Analytical Framework for Power System Strength

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Panel Session: Power System Strength in Electricity Systems Dominated by CIG: Concepts, Assessment, Challenges and Solutions

Motivation

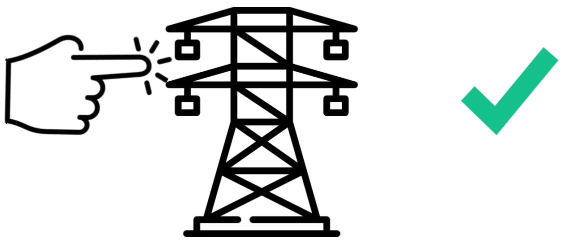
The concept of ‘strength’

How do we use it in power systems?

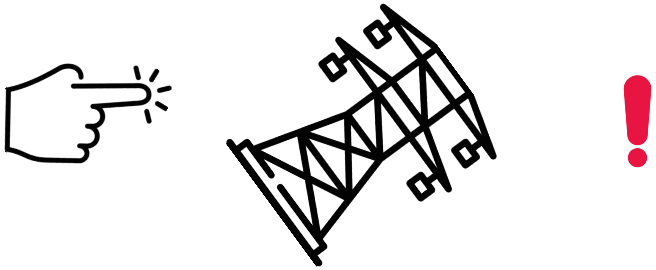
- Lack of standard definition.

- Refers to the system’s resistance to perturbations.

A **stronger** system is less sensitive to perturbations.



A **weaker** system is more sensitive to perturbations.



The concept of ‘strength’

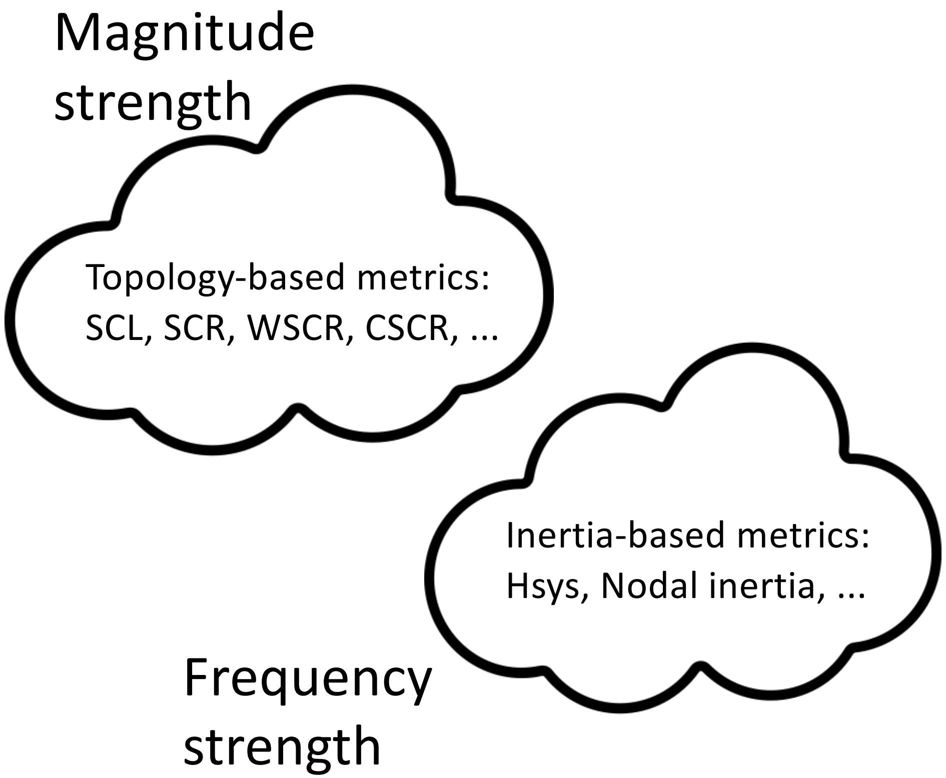
- In practice, it is typically **the voltage** the representative variable over which strength is evaluated.

- For example, the Australian Energy Market Comission (AEMC) defines system strength as: *“the power system’s ability to resist the changes in the **magnitude, phase angle, and waveform** of the voltage at any given location under different operating conditions”.*

Open question: **How to quantify this ability?**

Conventional strength assessment

Is this framework still adequate?



