The Rince Institute

Researching Innovative Engineering Technologies

Dublin City University, Ireland

Research Institute for Networks and Communications Engineering

Science Foundation Ireland

For what's next

Nonlinear analysis of dc-dc converters

Presented by
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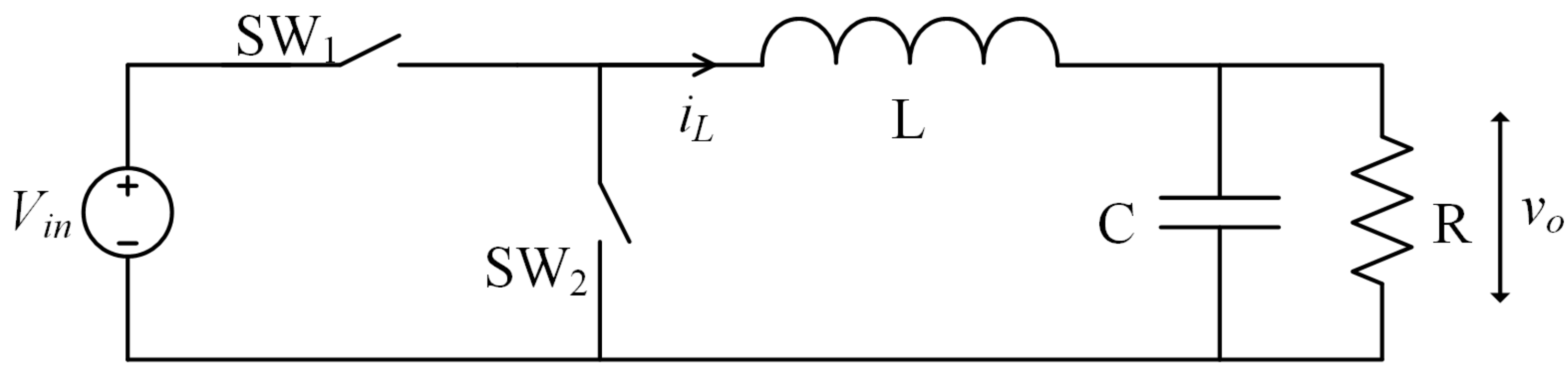
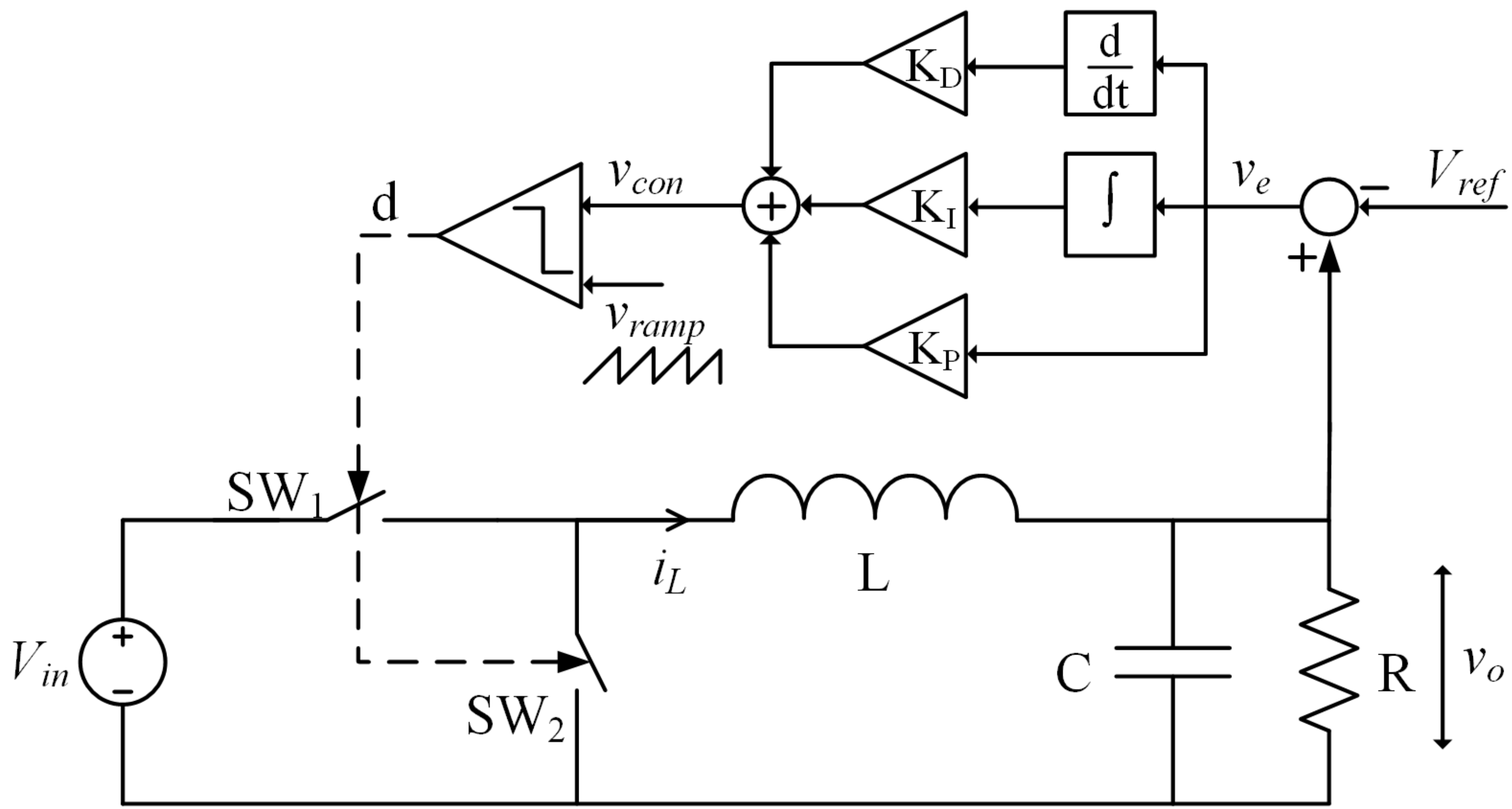
- Overview of discontinuous right hand side ODE
- How to perform stability analysis
- Analog PID buck converter
- Conclusions

