_p (\omega_n - \omega_{\text{VSM}}) \right)$$

Outer voltage loop:

$$\bar{v}_{\text{ref}} = j v_{q,\text{ref}} = j \psi_v \omega_{\text{VSM}} ,$$

$$\bar{\eta}_{v_{\text{ref}}} \bar{v}_{\text{ref}} = j (\dot{\psi}_v \omega_{\text{VSM}} + \psi_v \dot{\omega}_{\text{VSM}})$$

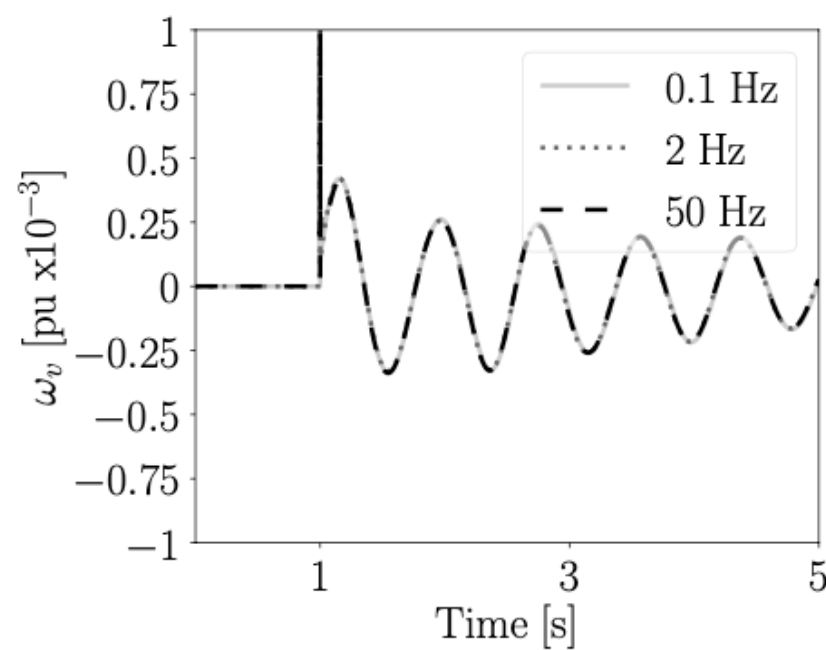
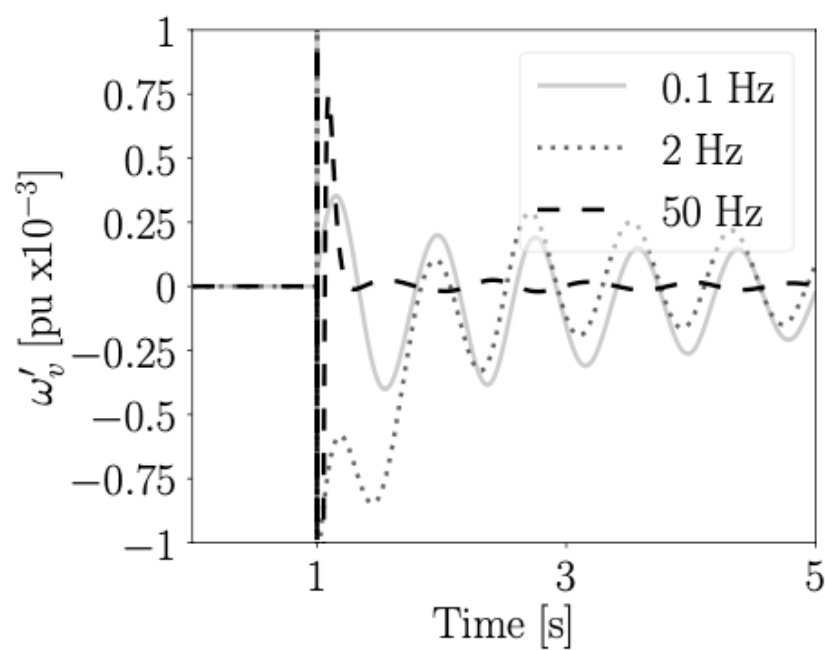
$$\bar{\eta}_{v_{\text{ref}}} = \rho_{v_{\text{ref}}}$$