1) Rely on strong approximations.
2) Often involve equations proposed rather empirically than derived analytically.
3) Do not capture the effect of heterogeneous devices.
4) Inconsistent assessment for voltage magnitude and frequency.

Our proposal

- I. Ponce, **F. Milano**, "Analytical Framework for Power System Strength", submitted to the IEEE Transactions on Power Systems.

Contributions:

1) A novel general analytical framework to evaluate system strength in steady-state and dynamic conditions.

2) A systematic methodology to study the effect of diverse device models on system strength.

3) Definition of a novel mathematical operator, called *Delta operator*, along with some of its properties and identities.

Mathematical Background

Complex Frequency (CF)

- Consider a three-phase voltage represented as a dynamic vector:

$$\bar{v} \in \mathbb{C} \,|\, \bar{v} = v\cos\theta + \jmath\, v \sin\theta$$

- The CF of the vector is:

$$\bar{\eta} = \frac{\dot{v}}{v} + \jmath\,\dot{\theta} = \rho + \jmath\,\omega, \quad v \neq 0\,.$$

- The CF acts as a time-derivative operator:

$$\dot{\bar{v}} = \bar{v}\,\bar{\eta}$$

Complex Frequency (CF)

- Relative motion with respect to a rotating reference frame:

$$\dot{\bar{v}} = \bar{v}\,(\bar{\eta} - \jmath\,\omega_0) = \bar{v}\,\bar{\eta}'$$

- Second order CF:

$$\ddot{\bar{v}} = \bar{v}\,\left((\bar{\eta} - \jmath\,\omega_0)^2 + \dot{\bar{\eta}}\right) = \bar{v}\,\bar{\eta}''$$

$$\bar{\eta}'' = \sigma + \jmath\,\gamma$$

$$\sigma = \rho^2 - (\omega - \omega_0)^2 + \dot{\rho}\,; \quad \gamma = 2\rho\,(\omega - \omega_0) + \dot{\omega}$$

Delta operator (Δ)

- Let *f*(*t*) be an algebraic variable of the set of DAEs of the system.

Definition 1: Delta (Δ) operator applied to $f(t)$:

$$\Delta f(t) := \lim_{\tau \to t^+} f(\tau) - \lim_{\tau \to t^-} f(\tau) = f^+ - f^-$$

Definition 2: The *instantaneous arithmetic mean* of f:

$$\widetilde{f}(t) := \frac{f^+ + f^-}{2} \, .$$

Definition 3: The *instantaneous geometric mean* of f:

$$\hat{f}(t) := \sqrt{f^+ f^-} \, .$$

Delta operator (Δ)

- Properties:

Property 1: Δ of a constant with time is null.

$$\Delta\alpha = 0\,.$$

Property 2: Linearity.

$$\Delta\{\alpha f(t) + \beta g(t)\} = \alpha\Delta f(t) + \beta\Delta g(t)\,.$$

Property 3: Multiplication rule.

$$\Delta\{f(t)g(t)\} = \Delta f(t)\widetilde{g}(t) + \widetilde{f}(t)\Delta g(t)\,.$$

Property 4: Division rule.

$$\Delta\left\{\frac{f(t)}{g(t)}\right\} = \frac{\Delta f(t)\widetilde{g}(t) - \widetilde{f}(t)\Delta g(t)}{\hat{g}(t)^2}\,.$$

Property 5: Chain rule of the complex exponential

$$\Delta e^{\jmath f(t)} = \widetilde{e^{\jmath f(t)}}\,\jmath\,\frac{\tan(\Delta f(t)/2)}{1/2}\,.$$

Proposed Framework

A more general and unifying framework for system strength

Preliminaries

- Strength is conceived as a property of each bus of the network. Particularly, of its voltage vector.
- It is evaluated with respect to changes in the current injected by a fictitious independent current source at the bus. **Not** restricted to infinitesimal changes; in turn applicable to large-signal events (using Δ operator).
- It evaluates (i) how much the voltage vector is expected to jump, and (ii) how fast it will continue to deviate right after the disturbance.

