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Impact of PLL Control on Small-Signal Stability of Wind DFIGs

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Introduction

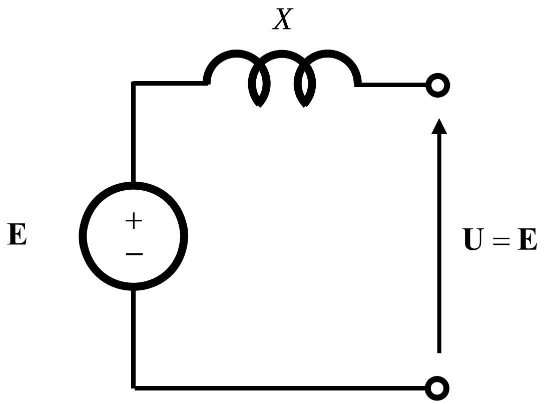
- Connecting wind generation to weak grids is becoming challenging because of stability issues.
- The strength of a grid is defined by the short circuit ratio.

$$SCR = \frac{S_{sc}}{P_n}$$

S_{sc} Grid short circuit capacity (MVA)

P_n Nominal active power of the plant (MW)

Thèvenin equivalent in per unit of the nominal active power of the plant



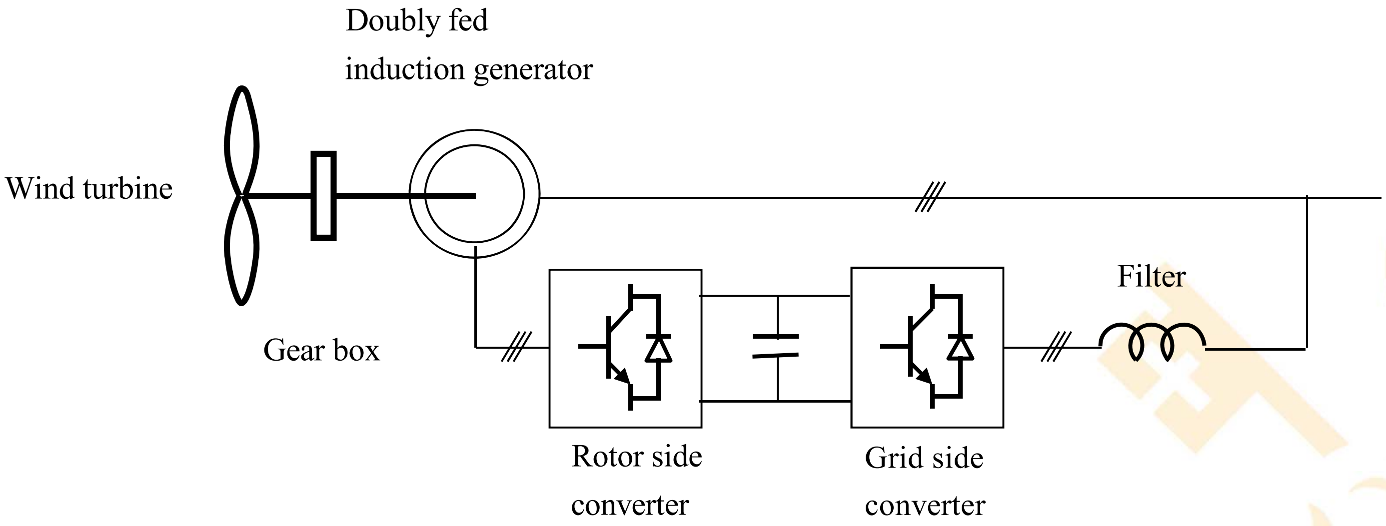
$$SCR = \frac{1}{X}$$

Introduction

- Wind DFIGs (Doubly Fed Induction Generators), also called type 3 wind turbine generators, are very much used in on-shore wind generation due to the compromise between performance and cost.
- It has been found that stability issues arising in the connection of wind DFIGs to weak grids can only be addressed using electromagnetic simulation.
- It has been also found that the Phase Locked Loop (PLL) control plays an important role in such instability phenomena.
- Hence, power system stability models of wind turbine generators do not characterize the phenomena os interest.
- This presentation discusses the nature of the instability of phenomena related with PLL control using a very detailed model of a wind DFIG.