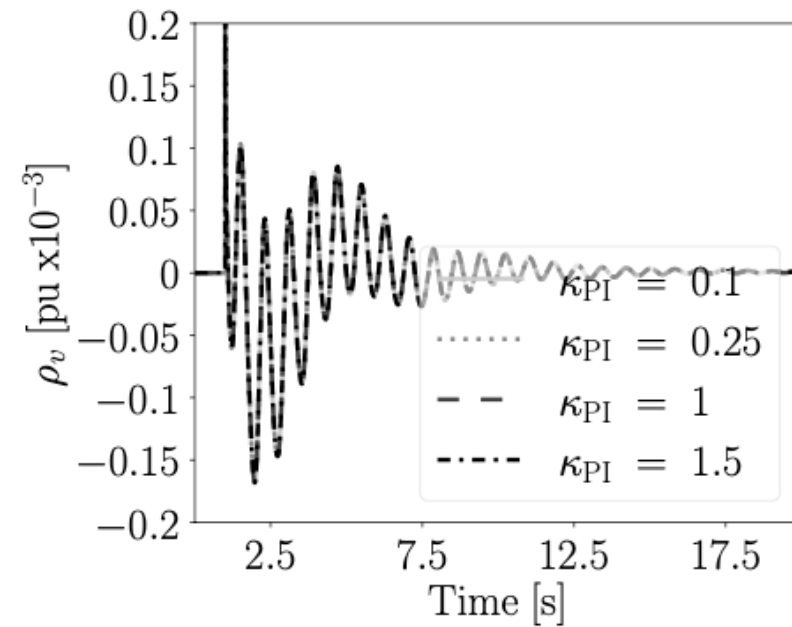
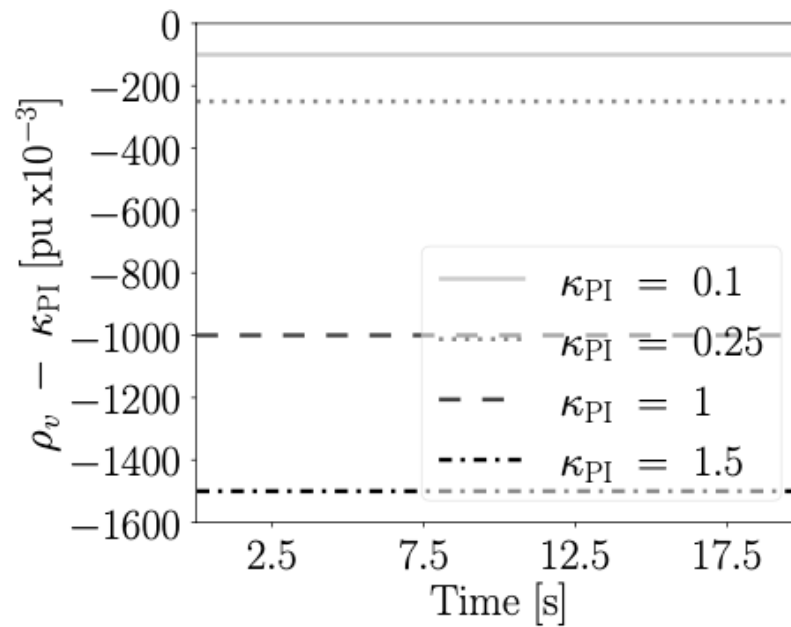
Case Study