A more general framework

- We propose a formulation composed of **three categories of indicators**, depending on the order of the time derivative involved:

Zero-order:

$$\begin{bmatrix} \Delta v/\widetilde{v} \\ 2\tan(\Delta\theta/2) \end{bmatrix} = \begin{bmatrix} S_{v\imath_\mathrm{p}} & S_{v\imath_\mathrm{q}} \\ S_{\theta\imath_\mathrm{p}} & S_{\theta\imath_\mathrm{q}} \end{bmatrix} \begin{bmatrix} \Delta\imath_\mathrm{p} \\ \Delta\imath_\mathrm{q} \end{bmatrix}$$

First-order:

$$\begin{bmatrix} \Delta\rho \\ \Delta\omega \end{bmatrix} = \begin{bmatrix} S_{\rho\imath_\mathrm{p}} & S_{\rho\imath_\mathrm{q}} \\ S_{\omega\imath_\mathrm{p}} & S_{\omega\imath_\mathrm{q}} \end{bmatrix} \begin{bmatrix} \Delta\imath_\mathrm{p} \\ \Delta\imath_\mathrm{q} \end{bmatrix}$$

Second-order:

$$\begin{bmatrix} \Delta\sigma \\ \Delta\gamma \end{bmatrix} = \begin{bmatrix} S_{\sigma\imath_\mathrm{p}} & S_{\sigma\imath_\mathrm{q}} \\ S_{\gamma\imath_\mathrm{p}} & S_{\gamma\imath_\mathrm{q}} \end{bmatrix} \begin{bmatrix} \Delta\imath_\mathrm{p} \\ \Delta\imath_\mathrm{q} \end{bmatrix}$$

12 strength indicators

How to calculate the metrics?

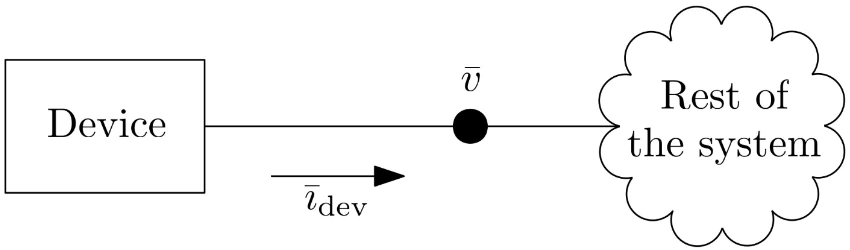
- We propose an **analytical approach** based on the dynamic model of the system.
- Therefore, the metrics depend on the parameters and variables of the system model.

$$\begin{aligned}\Delta \underline{\boldsymbol{v}}_{\mathrm{v}\theta} &= \underline{\underline{\boldsymbol{S}}}(\boldsymbol{p}, \boldsymbol{x}(t), \boldsymbol{y}(t))\Delta \underline{\boldsymbol{\imath}}_{\mathrm{pq}} \,, \\ \Delta \underline{\boldsymbol{\eta}}' &= \underline{\underline{\boldsymbol{S}}}'(\boldsymbol{p}, \boldsymbol{x}(t), \boldsymbol{y}(t))\Delta \underline{\boldsymbol{\imath}}_{\mathrm{pq}} \,, \\ \Delta \underline{\boldsymbol{\eta}}'' &= \underline{\underline{\boldsymbol{S}}}''(\boldsymbol{p}, \boldsymbol{x}(t), \boldsymbol{y}(t))\Delta \underline{\boldsymbol{\imath}}_{\mathrm{pq}}\end{aligned}$$

The goal is to find analytical expressions for $\underline{\underline{S}}$, $\underline{\underline{S}}'$ and $\underline{\underline{S}}''$.