Wind DFIG controls

- Wind DFIGs

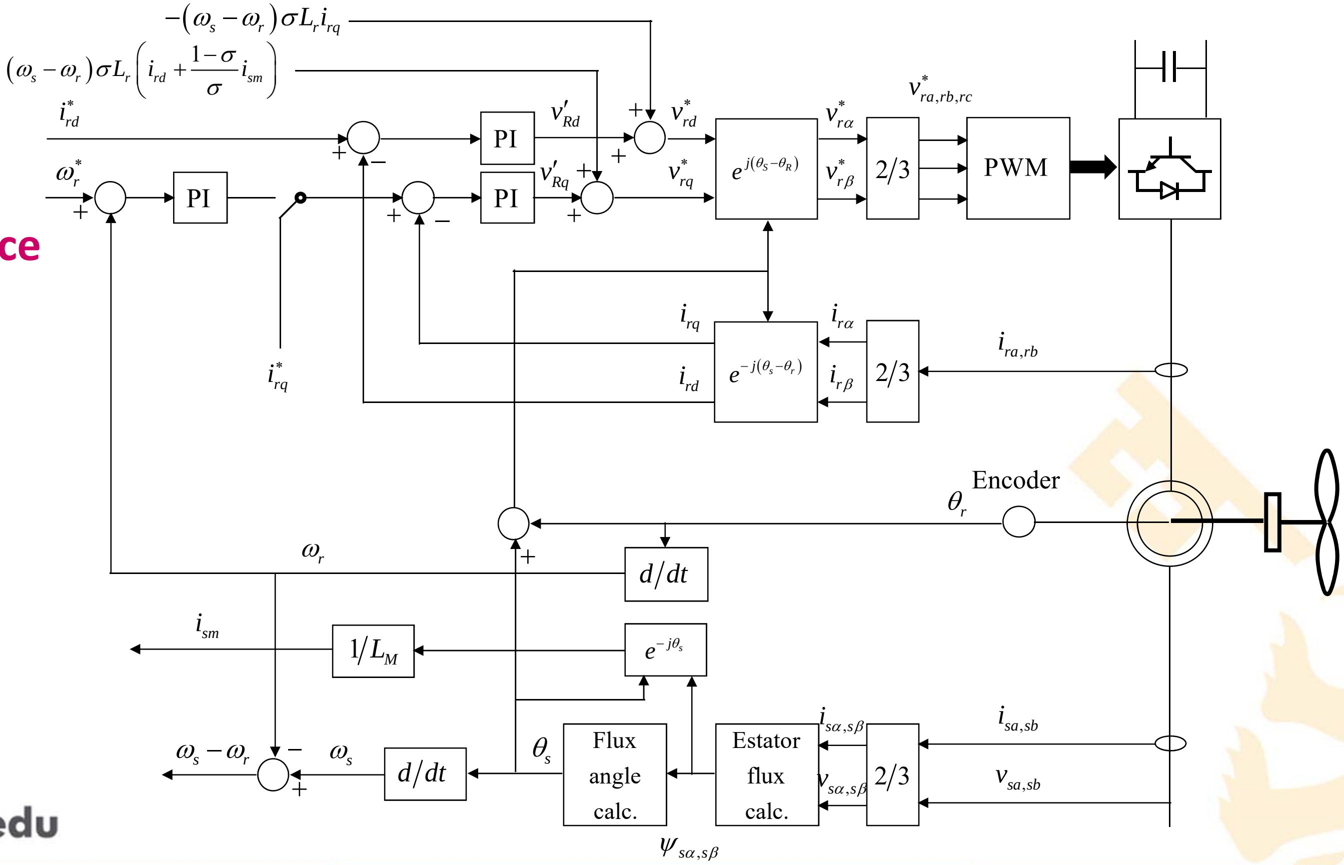


Wind DFIG controls

- Rotor side controls

Rotor reactive current reference

Speed reference

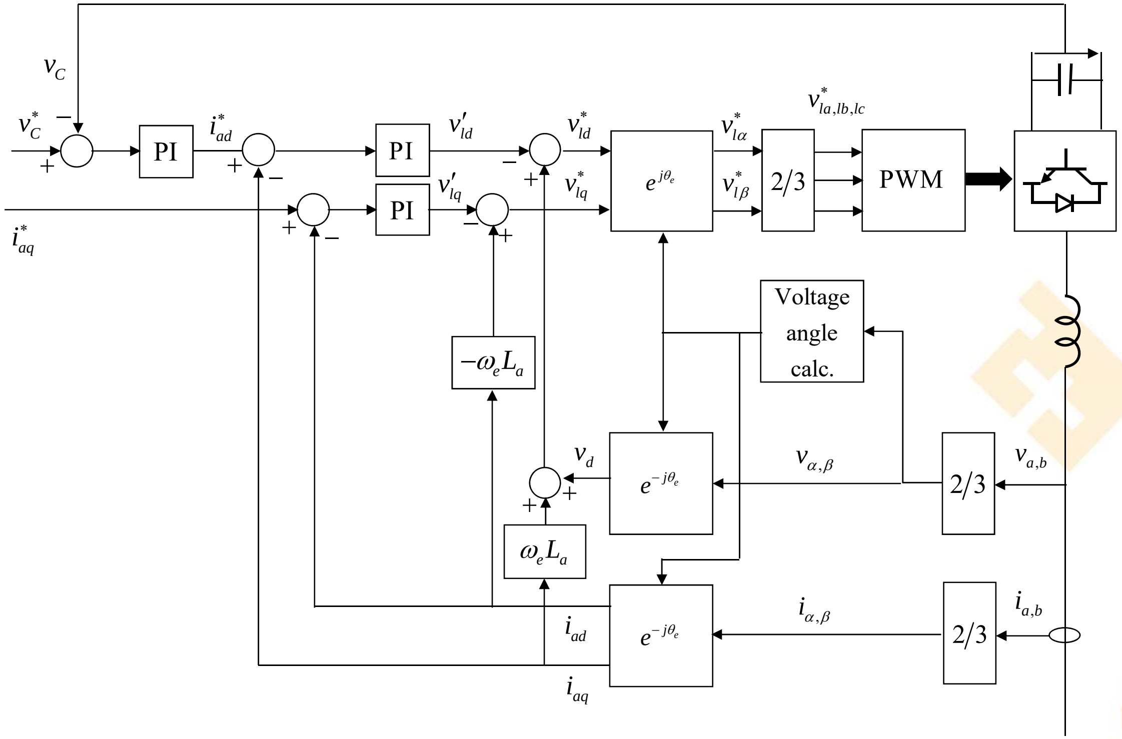


Wind DFIG controls

- Grid side controls

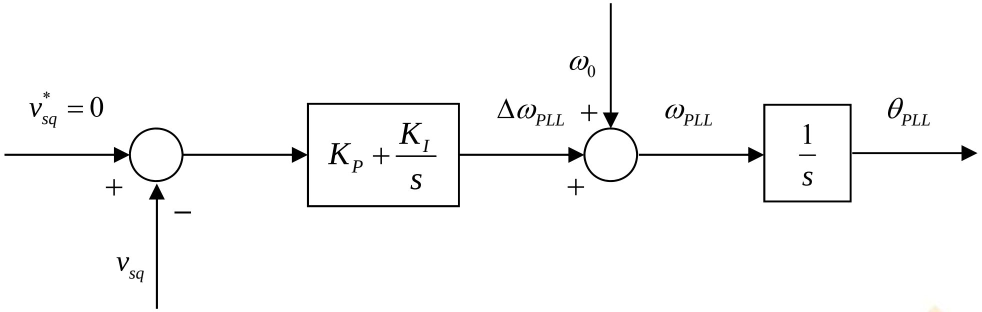
DC link capacitor voltage reference

Reactive current reference



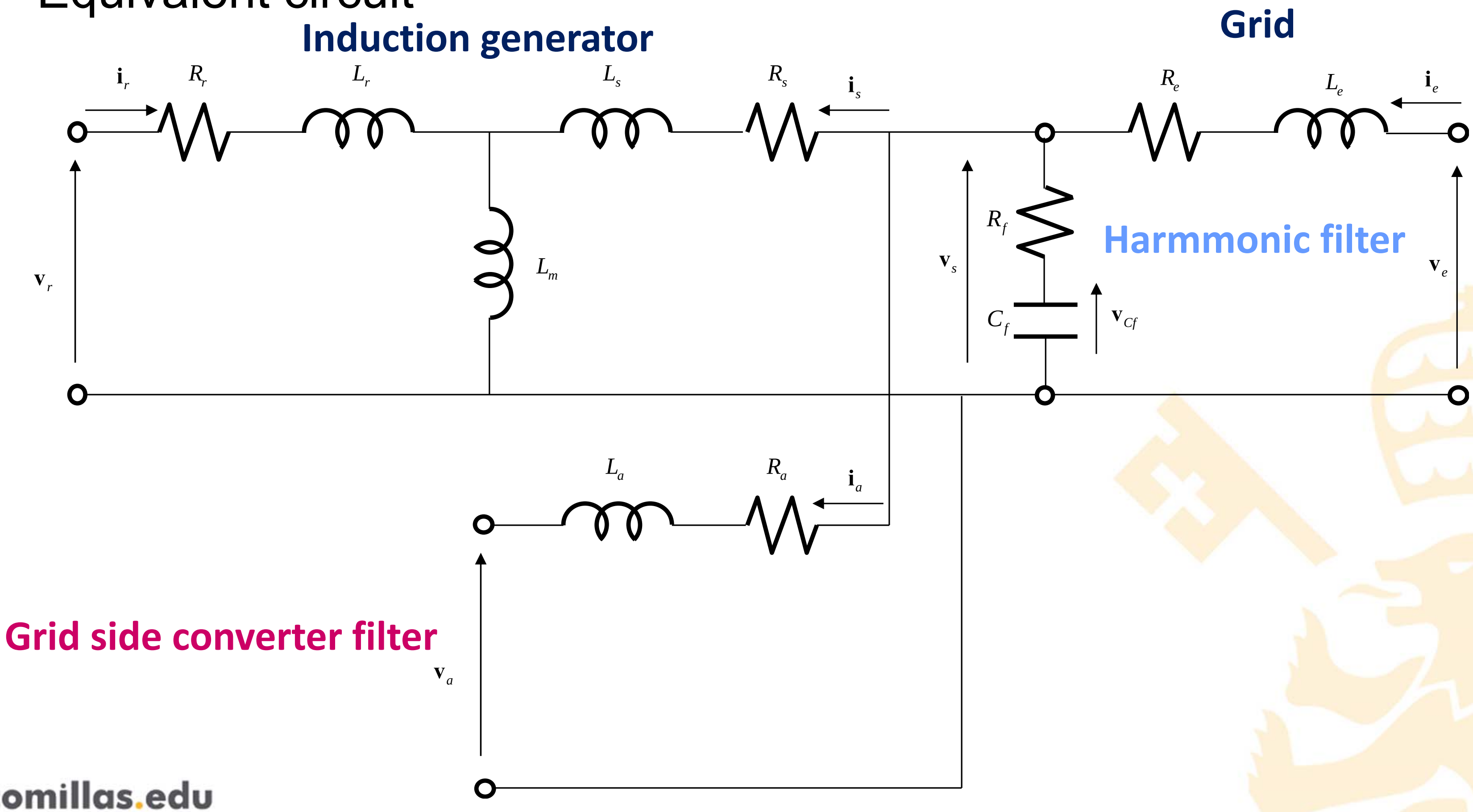
Wind DFIG controls

- Voltage angle calculation
  - Phase Locked Loop



Wind DFIG models

- Equivalent circuit



Wind DFIG models

- Induction machine
  - Stator

$$\frac{d}{dt}\begin{bmatrix}\psi_{sd}\\ \psi_{sq}\end{bmatrix} = -\omega_0\begin{bmatrix}R_s & 0\\ 0 & R_s\end{bmatrix}\begin{bmatrix}i_{sd}\\ i_{sq}\end{bmatrix} - \omega_0\omega_s\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}\begin{bmatrix}\psi_{sd}\\ \psi_{sq}\end{bmatrix} + \omega_0\begin{bmatrix}v_{sd}\\ v_{sq}\end{bmatrix}$$

$$\begin{bmatrix}\psi_{sd}\\ \psi_{sq}\end{bmatrix} = \begin{bmatrix}L_{ss} & 0\\ 0 & L_{ss}\end{bmatrix}\begin{bmatrix}i_{sd}\\ i_{sq}\end{bmatrix} + \begin{bmatrix}L_m & 0\\ 0 & L_m\end{bmatrix}\begin{bmatrix}i_{rd}\\ i_{rq}\end{bmatrix}$$