CF as a “metric” of the effectiveness of the control

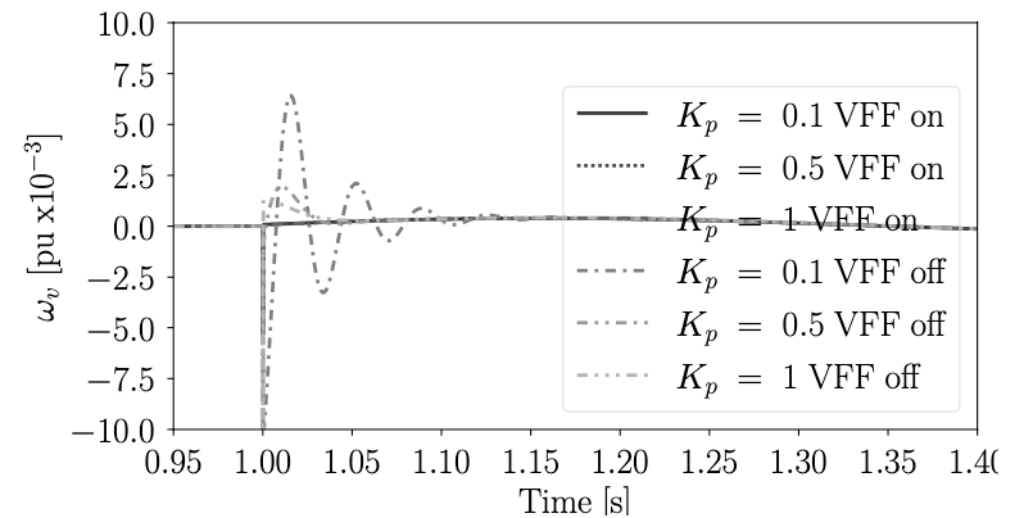
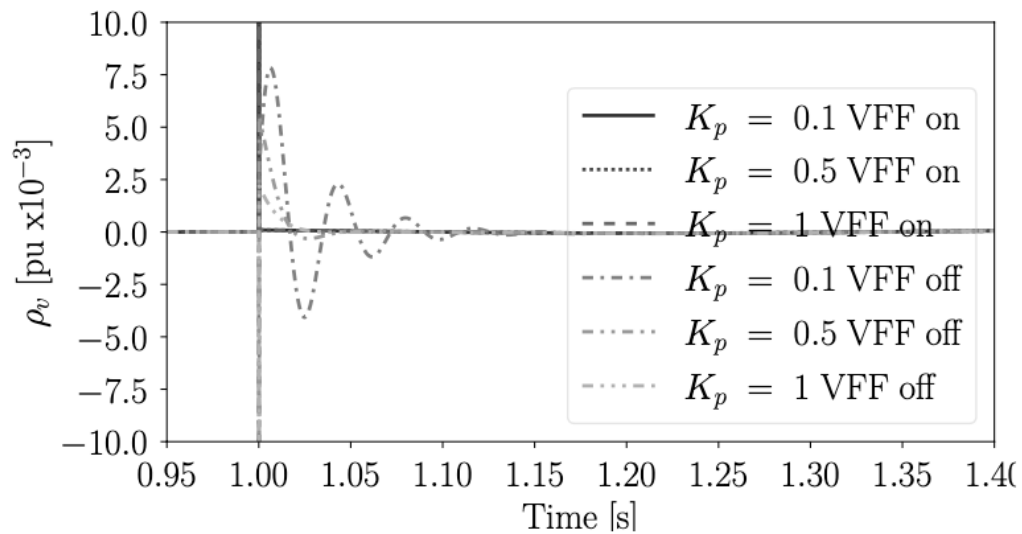
Effect of the bandwidth of the PLL