Effect of devices composing the grid

- The effect of the devices shunt-connected to buses is modeled as follows:



$$\begin{aligned}
\Delta \underline{\boldsymbol{\imath}}_{\mathbf{dev}} &= \underline{\underline{\boldsymbol{A}}}\, \Delta \underline{\boldsymbol{v}}\,, \\
\Delta \underline{\dot{\boldsymbol{\imath}}}_{\mathbf{dev}} &= \underline{\underline{\boldsymbol{A}}}'\, \Delta \underline{\boldsymbol{v}} + \underline{\underline{\boldsymbol{B}}}'\, \Delta \underline{\dot{\boldsymbol{v}}}\,, \\
\Delta \underline{\ddot{\boldsymbol{\imath}}}_{\mathbf{dev}} &= \underline{\underline{\boldsymbol{A}}}''\, \Delta \underline{\boldsymbol{v}} + \underline{\underline{\boldsymbol{B}}}''\, \Delta \underline{\dot{\boldsymbol{v}}} + \underline{\underline{\boldsymbol{C}}}''\, \Delta \underline{\ddot{\boldsymbol{v}}}\,.
\end{aligned}$$

- Their contribution is captured through matrices:

$$\underline{\underline{\boldsymbol{A}}},\ \underline{\underline{\boldsymbol{A}}}',\ \underline{\underline{\boldsymbol{A}}}'',\ \underline{\underline{\boldsymbol{B}}}',\ \underline{\underline{\boldsymbol{B}}}'' \text{ and } \underline{\underline{\boldsymbol{C}}}''$$

Derivation

- Starting from the algebraic equations of the network voltages and current injections, an strict analytical derivation is done by applying and exploiting the properties the Δ operator and the CF.
- The sought expressions for the strength metrics are:

Zero-order: $$\underline{\underline{S}} = \widetilde{\underline{\underline{v}}}^{-1} \left(\widetilde{\underline{\underline{e}}^{\jmath\,\theta}}\right)^{-1} \underline{\underline{Z}}_{\mathrm{eq}}\, \underline{\underline{e}}^{\jmath\,\theta^+}$$

First-order: $$\underline{\underline{S}}' = \widetilde{\underline{\underline{v}}}^{-1} \left(\underline{\underline{Z}}'_{\mathrm{eq}} - \widetilde{\underline{\underline{\eta}}'}\, \underline{\underline{Z}}_{\mathrm{eq}}\right) \underline{\underline{e}}^{\jmath\,\theta^+}$$

Second-order: $$\underline{\underline{S}}'' = \widetilde{\underline{\underline{v}}}^{-1} \left(\underline{\underline{Z}}''_{\mathrm{eq}} - \widetilde{\underline{\underline{\eta}}''}\, \underline{\underline{Z}}_{\mathrm{eq}}\right) \underline{\underline{e}}^{\jmath\,\theta^+}$$

where $\underline{\underline{Z}}_{\mathrm{eq}}$, $\underline{\underline{Z}}'_{\mathrm{eq}}$ and $\underline{\underline{Z}}''_{\mathrm{eq}}$ depend on device matrices: $\underline{\underline{A}}$, $\underline{\underline{A}}'$, $\underline{\underline{A}}''$, $\underline{\underline{B}}'$, $\underline{\underline{B}}''$ and $\underline{\underline{C}}''$

The problem reduces to find analytical expressions for these components for specific devices.

Device Models Examples

A systematic procedure to study their effect on system strength

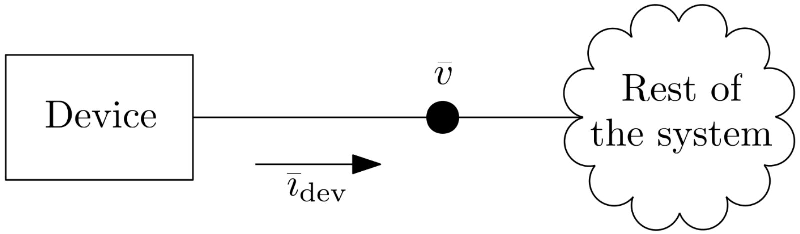
Procedure

- Starting from the set of DAEs of the device model,
- the goal is to find $\underline{\underline{a}}$, $\underline{\underline{a}}'$, $\underline{\underline{a}}''$, $\underline{\underline{b}}'$, $\underline{\underline{b}}''$ and $\underline{\underline{c}}''$

Zero-order: $\Delta \underline{i}_{\text{dev}} = \underline{\underline{a}}\, \Delta \underline{v}\,,$

First-order: $\Delta \underline{\dot{i}}_{\text{dev}} = \underline{\underline{a}}'\, \Delta \underline{v} + \underline{\underline{b}}'\, \Delta \underline{\dot{v}}\,,$