  - Rotor

$$\frac{d}{dt}\begin{bmatrix}\psi_{rd}\\ \psi_{rq}\end{bmatrix} = -\omega_0\begin{bmatrix}R_r & 0\\ 0 & R_r\end{bmatrix}\begin{bmatrix}i_{rd}\\ i_{rq}\end{bmatrix}^{v_s} - \omega_0 s\omega_s\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}\begin{bmatrix}\psi_{rd}\\ \psi_{rq}\end{bmatrix} + \omega_0\begin{bmatrix}v_{rd}\\ v_{rq}\end{bmatrix}^{v_s}$$

$$\begin{bmatrix}\psi_{rd}\\ \psi_{rq}\end{bmatrix} = \begin{bmatrix}L_m & 0\\ 0 & L_m\end{bmatrix}\begin{bmatrix}i_{sd}\\ i_{sq}\end{bmatrix} + \begin{bmatrix}L_{rr} & 0\\ 0 & L_{rr}\end{bmatrix}\begin{bmatrix}i_{rd}\\ i_{rq}\end{bmatrix}^{v_s}$$

Wind DFIG models

- Induction machine
  - Rotor dynamics

$$2H\omega_s \frac{ds}{dt} = t_m - t_e$$

$$t_e = \psi_{rq} i_{rd} - \psi_{rd} i_{rq}$$

Wind DFIG models

- Rotor side converter control

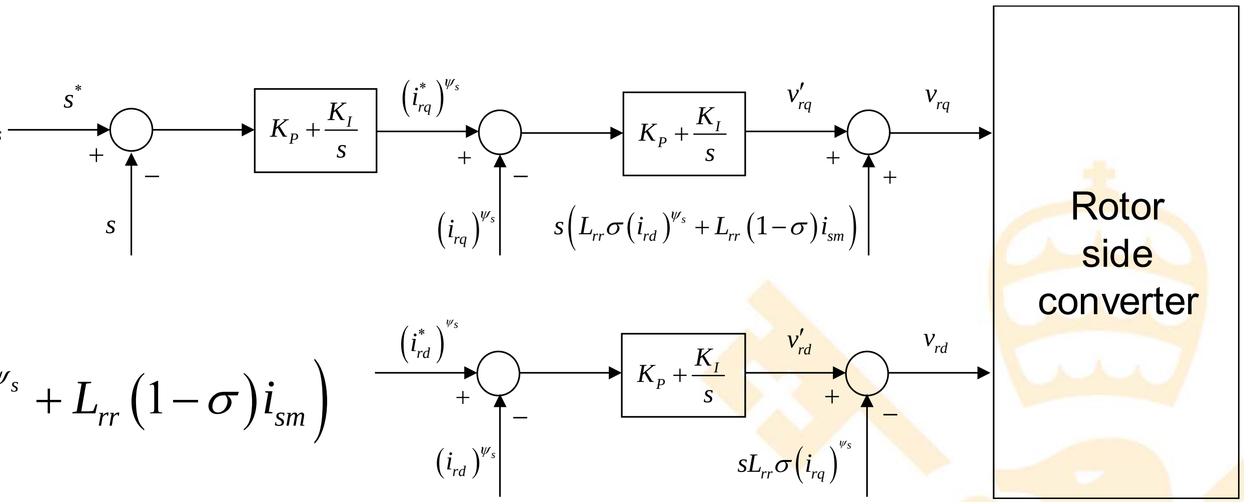
$$v'_{rd} = \left(K_P + \frac{K_I}{s}\right)\left(\left(i^*_{rd}\right)^{\psi_s} - \left(i_{rd}\right)^{\psi_s}\right)$$

$$\left(v_{rd}\right)^{\psi_s} = v'_{rd} - \left(\omega_s - \omega_r\right) L_{rr} \sigma \left(i_{rq}\right)^{\psi_s}$$

$$v'_{rq} = \left(K_P + \frac{K_I}{s}\right)\left(\left(i^*_{rq}\right)^{\psi_s} - \left(i_{rq}\right)^{\psi_s}\right)$$

$$\left(v_{rq}\right)^{\psi_s} = v'_{rq} + \left(\omega_s - \omega_r\right)\left(L_{rr}\sigma\left(i_{rd}\right)^{\psi_s} + L_{rr}\left(1-\sigma\right) i_{sm}\right)$$

$$\left(i^*_{rq}\right)^{\psi_s} = \left(K_{Ps} + \frac{K_{Is}}{s}\right)\left(s^* - s\right)$$



Wind DFIG models

- Rotor side converter

$$i_{sm} = \frac{\psi_s}{L_m} = \frac{\sqrt{\psi_{sd}^2 + \psi_{sq}^2}}{L_m}$$

$$\psi_s = \sqrt{\psi_{sd}^2 + \psi_{sq}^2}$$

$$\phi = \arctan \frac{\psi_{sd}}{\psi_{sq}}$$

$$\begin{bmatrix} v_{rd} \\ v_{rq} \end{bmatrix}^{v_s} = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} v_{rd} \\ v_{rq} \end{bmatrix}^{\psi_s}$$

$$\begin{bmatrix} i_{rd} \\ i_{rq} \end{bmatrix}^{v_s} = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} i_{rd} \\ i_{rq} \end{bmatrix}^{\psi_s}$$

Wind DFIG models

- Grid side converter filter

$$\frac{d}{dt}\begin{bmatrix}\psi_{ad}\\ \psi_{aq}\end{bmatrix} = -\omega_0\begin{bmatrix}R_a & 0\\ 0 & R_a\end{bmatrix}\begin{bmatrix}i_{ad}\\ i_{aq}\end{bmatrix} - \omega_0\omega_s\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}\begin{bmatrix}\psi_{ad}\\ \psi_{aq}\end{bmatrix} + \omega_0\begin{bmatrix}v_{sd}\\ v_{sq}\end{bmatrix} - \omega_0\begin{bmatrix}v_{ad}\\ v_{aq}\end{bmatrix}$$

$$\begin{bmatrix}\psi_{ad}\\ \psi_{aq}\end{bmatrix} = \begin{bmatrix}L_a & 0\\ 0 & L_a\end{bmatrix}\begin{bmatrix}i_{ad}\\ i_{aq}\end{bmatrix}$$

- DC link capacitor

$$\frac{d\left(v_C^2\right)}{dt} = \frac{2}{C}\left(p_r - p_a\right)$$

$$p_r = v_{rd}i_{rd} + v_{rq}i_{rq}$$
$$p_a = v_{ad}i_{ad} + v_{aq}i_{aq}$$

Wind DFIG models

- Grid side converter controls

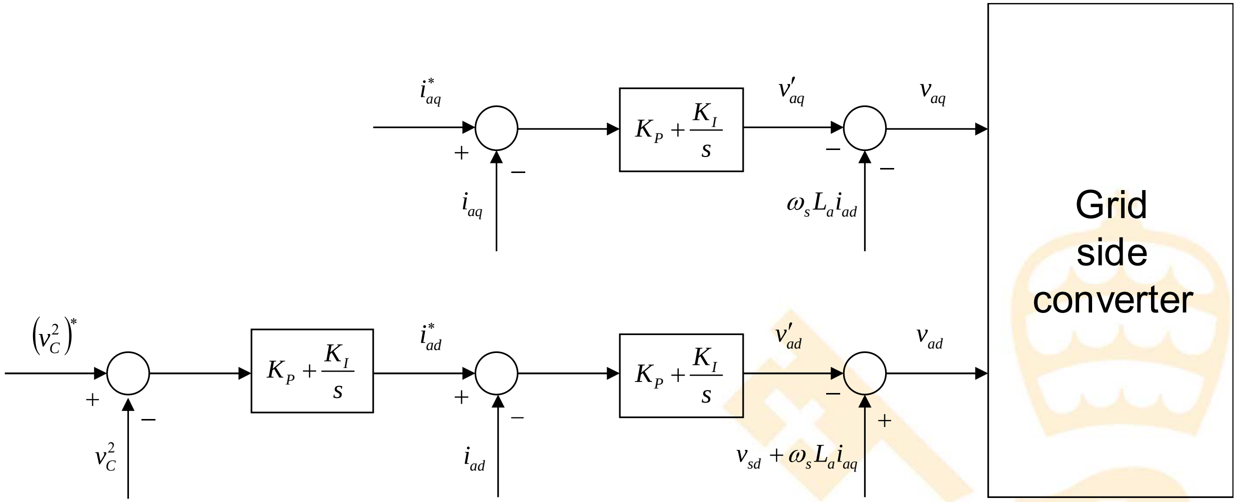
$$v'_{aq} = \left(K_{Pa} + \frac{K_{Ia}}{s}\right)\left(i^*_{aq} - i_{aq}\right)$$