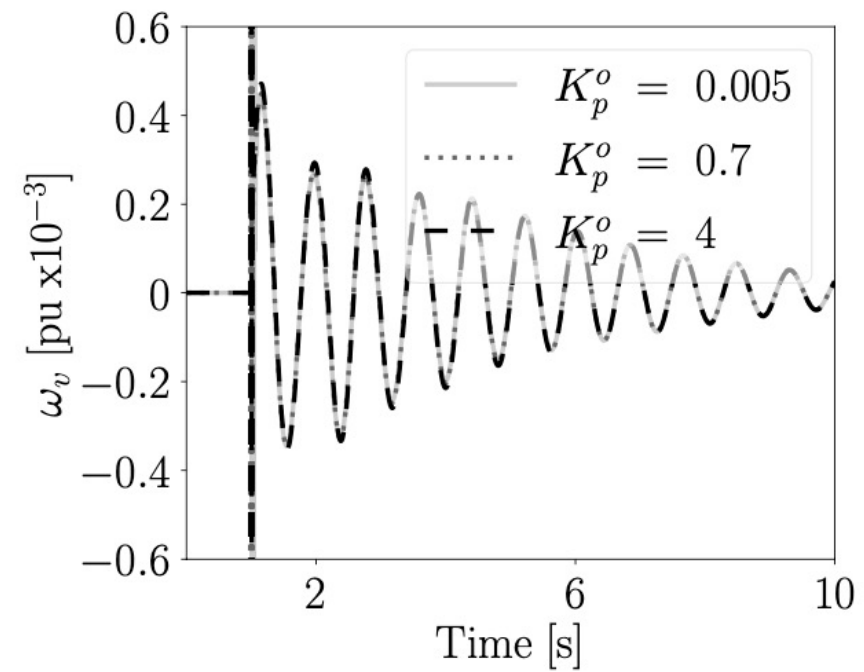
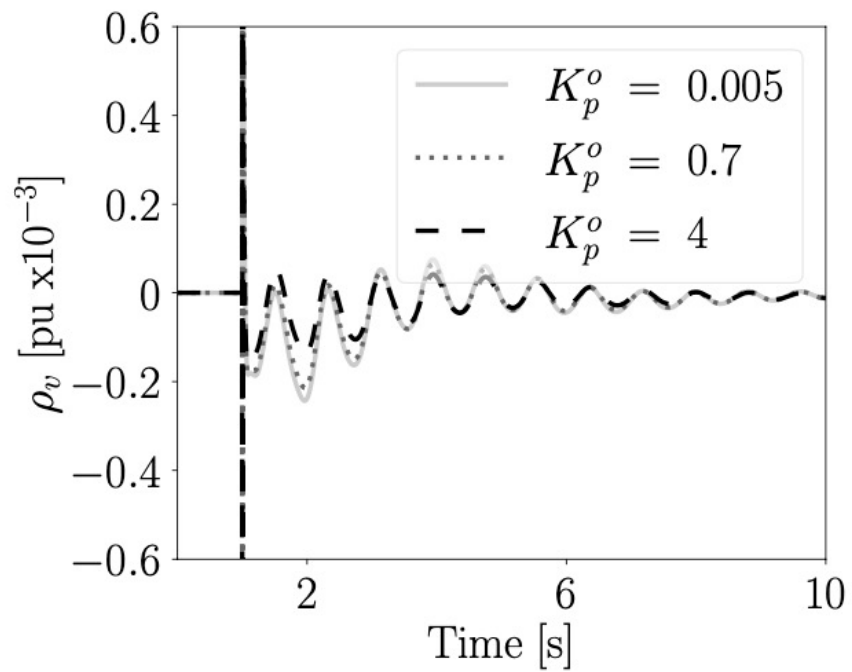
Effect Current Control Gains