Second-order: $\Delta \underline{\ddot{i}}_{\text{dev}} = \underline{\underline{a}}''\, \Delta \underline{v} + \underline{\underline{b}}''\, \Delta \underline{\dot{v}} + \underline{\underline{c}}''\, \Delta \underline{\ddot{v}}\,.$



Synchronous machines

- Classical model of a synchronous machine:

$$\underline{\underline{a}} = -\begin{bmatrix} r_{\mathrm{a}} & -x_{1\mathrm{d}} \\ x_{1\mathrm{d}} & r_{\mathrm{a}} \end{bmatrix}^{-1}$$

$$\underline{\underline{a}}' = 0\,; \qquad \underline{\underline{b}}' = \underline{\underline{a}}\,.$$

$$\underline{\underline{a}}'' = \jmath\, \underline{\underline{a}}\, \underline{\underline{E}}\, \frac{\Omega_{\mathrm{b}}}{M} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \underline{\underline{E}}^{*}\, \underline{\underline{a}}\,,$$

$$\underline{\underline{b}}'' = 0\,; \qquad \underline{\underline{c}}'' = \underline{\underline{a}}\,.$$

Loads

- Constant-impedance load:

$$\underline{\underline{a}} = \underline{\underline{b}}' = \underline{\underline{c}}'' = -\underline{\underline{z}}^{-1} \,,$$
$$\underline{\underline{a}}' = \underline{\underline{a}}'' = \underline{\underline{b}}'' = 0 \,.$$

Converters

- Standard model of a GFL converter with active and reactive power control, ideal synchronization, and a droop frequency control.

$$\underline{\underline{a}} = \underline{\underline{\imath}}\,\underline{\underline{\widetilde{\mathrm{e}^{\jmath\theta}}}} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \widetilde{v}^{-1} \left(\underline{\underline{\widetilde{\mathrm{e}^{\jmath\theta}}}}\right)^{-1} .$$

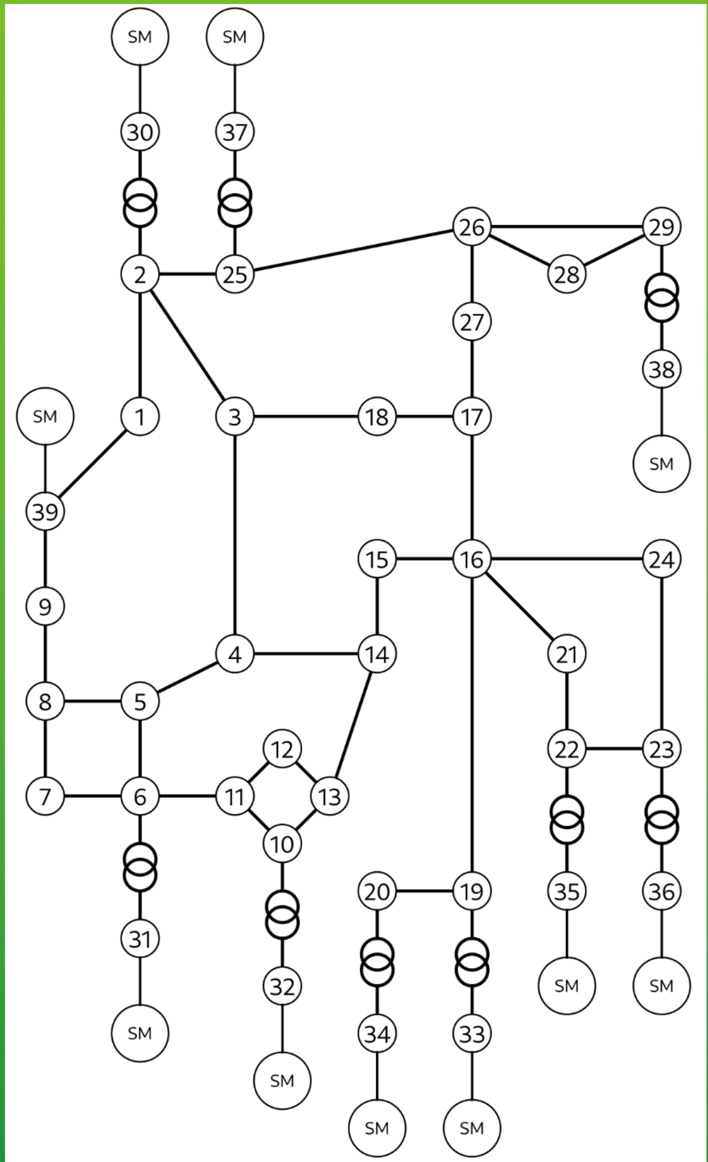
$$\underline{\underline{b}}' = \underline{\underline{\imath}}\,\underline{\underline{\widetilde{\mathrm{e}^{\jmath\theta}}}} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \underline{\underline{\widetilde{v}}}^{-1} ,$$