$$v_{aq} = -\omega_s L_a i_{ad} - v'_{aq}$$

$$v'_{ad} = \left(K_{Pa} + \frac{K_{Ia}}{s}\right)\left(i^*_{ad} - i_{ad}\right)$$

$$v_{ad} = v_{sd} + \omega_s L_a i_{aq} - v'_{ad}$$

$$i^*_{ad} = \left(K_{Pv} + \frac{K_{Iv}}{s}\right)\left(\left(v_C^2\right)^* - v_C^2\right)$$



Wind DFIG models

- Harmmonic filter

$$\begin{bmatrix} i_{Cfd} \\ i_{Cfq} \end{bmatrix} = \frac{1}{\omega_0} \begin{bmatrix} C_f & 0 \\ 0 & C_f \end{bmatrix} \frac{d}{dt} \begin{bmatrix} v_{Cfd} \\ v_{Cfq} \end{bmatrix} + \omega_s \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} v_{Cfd} \\ v_{Cfq} \end{bmatrix}$$

$$\begin{bmatrix} v_{sd} \\ v_{sd} \end{bmatrix} = \begin{bmatrix} R_f & 0 \\ 0 & R_f \end{bmatrix} \begin{bmatrix} i_{fd} \\ i_{fq} \end{bmatrix} + \begin{bmatrix} v_{Cfd} \\ v_{Cfq} \end{bmatrix}$$

Wind DFIG models

- Grid

$$\begin{bmatrix} i_{ed} \\ i_{eq} \end{bmatrix} = \begin{bmatrix} \cos\theta_{PLL} & \sin\theta_{PLL} \\ -\sin\theta_{PLL} & \cos\theta_{PLL} \end{bmatrix} \begin{bmatrix} i_{eR} \\ i_{eI} \end{bmatrix}$$

$$\begin{bmatrix} i_{ed} \\ i_{eq} \end{bmatrix} = \begin{bmatrix} i_{sd} \\ i_{sq} \end{bmatrix} + \begin{bmatrix} i_{ad} \\ i_{aq} \end{bmatrix} + \begin{bmatrix} i_{fd} \\ i_{fq} \end{bmatrix}$$

$$\begin{bmatrix} v_{eR} \\ v_{eI} \end{bmatrix} = \begin{bmatrix} R_e & 0 \\ 0 & R_e \end{bmatrix} \begin{bmatrix} i_{ed} \\ i_{eq} \end{bmatrix} + \frac{1}{\omega_0}\frac{d}{dt} \begin{bmatrix} \psi_{eR} \\ \psi_{eI} \end{bmatrix} + \omega_s \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \psi_{eR} \\ \psi_{eI} \end{bmatrix} + \begin{bmatrix} v_{sR} \\ v_{sI} \end{bmatrix}$$

$$\begin{bmatrix} \psi_{eR} \\ \psi_{eI} \end{bmatrix} = \begin{bmatrix} L_a & 0 \\ 0 & L_a \end{bmatrix} \begin{bmatrix} i_{eR} \\ i_{eI} \end{bmatrix}$$

$$\begin{bmatrix} v_{sR} \\ v_{sI} \end{bmatrix} = \begin{bmatrix} \cos\theta_{PLL} & \sin\theta_{PLL} \\ -\sin\theta_{PLL} & \cos\theta_{PLL} \end{bmatrix}^{-1} \begin{bmatrix} v_{sd} \\ v_{sq} \end{bmatrix}$$

Wind DFIG models

- PLL

$$\Delta\omega_{PLL} = \left(K_{P,PLL} + \frac{K_{I,PLL}}{s} \right)\left(v_{sq}^{*} - v_{sq} \right)$$

$$\theta_{PLL} = \frac{1}{s}\left(\Delta\omega_{PLL} + \omega_{0} \right)$$

Wind DFIG models

- Non linear model (20th order)

$$\dot{\mathbf{x}} = \mathbf{f}\left(\mathbf{x}, \mathbf{z}, \mathbf{u}\right)$$

$$\mathbf{0} = \mathbf{g}\left(\mathbf{x}, \mathbf{z}, \mathbf{u}\right)$$

$$\mathbf{x}^T = \Big[\psi_{sd}\ \psi_{sq}\ \psi_{rd}\ \psi_{rq}\ \psi_{ad}\ \psi_{aq}\ s\ v_C^2\ x_{a1}\ x_{a2}\ x_{a3}\ x_{r1}\ x_{r2}\ x_s$$
$$v_{Cfd}\ v_{Cfq}\ \psi_{tR}\ \psi_{tI}\ \omega_{PLL}\ \theta_{PLL}\Big]$$

$$\mathbf{u} = \Big[t_m\ i_{aq}^*\ v_C^*\ \left(i_{rd}^*\right)^{\psi_s}\ s^*\ v_{eR}\ v_{eI} v_{sq}^*\Big]$$

$$\mathbf{z}^T = \Big[i_{sd}\ i_{sq}\ i_{rd}\ i_{rq}\ i_{ad}\ i_{aq}\ v_{ad}\ v_{aq}\ i_{ad}^*\ i_{sm}\ \psi_s\ \phi$$
$$v_{sd}\ v_{sq}\ v_{sR}\ v_{sI}\ i_{fd}\ i_{fq}\ i_{ed}\ i_{eq}\ i_{eR}\ i_{eI}$$
$$\left(v_{rd}\right)^{\psi_s}\ \left(v_{rq}\right)^{\psi_s}\ v_{rd}\ v_{rd}\ \left(i_{rd}\right)^{\psi_s}\ \left(i_{rq}\right)^{\psi_s}\ t_e\ p_r\ p_a\ \left(i_{rq}^*\right)^{\psi_s}\Big]$$

Wind DFIG models

- Linear model

$$\begin{bmatrix} \Delta\dot{\mathbf{x}} \\ \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{bmatrix} \begin{bmatrix} \Delta\mathbf{x} \\ \Delta\mathbf{z} \end{bmatrix} + \begin{bmatrix} \mathbf{B}_1 \\ \mathbf{B}_2 \end{bmatrix} \Delta\mathbf{u}$$

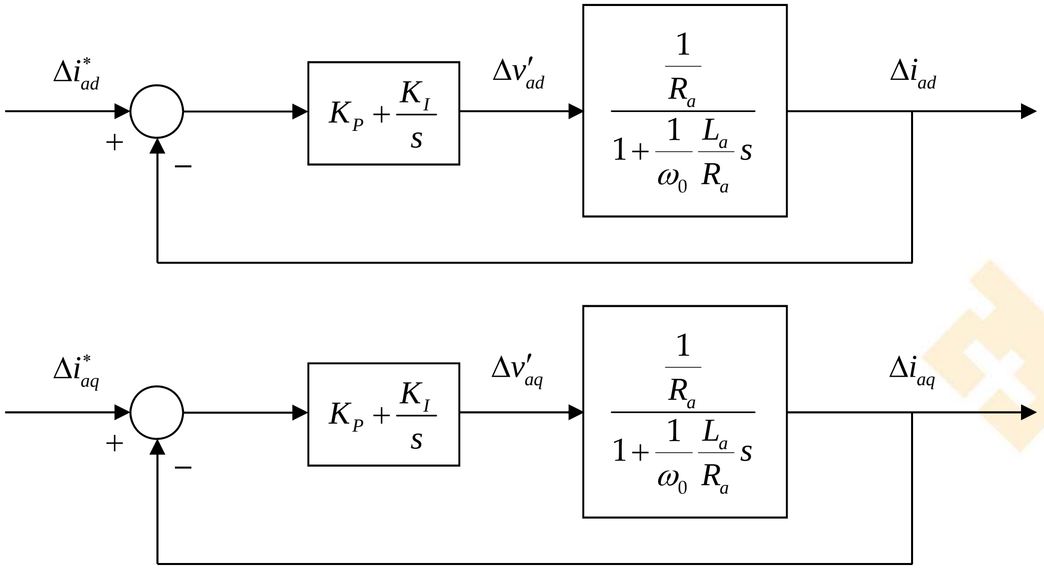
$$\Delta\mathbf{y} = \begin{bmatrix} \mathbf{C}_1 & \mathbf{C}_2 \end{bmatrix} \begin{bmatrix} \Delta\mathbf{x} \\ \Delta\mathbf{z} \end{bmatrix}$$