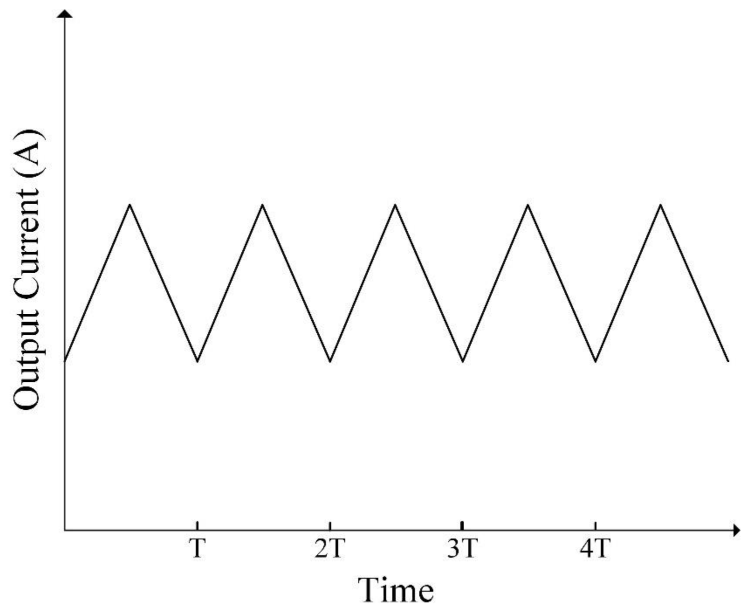
The buck converter



The buck converter

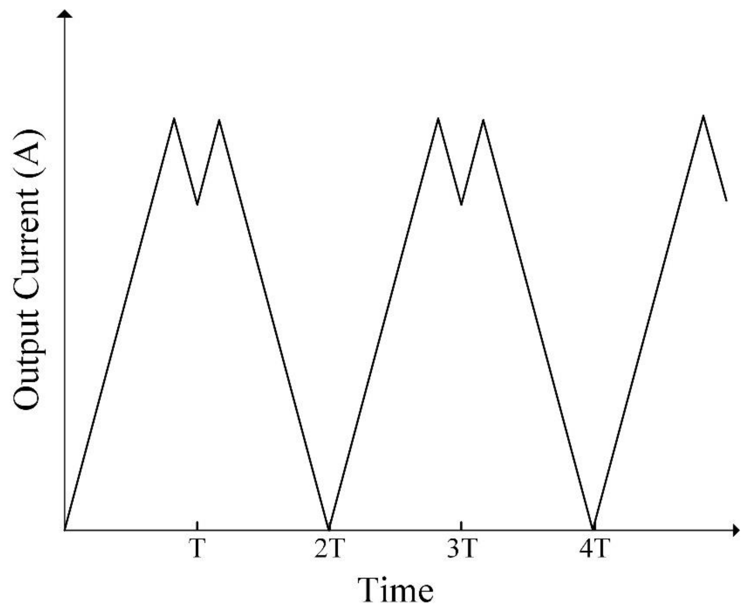


Stable Operation



- Output repeats with a frequency $f = 1/T$
- Termed a period-1 orbit
- System is stable

Unstable Operation



- Output repeats with a frequency $f = 1/2T$
- Termed a period-2 orbit
- System is **un**stable

System model

$$\frac{d}{dt}\begin{bmatrix} v_o \\ u \\ v_i \end{bmatrix} = \begin{bmatrix} -\sigma & \omega & 0 \\ -\omega & -\sigma & 0 \\ K_I & 0 & 0 \end{bmatrix}\begin{bmatrix} v_o \\ u \\ v_I \end{bmatrix} + \delta\begin{bmatrix} 0 \\ \frac{\sigma^2+\omega^2}{\omega} V_{in} \\ V_{ref} \end{bmatrix}$$

- Integral term must be included in state-space
- SW_1 closed for dT
- SW_1 open for $(1-d)T$