$$\underline{\underline{a}}' = \frac{1}{T} \underline{\underline{\widetilde{\mathrm{e}^{\jmath\theta}}}} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \hat{v}^{-2} \left(\underline{\underline{\widetilde{\mathrm{e}^{\jmath\theta}}}}\right)^{-1} \underline{\underline{s_{\mathrm{ref}}}} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \left(\underline{\underline{\widetilde{\mathrm{e}^{\jmath\theta}}}}\right)^{-1} .$$

$$\underline{\underline{c}}'' = \underline{\underline{b}}' ,$$

$$\underline{\underline{a}}'' = \frac{1}{T^2} \widetilde{v}\,\underline{\underline{\widetilde{\mathrm{e}^{\jmath\theta}}}}\,\hat{v}^{-4} \underline{\underline{\imath}}^{-1} \underline{\underline{s_{\mathrm{ref}}}}^{*2} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \left(\underline{\underline{\widetilde{\mathrm{e}^{\jmath\theta}}}}\right)^{-1} ,$$

$$\underline{\underline{b}}'' = \frac{1}{T} \hat{v}^{-2} \underline{\underline{\widetilde{\mathrm{e}^{\jmath\theta}}}} \left(\frac{1}{T_{\mathrm{f}} R} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} - \underline{\underline{s_{\mathrm{ref}}}}^{*} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \right) \left(\underline{\underline{\widetilde{\mathrm{e}^{\jmath\theta}}}}\right)^{-1} .$$



