Dynamic Workshop

Small-Signal Stability Analysis of Delay System

Muyang Liu

muyang.liu@ucdconnect.ie

Outline

- Delay System Modelling
- Small-Signal Stability
- Eigenvalue-based Approach
- Analytical Approach
- General Cases

Definitions in Delay System

- Delay-independent System

  The system remains stable for any positive value of delay.

- Delay-dependent System

  The system is stable without delay, but it will become unstable due to some specific delays.

- Delay Margin

  Delay margin is a constant value. In a delay-dependent system, if the magnitude of the delay is larger than the delay margin, the system collapse.

Small-Signal Stability

- Index 1 Hessenberg Form System

$$\dot{x} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{y}_d, \boldsymbol{u})$$
$$\boldsymbol{0} = \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{u})$$
$$\boldsymbol{x}_d = \boldsymbol{x}(t - \tau)$$
$$\boldsymbol{y}_d = \boldsymbol{y}(t - \tau)$$

- Stationary Point:

$$\boldsymbol{0} = \boldsymbol{f}(\boldsymbol{x}_0, \boldsymbol{y}_0, \boldsymbol{x}_0, \boldsymbol{y}_0, \boldsymbol{u}_0)$$
$$\boldsymbol{0} = \boldsymbol{g}(\boldsymbol{x}_0, \boldsymbol{y}_0, \boldsymbol{u}_0)$$

Small-Signal Stability

- Linearzing the system at the stationary solution yields:

$$\Delta \dot{\boldsymbol{x}} = \boldsymbol{f}_{\boldsymbol{x}} \Delta \boldsymbol{x} + \boldsymbol{f}_{\boldsymbol{y}} \Delta \boldsymbol{y} + \boldsymbol{f}_{\boldsymbol{y}_d} \Delta \boldsymbol{y}_d$$

$$\boldsymbol{0} = \boldsymbol{g}_{\boldsymbol{x}} \Delta \boldsymbol{x} + \boldsymbol{g}_{\boldsymbol{y}} \Delta \boldsymbol{y}$$

Eliminating Δy:

$$\Delta \dot{\boldsymbol{x}} = \boldsymbol{A}_0 \Delta \boldsymbol{x} + \boldsymbol{A}_1 \Delta \boldsymbol{x}(t-\tau)$$

$$\boldsymbol{A}_0 = \boldsymbol{f}_{\boldsymbol{x}} - \boldsymbol{f}_{\boldsymbol{y}} \boldsymbol{g}_{\boldsymbol{y}}^{-1} \boldsymbol{g}_{\boldsymbol{x}}$$

$$\boldsymbol{A}_1 = -\boldsymbol{f}_{\boldsymbol{y}_d} \boldsymbol{g}_{\boldsymbol{y}}^{-1} \boldsymbol{g}_{\boldsymbol{x}}$$

- Characteristic equation:

$$\Delta(\lambda) = \lambda \boldsymbol{I} - \boldsymbol{A}_0 - \boldsymbol{A}_1 e^{-\lambda \tau}$$

Transcendental!

Eigenvalue-based Approach

Characteristic Equation:

$$\Delta(\lambda) = \lambda \boldsymbol{I} - \boldsymbol{A_0} - \boldsymbol{A_1} e^{-\lambda\tau}$$

Discretization:

F. Milano, M. Anghel, "Impact of Time Delays on Power System Stability", IEEE Transactions on Circuits and Systems-I: Regular Papers, Vol. 59, No. 4, pp. 889-900, April 2012.

Transform the differential algebraic equations in a continuous boundard value problem with partial derivatives and using Chebyshev's discretization scheme to calculate the eigenvalues.