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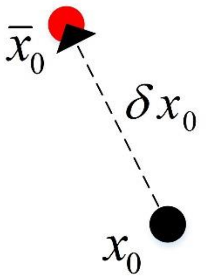
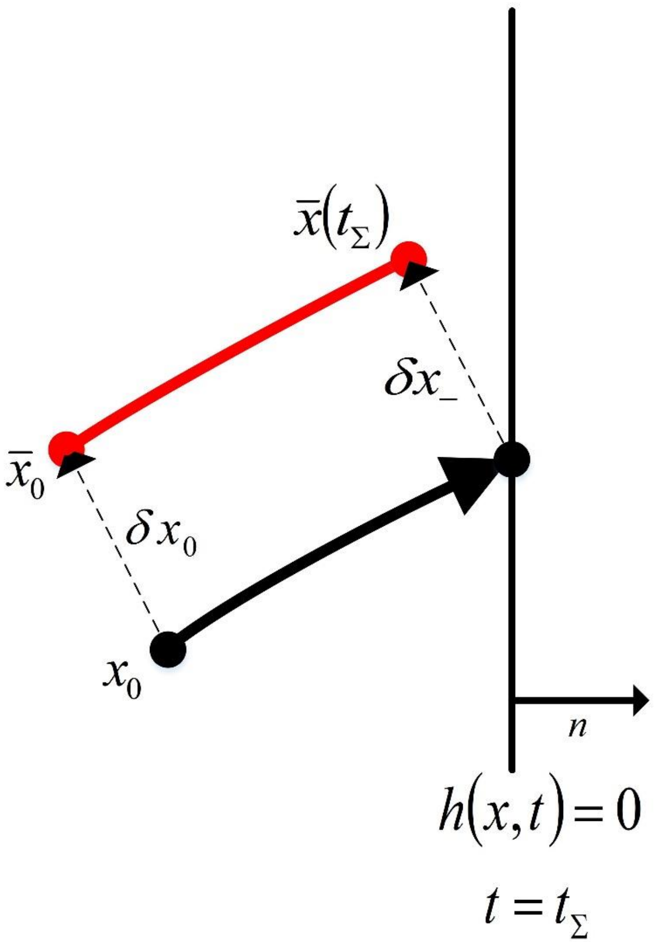
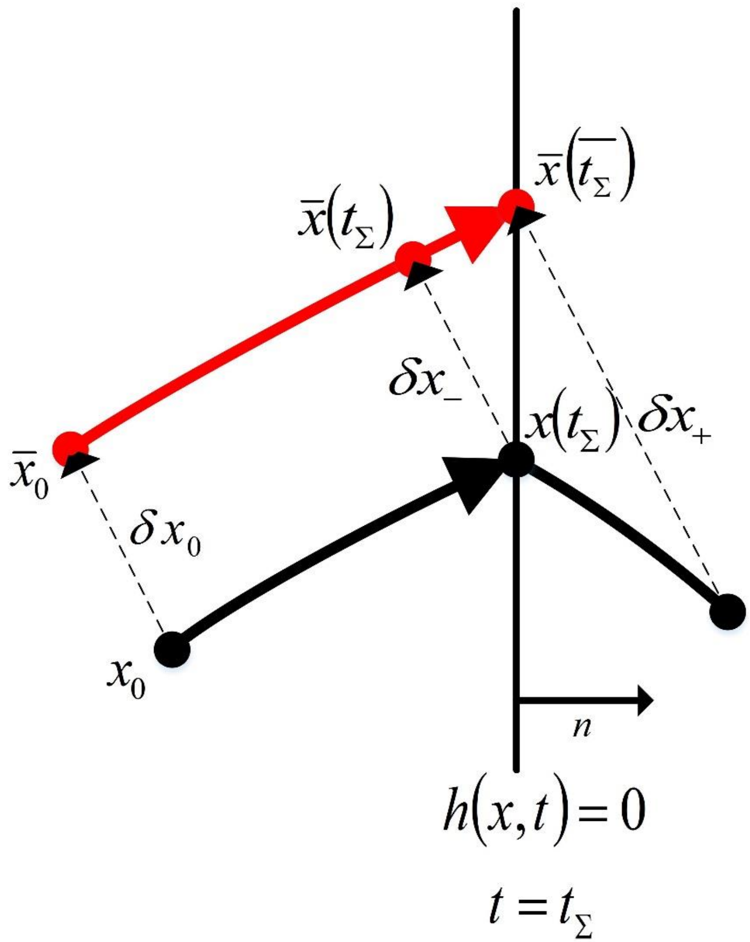