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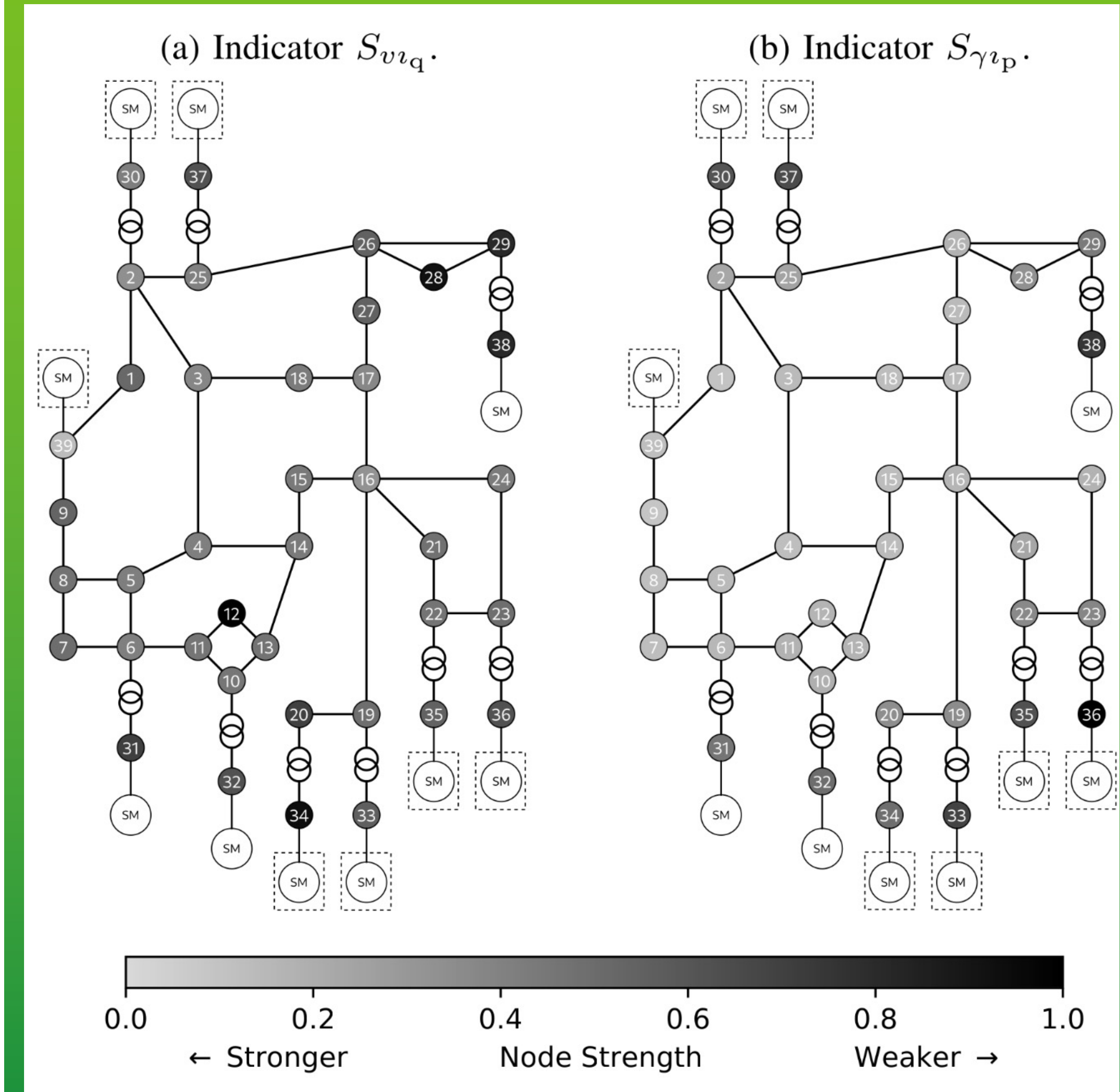
Case Study

- IEEE 39 Bus System



Strength metrics results

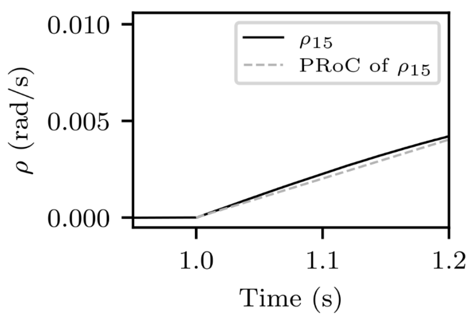
- Zero-order metric in accordance with SCL.
- First-order metrics are null (CF is infinitely strong).
- Second-order metric dominated by inertia distribution.

Dynamic validation

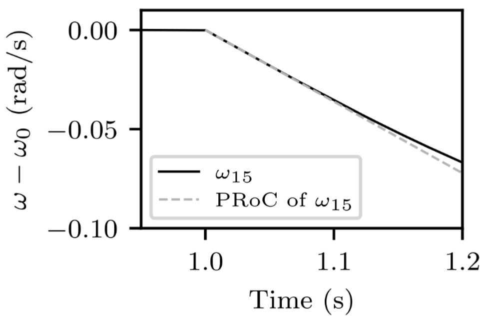
- Given a known perturbation, we use the strength metrics to predict $\Delta\bar{v}$, $\Delta\bar{\eta}'$ and $\Delta\bar{\eta}''$.
- The results are compared with the actual jump of these variables observed in the trajectories after a time-domain simulation.

Variable	Predicted using $\underline{\underline{S}}'_{15}$	Read from TDS	Error
Δv_{15} (pu)	-0.00461526	-0.00461524	-1.273e-08
$\Delta\theta_{15}$ (rad)	-0.01730211	-0.01730211	6.545e-09
$\Delta\rho_{15}$ (pu/s)	0.0	2.984e-06	-2.984e-06
$\Delta\omega_{15}$ (pu/s)	0.0	-4.067e-05	4.067e-05

(a) ρ_{15} and its PRoC.