Eigenvalue-based Method

- **Two useful properties:**

  - The characteristic equation only has a finite number of characteristic roots in any vertical strip of the complex plane:

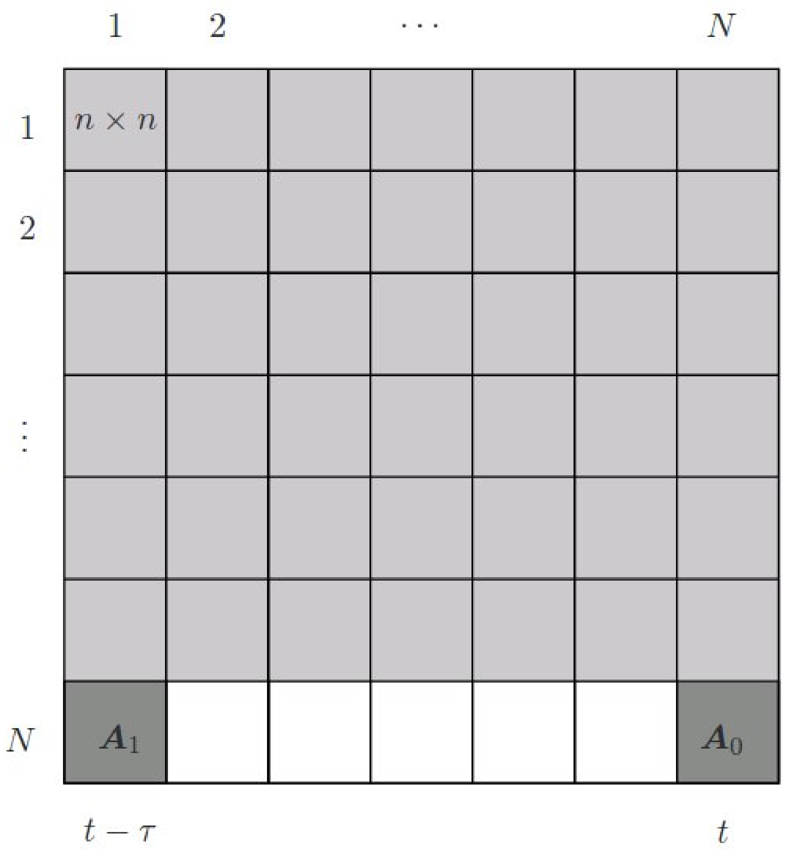
$$\lambda \in \boldsymbol{C} : \alpha < \Re(\lambda) < \beta$$

  - There exists a number such that all characteristic roots are confined to the half-plane:

$$\gamma \in \boldsymbol{R}, \lambda \in \boldsymbol{C} : \Re(\lambda) < \gamma$$

Eigenvalue-based Method

$$\Delta(\lambda) = \lambda \boldsymbol{I} - \boldsymbol{A}_0 - \boldsymbol{A}_1 e^{-\lambda\tau}$$



- **Do we really have to calculate the approximate eigenvalues of the delay system to find the delay margin?**

Analytical Approach

- Example

$$\dot{\boldsymbol{x}}(t)=\begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}\boldsymbol{x}(t)+\begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}\boldsymbol{x}(t-\tau)$$

At the delay margin the system is supposed to have two eigenvalues on the imaginary aix, let's assuming:

$$\lambda_1=j\omega,\lambda_2=-j\omega$$

Then we have:

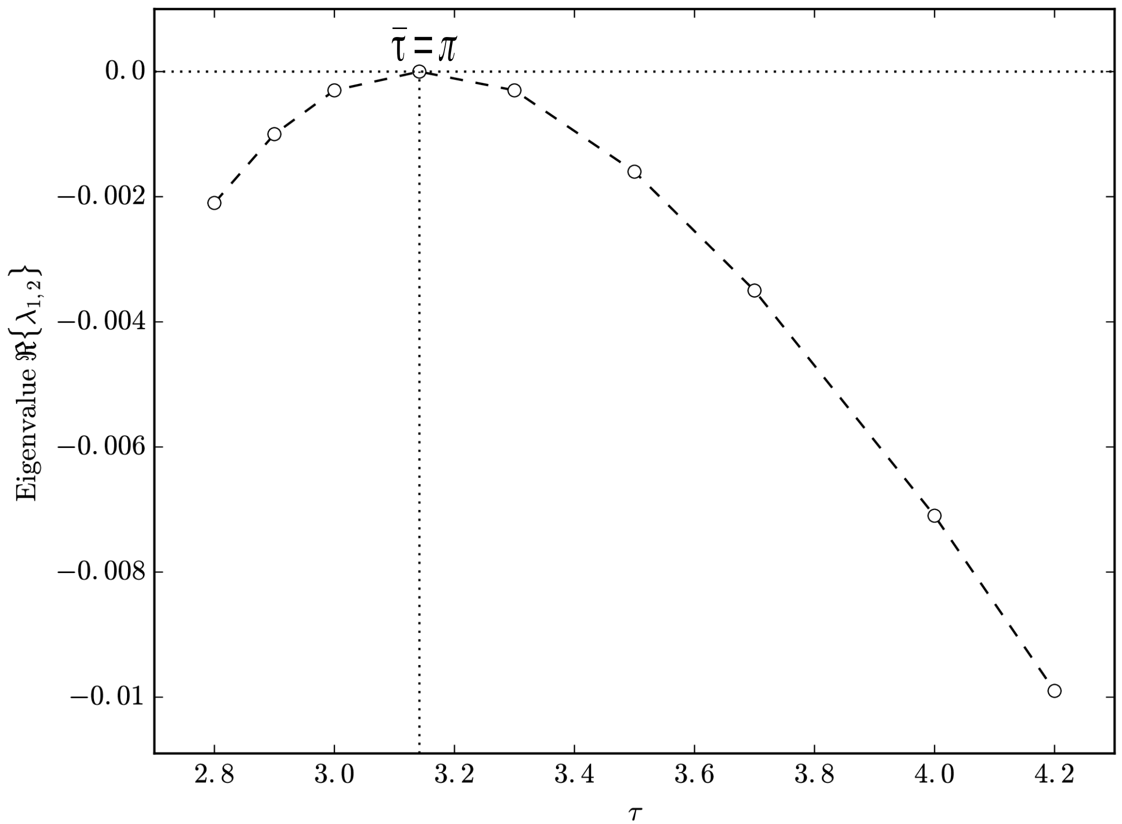
$$\Delta(\lambda_1)=\lambda_1\boldsymbol{I}-\boldsymbol{A_0}-\boldsymbol{A_1}e^{-\lambda_1\tau}=0$$
$$\Delta(\lambda_2)=\lambda_2\boldsymbol{I}-\boldsymbol{A_0}-\boldsymbol{A_1}e^{-\lambda_2\tau}=0$$

According to the above equations, we have:

$$\lambda=\pm j,\quad \bar{\tau}=\pi$$

Eigenvalue-based Approach

- Real part of the dominate eigenvalue as a function of delay:



Analytical Approach

- Decision function of the crossing direction:

$$\boldsymbol{sign}\left\{\Re\left[\frac{d\lambda}{d\tau}\right]_{\lambda=j\omega}\right\}$$

If positive (+), crossing towards instability

If negative (-), crossing towards stability

Advantages of Eigenvalue-based Approach

- Comparing with analytical method:

  Easy and less trick!

- Comparing with time-domain method:

  Lyapunove second method, linear matrix inequality method and etc.

  Easy and fast.

  Less conservative!

Delay System Modelling

- Retarded Delay Differential Equation (RDDE):

$$\dot{x} = f(x, x_d, y, y_d, u)$$
$$0 = g(x, x_d, y, y_d, u)$$

- Neutral Delay Differential Equation (NDDE):

$$\dot{x} = f(x, x_d, y, y_d, \dot{x}_d, u)$$
$$0 = g(x, x_d, y, y_d, u)$$

Characteristic Equations

- Index-1 Hessenerg Form System

$$\Delta(\lambda) = \lambda \boldsymbol{I} - \boldsymbol{A_0} - \boldsymbol{A_1} e^{-\lambda \tau}$$

- Non Index-1 Hessenerg Form System

$$\Delta(\lambda) = \lambda \boldsymbol{I} - \boldsymbol{A_0} - \Sigma_k \boldsymbol{A_k} e^{-\lambda k \tau}$$

- Neutral Delay System

$$\Delta(\lambda) = \lambda I - \boldsymbol{A_0} - \Sigma_k \boldsymbol{A_k} e^{-\lambda k \tau} - \boldsymbol{B} e^{-\lambda \tau}$$