$$\Delta\dot{\mathbf{x}} = \mathbf{A}\Delta\mathbf{x} + \mathbf{B}\Delta\mathbf{u}$$

$$\Delta\mathbf{y} = \mathbf{C}\Delta\mathbf{x} + \mathbf{D}\Delta\mathbf{u}$$

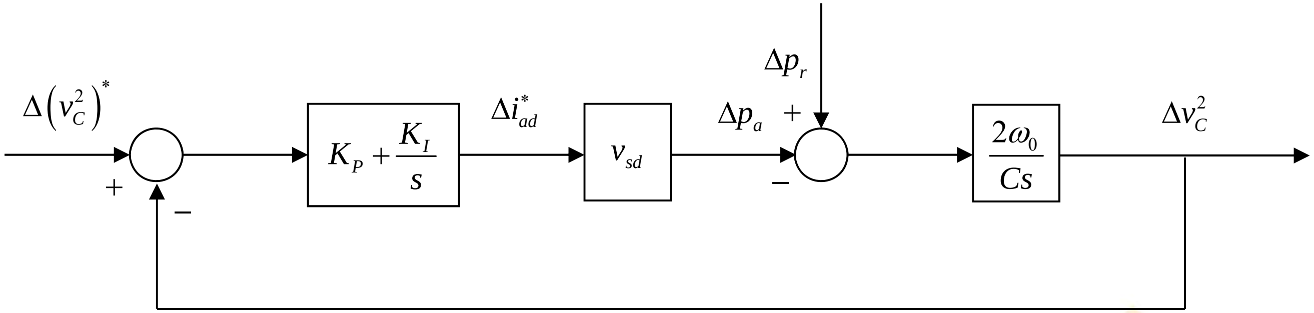
Wind DFIG controller design

- Grid side converter current controls



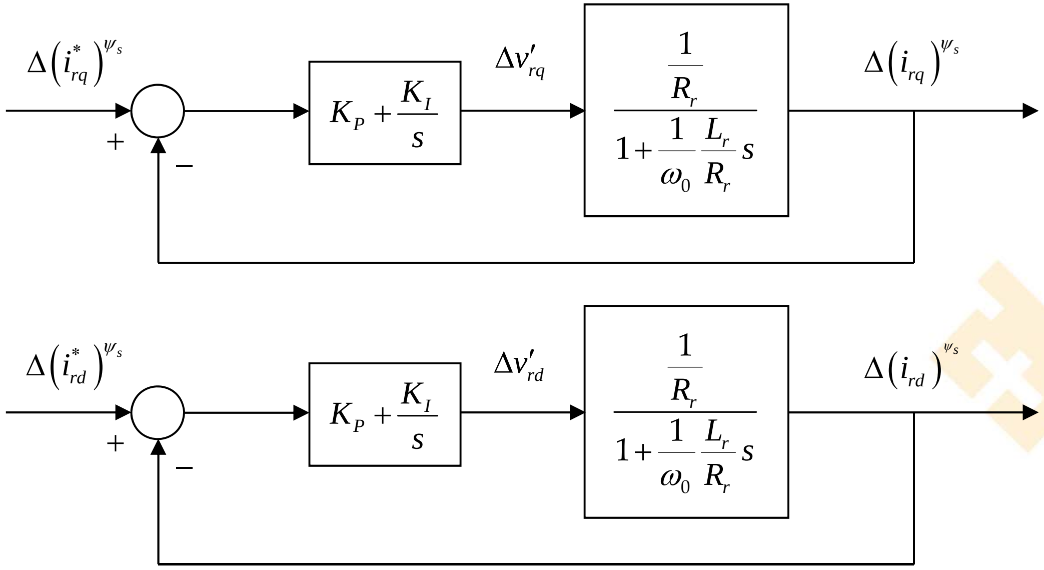
Wind DFIG controller design

- DC link capacitor voltage control



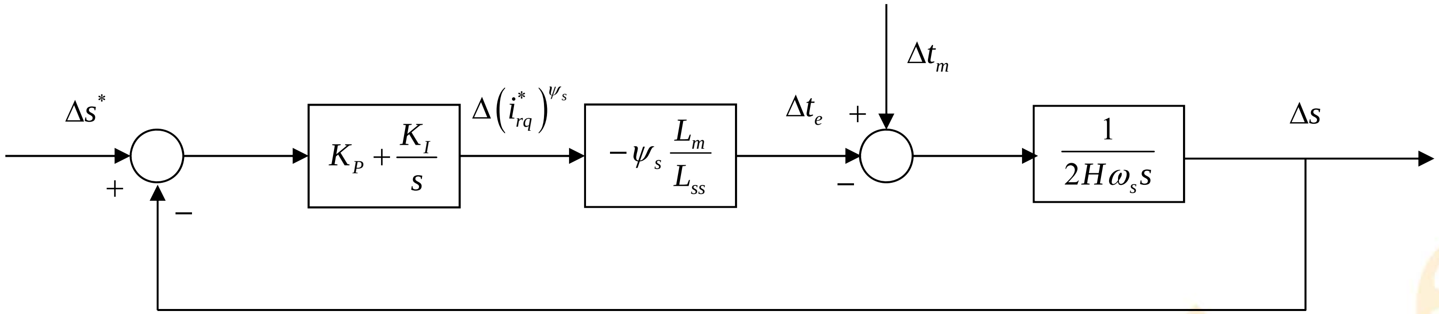
Wind DFIG controller design

- Rotor side converter current controls



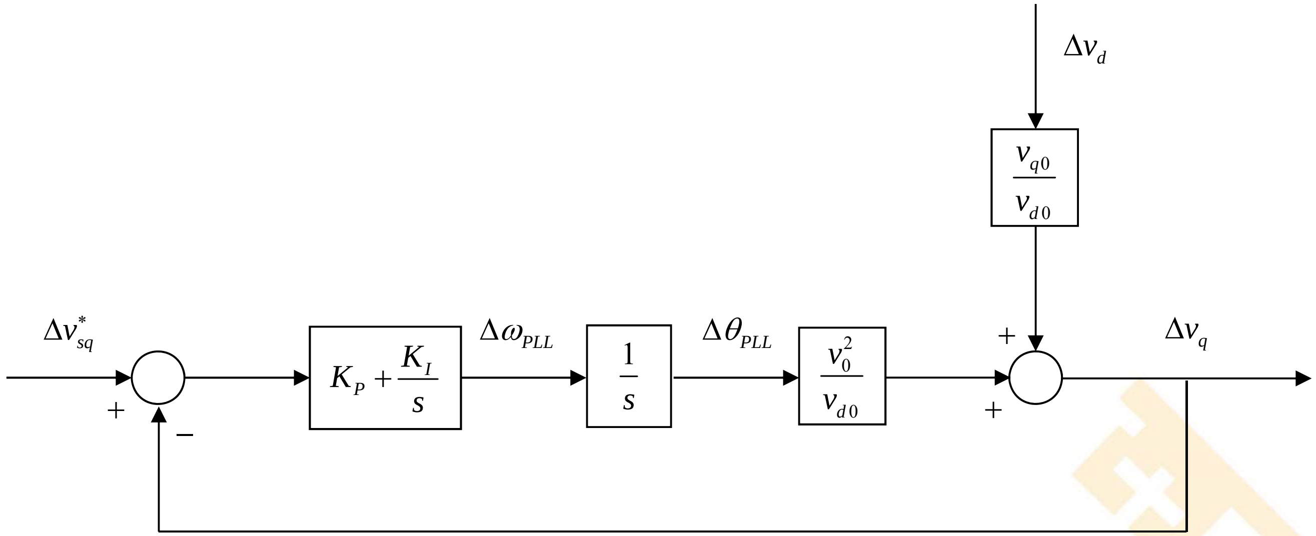
Wind DFIG controller design

- Rotor speed control



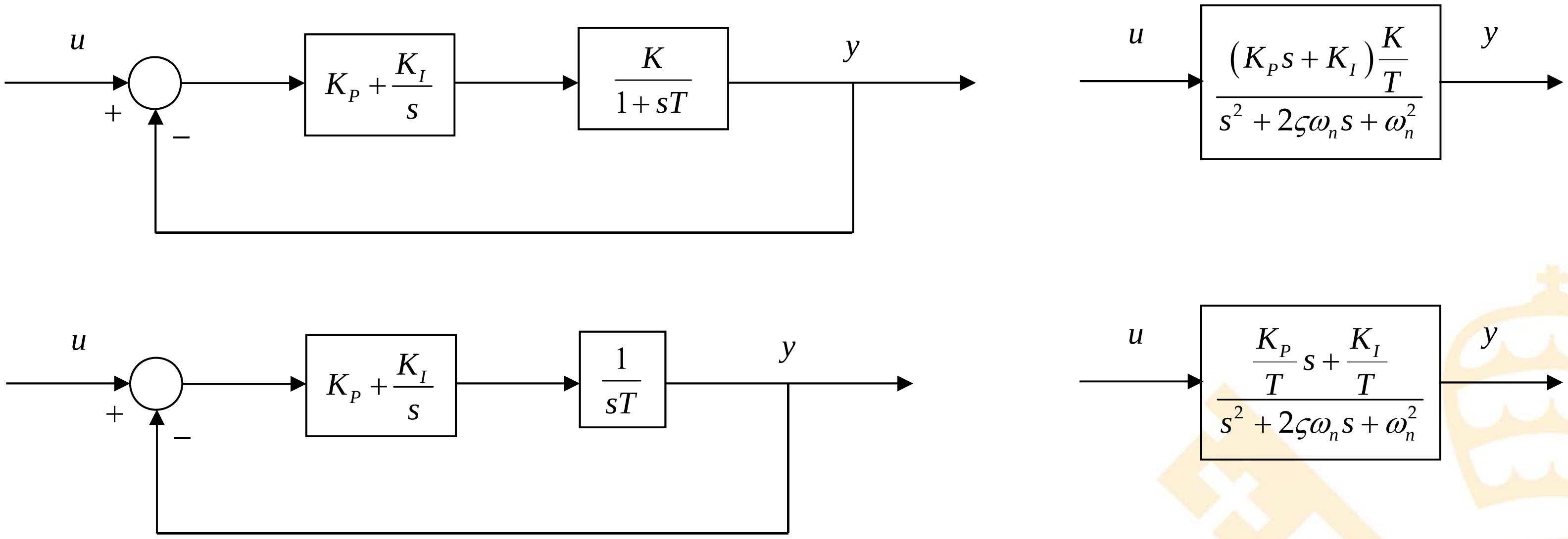
Wind DFIG controller design

- PLL control



Wind DFIG controller design

- PI controller design approach



Modal analysis

- Eigenvalues and right and left eigenvectors

$$\mathbf{A}\mathbf{v}_i = \mathbf{v}_i \lambda_i$$

$$\mathbf{w}_i^T \mathbf{A} = \lambda_i \mathbf{w}_i^T$$

$$\mathbf{w}_i^T \mathbf{v}_i = 1$$

- Participation factors

$$p_{ji} = v_{ji} w_{ij}$$

Modal analysis

- Test cases
  - Grid impedance
    - Low
    - High
  - Current controller bandwidth
    - 1250 rad/s
    - 125 rad/s

Modal analysis

- Low grid impedance
  - Bandwidth of current controllers: 1250 rad/s
  - Bandwidth of PLL controller: 650 rad/s
    - Eigenvalues

Number	Real	Imaginary	Damping (%)	Frequency (Hz)
1	-262.29	6796.40	3.86	1082.49
2	-126.74	5487.10	2.31	873.53
3	-637.08	1136.70	48.89	207.39
4	-240.45	1093.77	21.47	178.24
5	-875.00	892.68	70.00	198.94
6	-683.00	814.64	64.25	169.19
7	-525.44	416.23	78.39	106.69
8	-26.68	307.07	8.66	49.06
9	-93.23	90.96	71.58	20.73