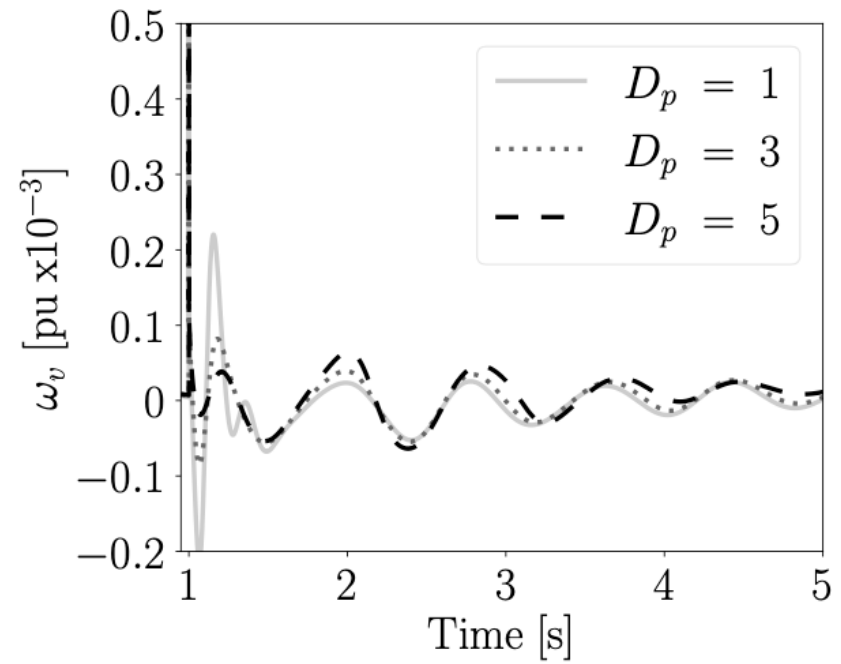
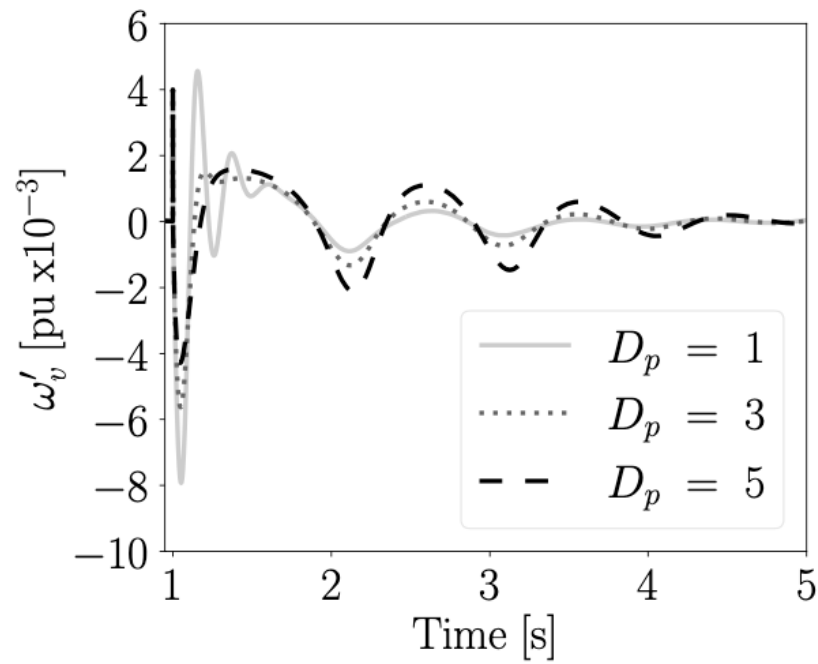
Effect of VFF