Implements in Dome

- Retarded Delay System

$$\dot{x} = f(x, y, x_d, y_d, u)$$
$$0 = g(x, y, x_d, y_d, u)$$

- Netural Delay System

$$\dot{x} = f(x, y, x_d, \dot{x}_d y_d, u)$$
$$0 = g(x, y, x_d, y_d, u)$$

$$\dot{x} = y$$
$$0 = f(x, y, x_d, y_d, u) - y$$

Delay-Independent Netrual System

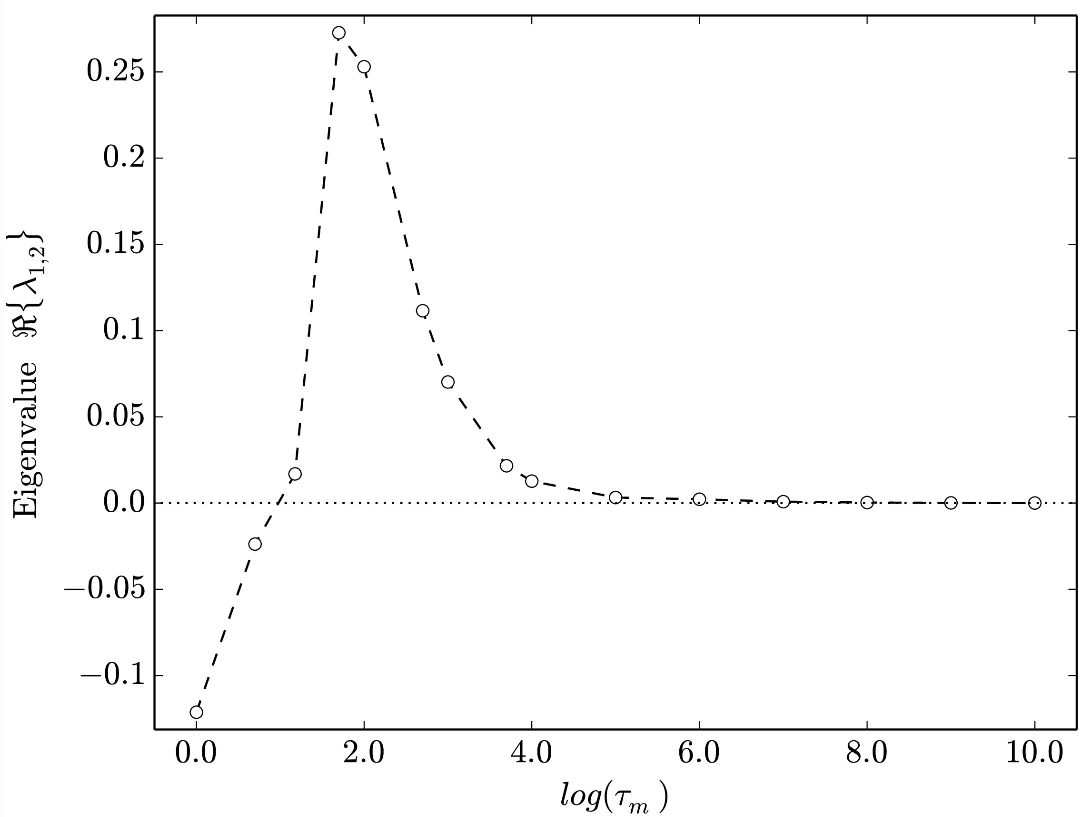
- Food-limited Dynamic Population Model

$$\dot{S}(t) = r\,S(t)\left[1 - \frac{S(t-\tau) + c\,\dot{S}(t-\tau)}{K}\right]$$