10	-80.60	81.85	70.16	18.28

Current controls

PLL control

Outer controls

Modal analysis

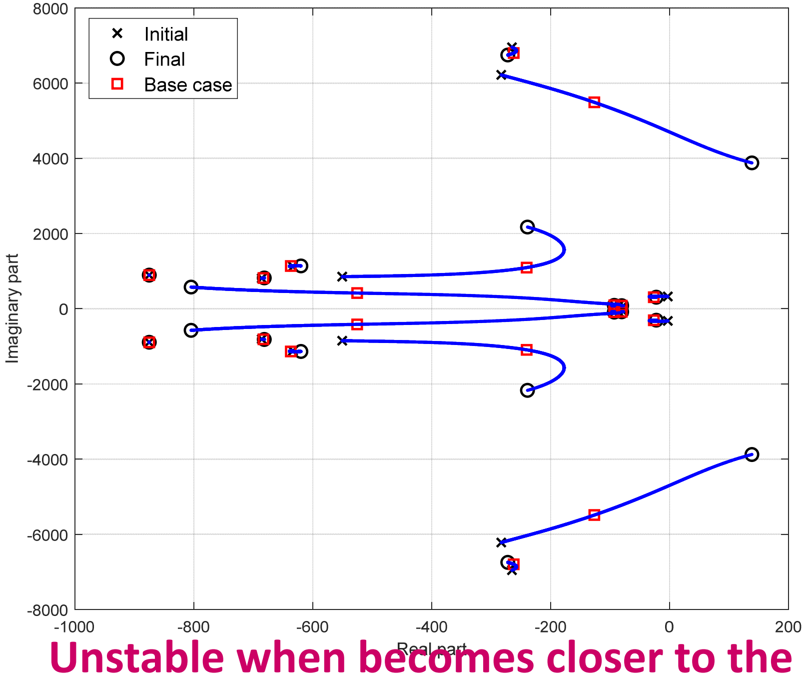
- Low grid impedance
  - Bandwidth of current controllers: 1250 rad/s
  - Bandwidth of PLL controller: 650 rad/s
    - Participation factors

	Mode									
Variable	1	2	3	4	5	6	7	8	9	10
Psi_sd	0.1583	0.0126	0.0749	0.0554	0	0.0810	0.0350	0.4482	0.0039	0.0082
Psi_sq	0.0206	0.2328	0.0145	0.2910	0	0.0050	0.0988	0.5067	0.0053	0.0019
Psi_rd	0.0380	0.0061	0.3809	0.0467	0	0.3482	0.0151	0.0455	0.0267	0.0265
Psi_rq	0.0057	0.0744	0.0329	0.4680	0	0.0136	0.1584	0.0197	0.0001	0.0002
Psi_ad	0.0022	0.0006	0.4559	0.0337	0	0.4280	0.0055	0.0036	0.0082	0.0033
Psi_aq	0	0	0	0	0.7001	0	0	0	0	0
s	0.0010	0.0003	0.0667	0.0205	0	0.0751	0.0053	0.0482	0.2746	0.3796
v_c2	0.0022	0.0006	0.1890	0.0379	0	0.1365	0.0098	0.0688	0.4754	0.2166
x_a1	0	0	0	0	0.7001	0	0	0	0	0
x_a2	0.0003	0.0001	0.3544	0.0242	0	0.4571	0.0085	0.0042	0.0054	0.0018
x_a3	0	0	0.0134	0.0031	0	0.0121	0.0015	0.0196	0.4662	0.2350
x_r1	0.0007	0.0120	0.0254	0.3279	0	0.0143	0.2549	0.0192	0.0001	0.0003
x_r2	0.0050	0.0010	0.3653	0.0514	0	0.4607	0.0290	0.0179	0.0034	0.0037
x_s	0	0	0.0047	0.0017	0	0.0066	0.0008	0.0137	0.2691	0.4120
v_cd	0.4579	0.0509	0.0042	0.0028	0	0.0027	0.0003	0.0003	0	0
v_cq	0.0475	0.4739	0.0011	0.0258	0	0.0002	0.0081	0	0	0
Psi_tr	0.2980	0.0436	0.0941	0.0155	0	0.0801	0.0040	0.0199	0.0144	0.0455
Psi_ti	0.0452	0.4422	0.0115	0.5045	0	0.0045	0.1261	0.0176	0.0002	0
x_w_pll	0.0009	0.0170	0.0077	0.2246	0	0.0020	0.6015	0.0194	0.0022	0.0006
x_th_pll	0.0137	0.2098	0.0200	0.5600	0	0.0037	0.5709	0.0226	0.0018	0.0005