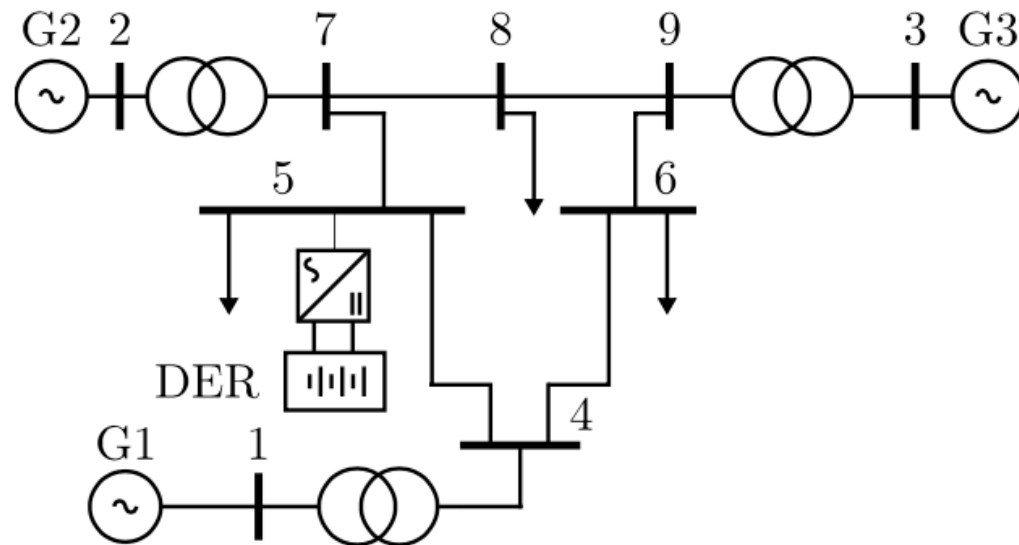


GFL current control



GFM virtual shaft

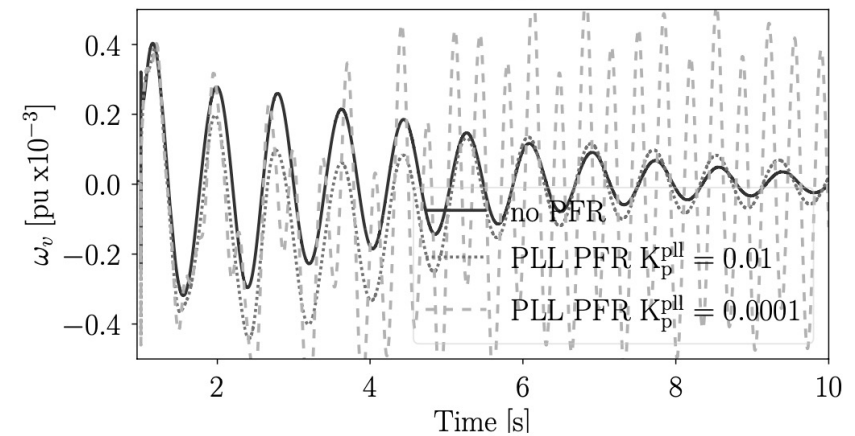
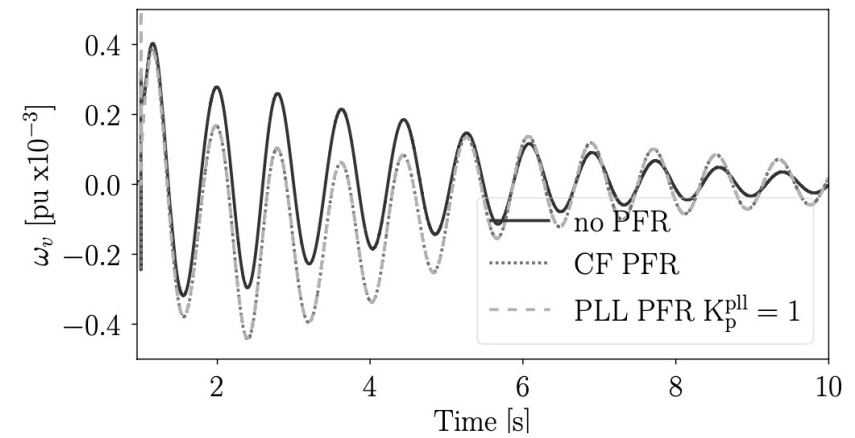
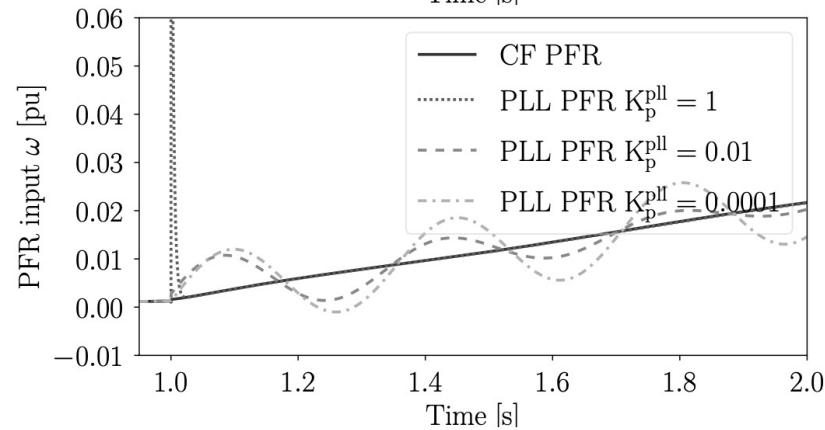
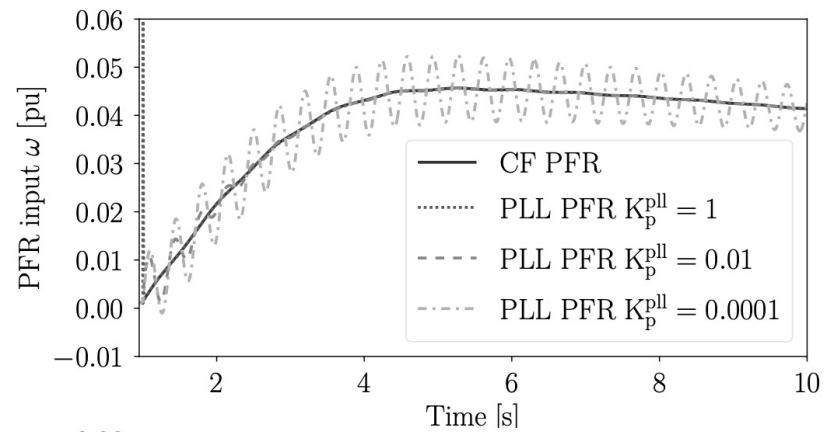