$$r = \frac{\pi}{\sqrt{3}} + \frac{1}{20}$$

$$c = \frac{\sqrt{3}}{2\pi} - \frac{1}{25}$$

$$K = 1$$



Delay-Independent Netrual System

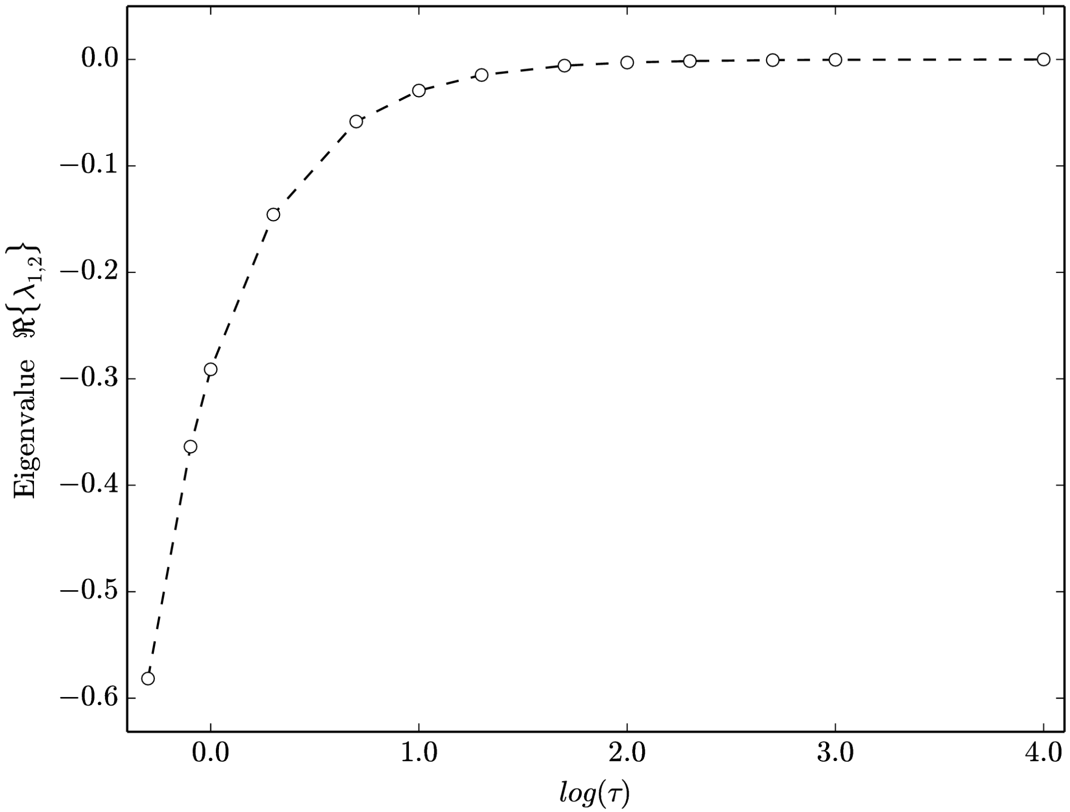
- Linear Partial Element Equivalent Circuit (PEEC)

$$\dot{\boldsymbol{x}} = \boldsymbol{L}\,\boldsymbol{x}(t) + \boldsymbol{M}\,\boldsymbol{x}(t-\tau) + \boldsymbol{N}\,\dot{\boldsymbol{x}}(t-\tau)$$

$$\frac{\boldsymbol{L}}{100} = \begin{bmatrix} -7 & 1 & 2 \\ 3 & -9 & 0 \\ 1 & 2 & -6 \end{bmatrix}$$

$$\frac{\boldsymbol{M}}{100} = \begin{bmatrix} 1 & 0 & -3 \\ -0.5 & -0.5 & -1 \\ -0.5 & -1.5 & 0 \end{bmatrix}$$

$$72\,\boldsymbol{N} = \begin{bmatrix} -7 & 1 & 2 \\ 3 & -9 & 0 \\ 1 & 2 & -6 \end{bmatrix}$$



Delay Modelling

- Constant Delay
- Distributed Delay

$$\tau = \int_b^a w(\theta)\, x(t-\theta)\, d\theta$$

- Periodic Time-varying Delay

$$\tau(t) = \tau_0 + \delta f(\Omega t)$$

- Stochastic Delay

References

[1]F. Milano, M. Anghel, Impact of Time Delays on Power System Stability, IEEE Transactions on Circuits and Systems-I: Regular Papers, Vol. 59, No. 4, pp. 889-900, April 2012.

[2]F. Milano, I. Dassios, Small-Signal Stability Analysis for Non-Index 1 Hessenberg Form Systems of Delay Differential-Algebraic Equations, IEEE Transactions on Circuits and Systems-I: Regular Papers, Vol. 63, No. 9, pp. 1521-1530, July 2016.

[3]M. Liu, I. Dassios and F. Milano, Small-Signal Stability Analysis of Neutral Delay Differential Equations

[4] K. Gu, V.L. Kharitonov and J. Chen, Stability of Time-Delay Systems, Springer, 2003.

Thanks for your attention!

Any Questions?