Modal analysis

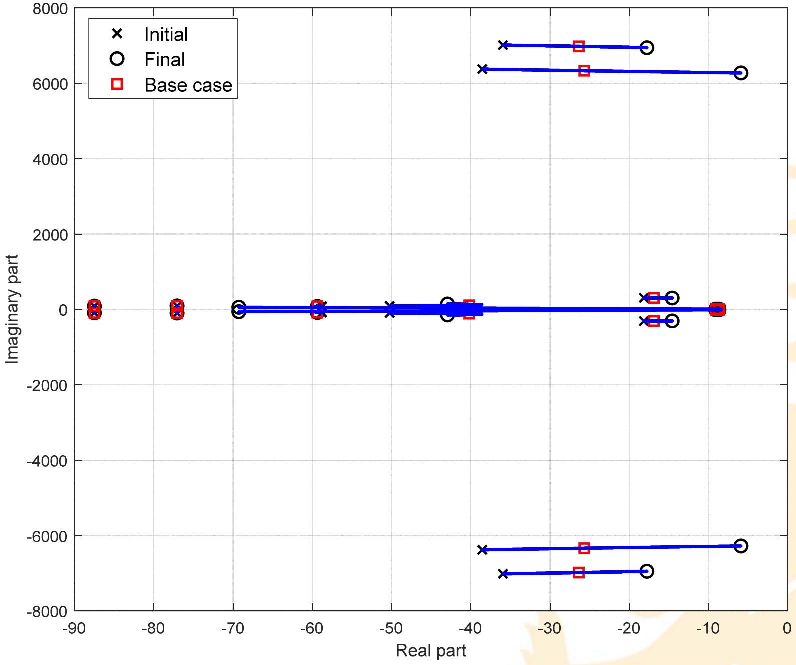
- Low grid impedance
  - Root locus changing the bandwidth of the PLL control from 1/10 to 1 of the bandwidth of the current controllers

Bandwidth current controllers=1250 rad/s



Unstable when becomes closer to the bandwidth of the current controllers

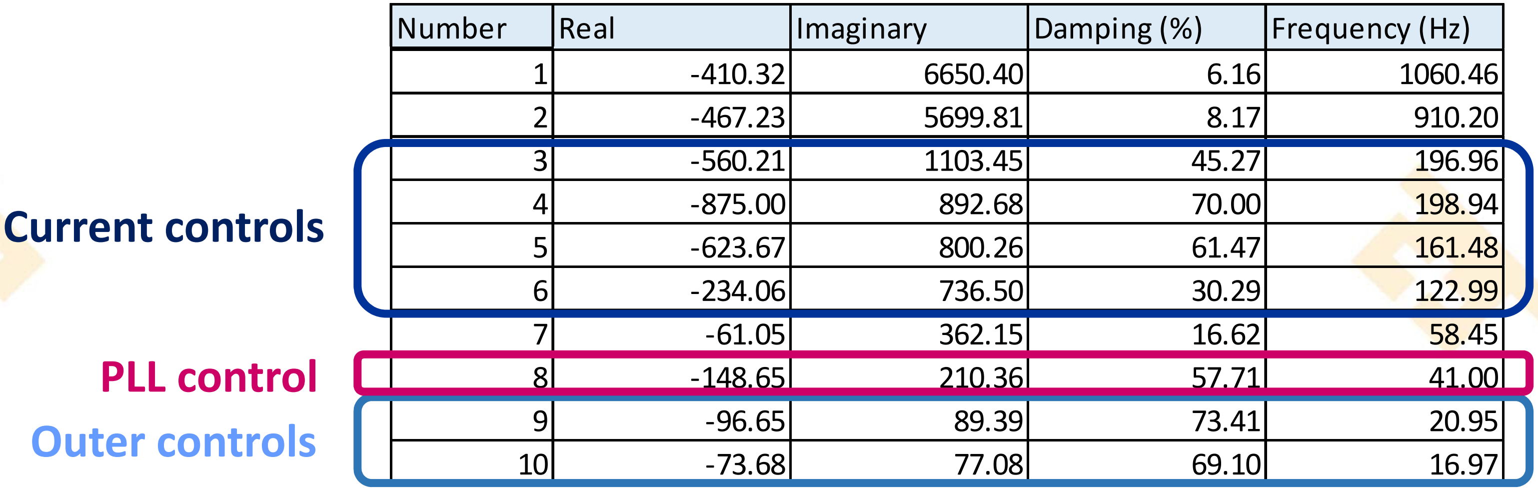
Bandwidth current controllers=125 rad/s



Stable for all cases

Modal analysis

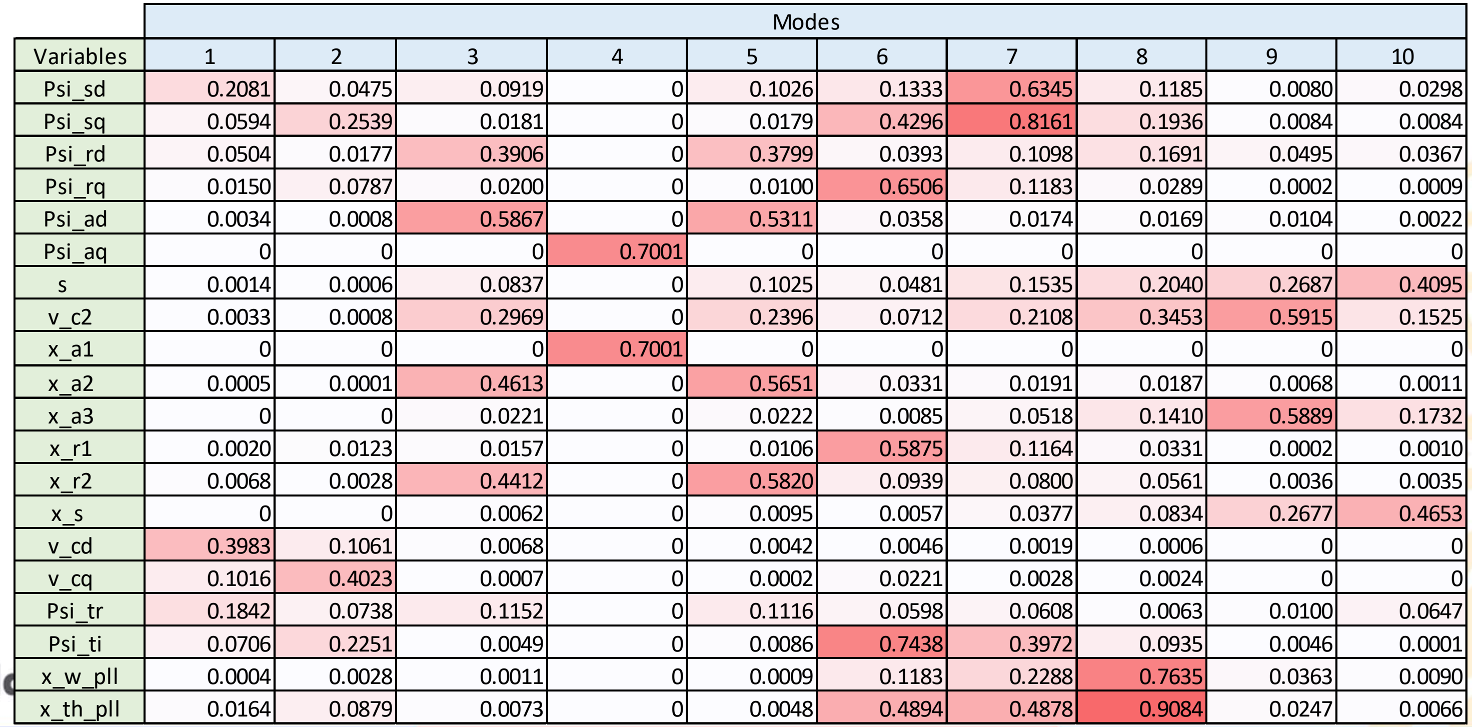
- High grid impedance
  - Bandwidth of current controllers: 1250 rad/s
  - Bandwidth of PLL controller: 250 rad/s
    - Eigenvalues