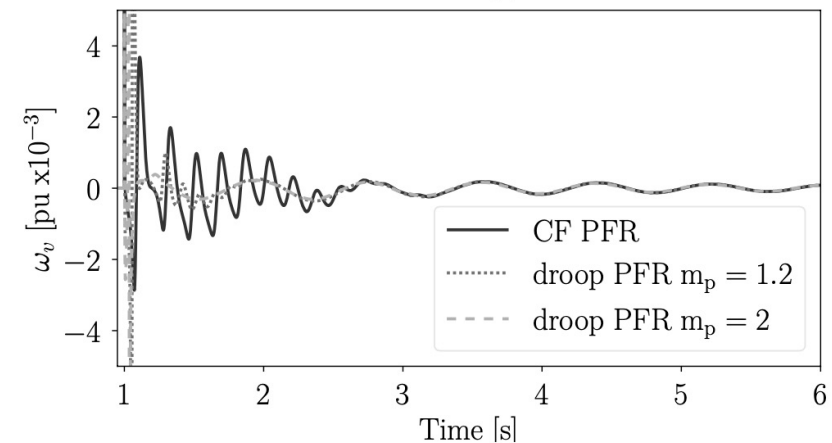
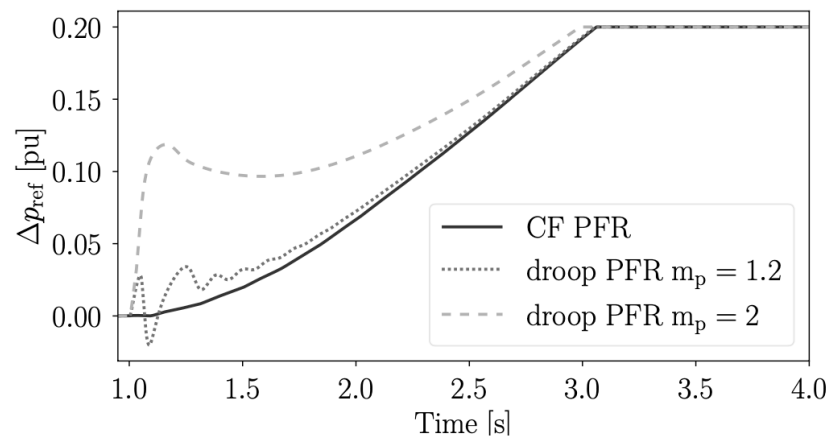
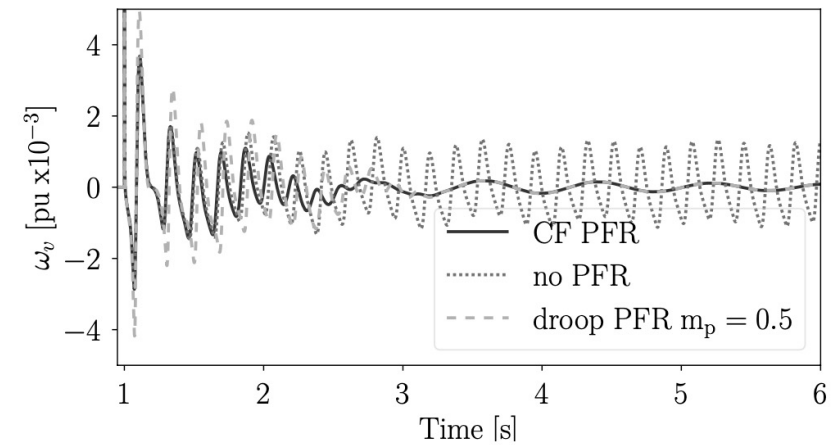
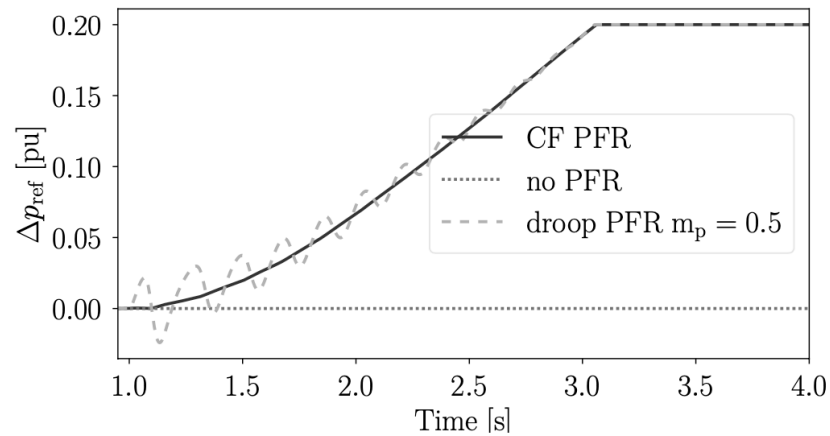
Case Study