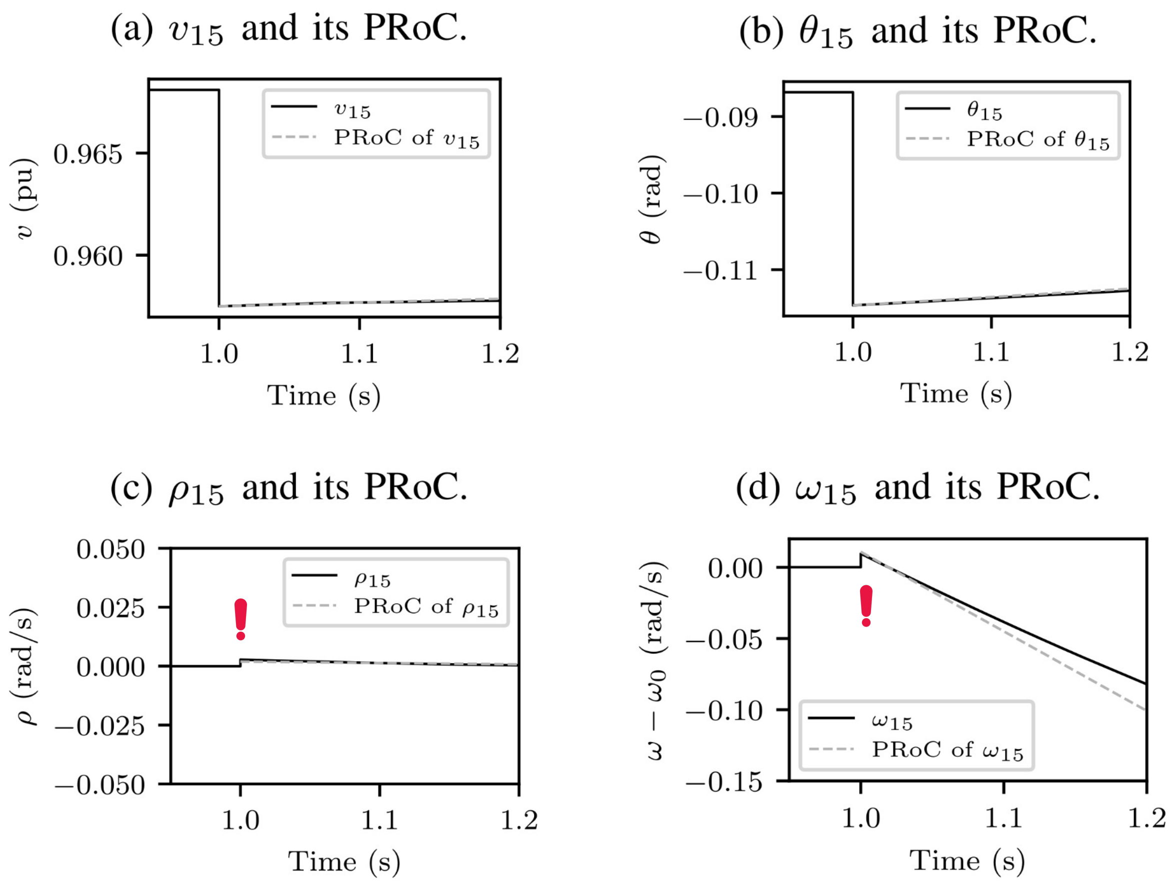


(b) ω_{15} and its PRoC.



Dynamic validation

- The system is modified replacing synchronous generation by GFLs, which introduce a first-order component to the system.
- The CF is not continuous anymore.
- The prediction remains accurate.



Conclusions

Conclusions

- Theoretical foundations established for a general and unifying framework for power system strength.
- It features a set of 12 indicators organized in three dynamical orders that capture the voltage strength when subjected to current injection changes.
- Systematic way to study the impact of different devices on strength. For instance, the key parameters of SMs are the internal reactance, mostly affecting the zero-order strength, and the inertia, dominating the second-order strength.

Conclusions

- A network composed exclusively of SMs forces the CF to be continuous.
- This is lost under presence of GFLs due to their first-order component.
- Future work will focus on applications and practical aspects of the calculation of the proposed metrics.

Thank you

- Questions?