Number	Real	Imaginary	Damping (%)	Frequency (Hz)
1	-410.32	6650.40	6.16	1060.46
2	-467.23	5699.81	8.17	910.20
3	-560.21	1103.45	45.27	196.96
4	-875.00	892.68	70.00	198.94
5	-623.67	800.26	61.47	161.48
6	-234.06	736.50	30.29	122.99
7	-61.05	362.15	16.62	58.45
8	-148.65	210.36	57.71	41.00
9	-96.65	89.39	73.41	20.95
10	-73.68	77.08	69.10	16.97

Modal analysis

- High grid impedance
  - Bandwidth of current controllers: 1250 rad/s
  - Bandwidth of PLL controller: 250 rad/s
    - Participation factors

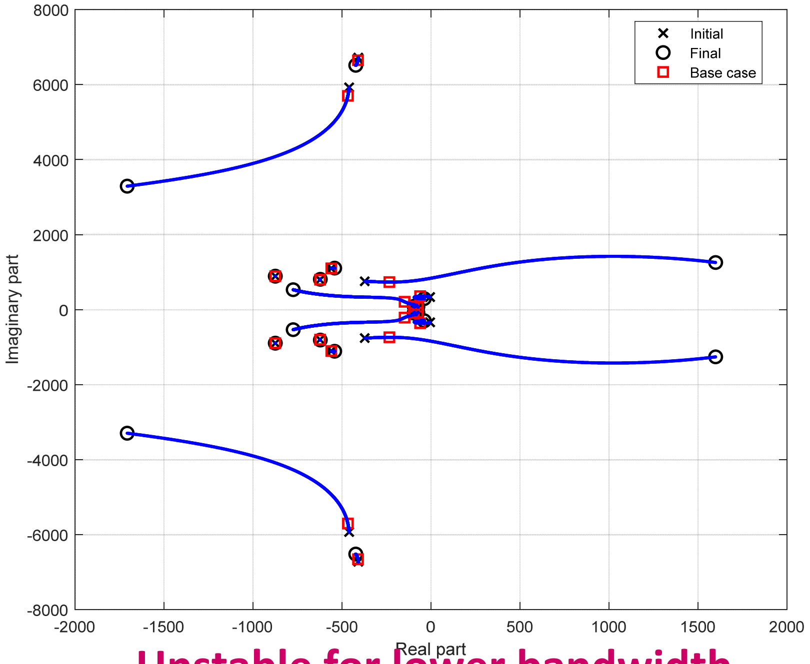


	Modes									
Variables	1	2	3	4	5	6	7	8	9	10
Psi_sd	0.2081	0.0475	0.0919	0	0.1026	0.1333	0.6345	0.1185	0.0080	0.0298
Psi_sq	0.0594	0.2539	0.0181	0	0.0179	0.4296	0.8161	0.1936	0.0084	0.0084
Psi_rd	0.0504	0.0177	0.3906	0	0.3799	0.0393	0.1098	0.1691	0.0495	0.0367
Psi_rq	0.0150	0.0787	0.0200	0	0.0100	0.6506	0.1183	0.0289	0.0002	0.0009
Psi_ad	0.0034	0.0008	0.5867	0	0.5311	0.0358	0.0174	0.0169	0.0104	0.0022
Psi_aq	0	0	0	0.7001	0	0	0	0	0	0
s	0.0014	0.0006	0.0837	0	0.1025	0.0481	0.1535	0.2040	0.2687	0.4095
v_c2	0.0033	0.0008	0.2969	0	0.2396	0.0712	0.2108	0.3453	0.5915	0.1525
x_a1	0	0	0	0.7001	0	0	0	0	0	0
x_a2	0.0005	0.0001	0.4613	0	0.5651	0.0331	0.0191	0.0187	0.0068	0.0011
x_a3	0	0	0.0221	0	0.0222	0.0085	0.0518	0.1410	0.5889	0.1732
x_r1	0.0020	0.0123	0.0157	0	0.0106	0.5875	0.1164	0.0331	0.0002	0.0010
x_r2	0.0068	0.0028	0.4412	0	0.5820	0.0939	0.0800	0.0561	0.0036	0.0035
x_s	0	0	0.0062	0	0.0095	0.0057	0.0377	0.0834	0.2677	0.4653
v_cd	0.3983	0.1061	0.0068	0	0.0042	0.0046	0.0019	0.0006	0	0
v_cq	0.1016	0.4023	0.0007	0	0.0002	0.0221	0.0028	0.0024	0	0
Psi_tr	0.1842	0.0738	0.1152	0	0.1116	0.0598	0.0608	0.0063	0.0100	0.0647
Psi_ti	0.0706	0.2251	0.0049	0	0.0086	0.7438	0.3972	0.0935	0.0046	0.0001
x_w_pll	0.0004	0.0028	0.0011	0	0.0009	0.1183	0.2288	0.7635	0.0363	0.0090
x_th_pll	0.0164	0.0879	0.0073	0	0.0048	0.4894	0.4878	0.9084	0.0247	0.0066

Modal analysis

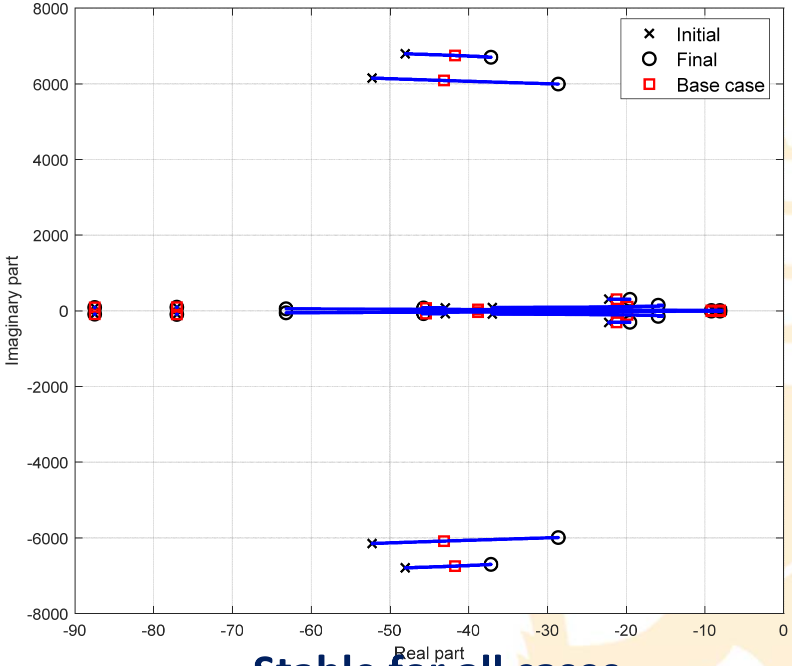
- High grid impedance
  - Root locus changing the bandwidth of the PLL control from 1/10 to 1 of the bandwidth of the current controllers

Bandwidth current controllers=1250 rad/s



Unstable for lower bandwidth

Bandwidth current controllers=125 rad/s



Stable for all cases

Conclusions

- If the DFIG is connected to a strong grid (low grid reactance) and the bandwidth of the current controllers is high (i.e. 1250 rad/s),
  - The system is stable if the bandwidth of PLL controller is smaller that the bandwidth of the current controllers (i.e. 50%).
- If the bandwidth of the current controllers is low (i.e. 125 rad/s), the bandwidth of the PLL controller can be even equal to the bandwidth of the current controllers no matter the strength of the grid.
- The mode that becomes unstable depends on the bandwidth of the controllers.

Acknowledgements

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    - Régulo Ávila-Martínez, Luis Rouco, Javier García-Aguilar, Javier Renedo, Lukas Sigrist, “Impact of PLL Control on Small-Signal Stability of Wind DFIGs”.