CF as a tool to design more effective controllers

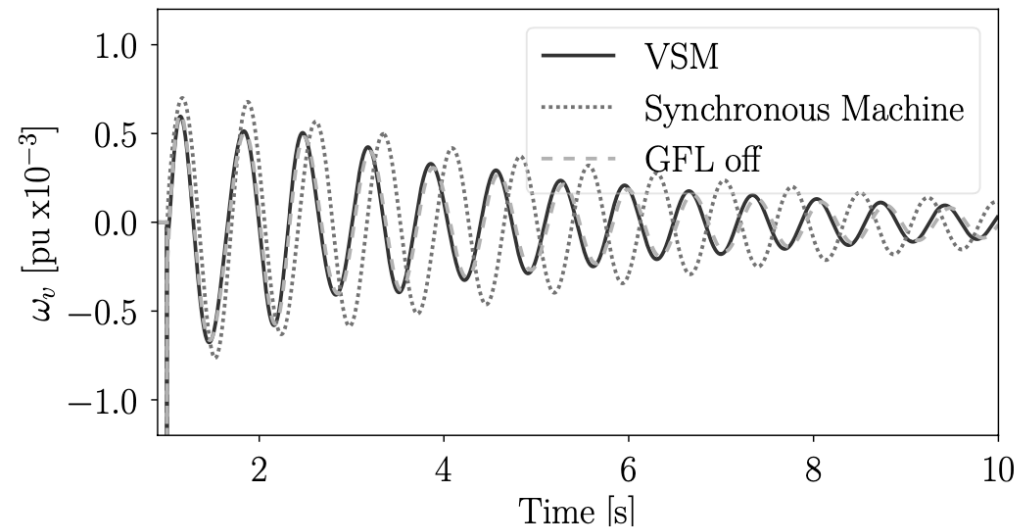
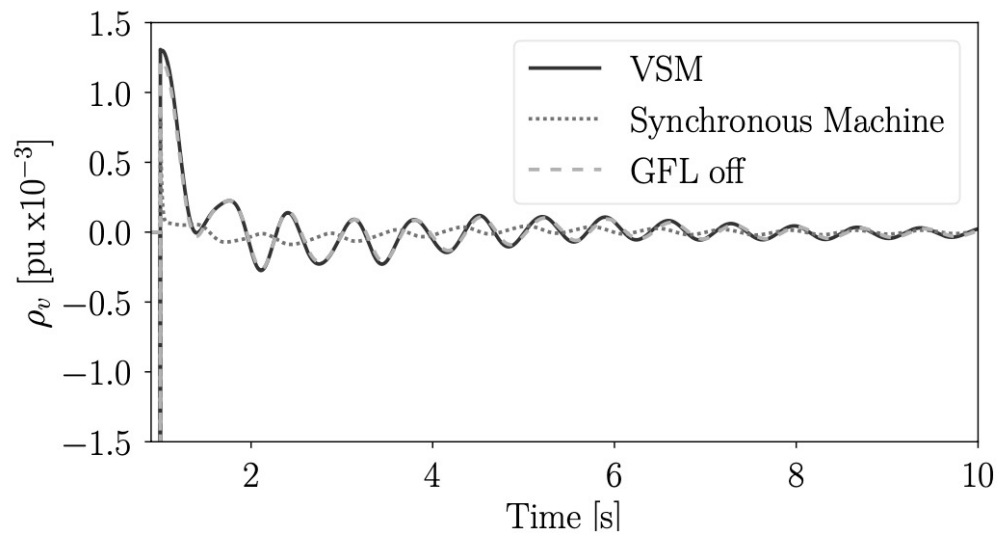
Design of the PLL



Design of the droop of GFL



Interaction among SM, VSM and GFL



Conclusions

Conclusions

- CF approach decouples the contribution on the local frequency of each sub-controller and identifies critical control parameters.
- The current controller is shown to represent a constant translation of the real part of the CF while the synchronization control, regardless of its type, affects the imaginary part.
- For GFL configurations, the PLL parameters are shown to have the largest impact on the local frequency.
- For GFM, active power droop parameter as well as VSM damping parameter are shown to affect the frequency response after a contingency.
- For the GFM case, the internal frequency of the controller achieves a better transient response than the exact frequency.

Future Work

- It seems to be relevant to extend the use of the calculated internal frequencies of the converters for control applications.
- There seem to be a potential of using non-conventional controllers based on CF or controllers based on non-conventional input signals (based on the real part of CF).
- The effect on CF of multiple converters, their dynamic interaction and the impact of this interaction on converter frequency control will also be studied.

Thank you!

Questions?