$\bar{x}_0$
δx_0
x_0


$\bar{x}(t_\Sigma)$
δx_-
$\bar{x}_0$
δx_0
x_0
n
$h(x,t)=0$
$t=t_\Sigma$


$\bar{x}(\overline{t_\Sigma})$
$\bar{x}(t_\Sigma)$
δx_-
$x(t_\Sigma)$
δx_+
$\bar{x}_0$
δx_0
x_0
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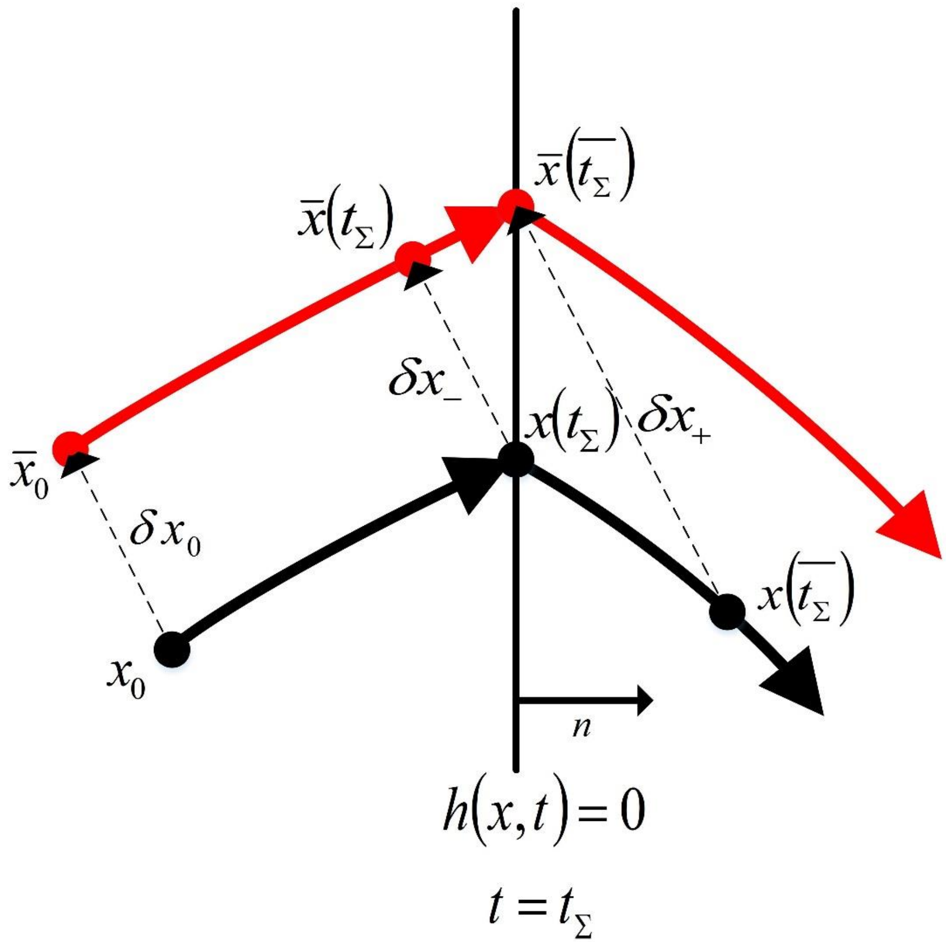
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Conventional stability approach

- Find state transition matrix, $\Phi(t, t_0)$:

$$\Phi(T, 0) \neq e^{A(t-t_0)}$$

- Previous methods miss fast-scale instabilities
  - Average the output over one period
  - e.g. Bode plots, root locus
- Must link equations before and after switching

Saltation Matrix

- Relates perturbation before and after switching

$$S = \frac{\delta x_+}{\delta x_-} = \frac{(f_+ - f_-)n^T}{n^T f_- + \frac{\partial h}{\partial t}}\Bigg|_{t=t_\Sigma}$$

f_+	– RHS of ODE after switching
f_-	– RHS of ODE before switching
h	– Switching manifold
n	– normal to switching manifold
t_Σ	– time when switching occurs

Monodromy matrix

$$\Phi_M(T,0) = S_2\Phi_{\mathrm{off}}(T,dT)S_1\Phi_{on}(dT,0)$$

$$\text{where } \Phi_{off} = e^{A(1-d)T} \text{ and } \Phi_{on} = e^{AdT}$$

- d is the "duty cycle"
  - Proportion of switching period in "ON" phase
- Eigenvalues of $\Phi_M(T,0)$ indicate stability
- $\Phi_{\mathrm{off}}(T,0)$ termed the Monodromy matrix

Research Motivation

- Transition from stable operation to unstable operation is termed a bifurcation
  - Saddle-node bifurcation $\lambda = +1$
  - Period-doubling bifurcation $\lambda = -1$
  - Hopf bifurcation $|\lambda| = 1$
- Unstable operation can damage the converter or interconnecting devices

Research Aims

- Stability analysis of converters nominal periodic motion is crucial
- Tools must be developed:
  - Predict when these bifurcations will occur
  - Determine underlying mechanism for their creation
  - Determine how they can be avoided

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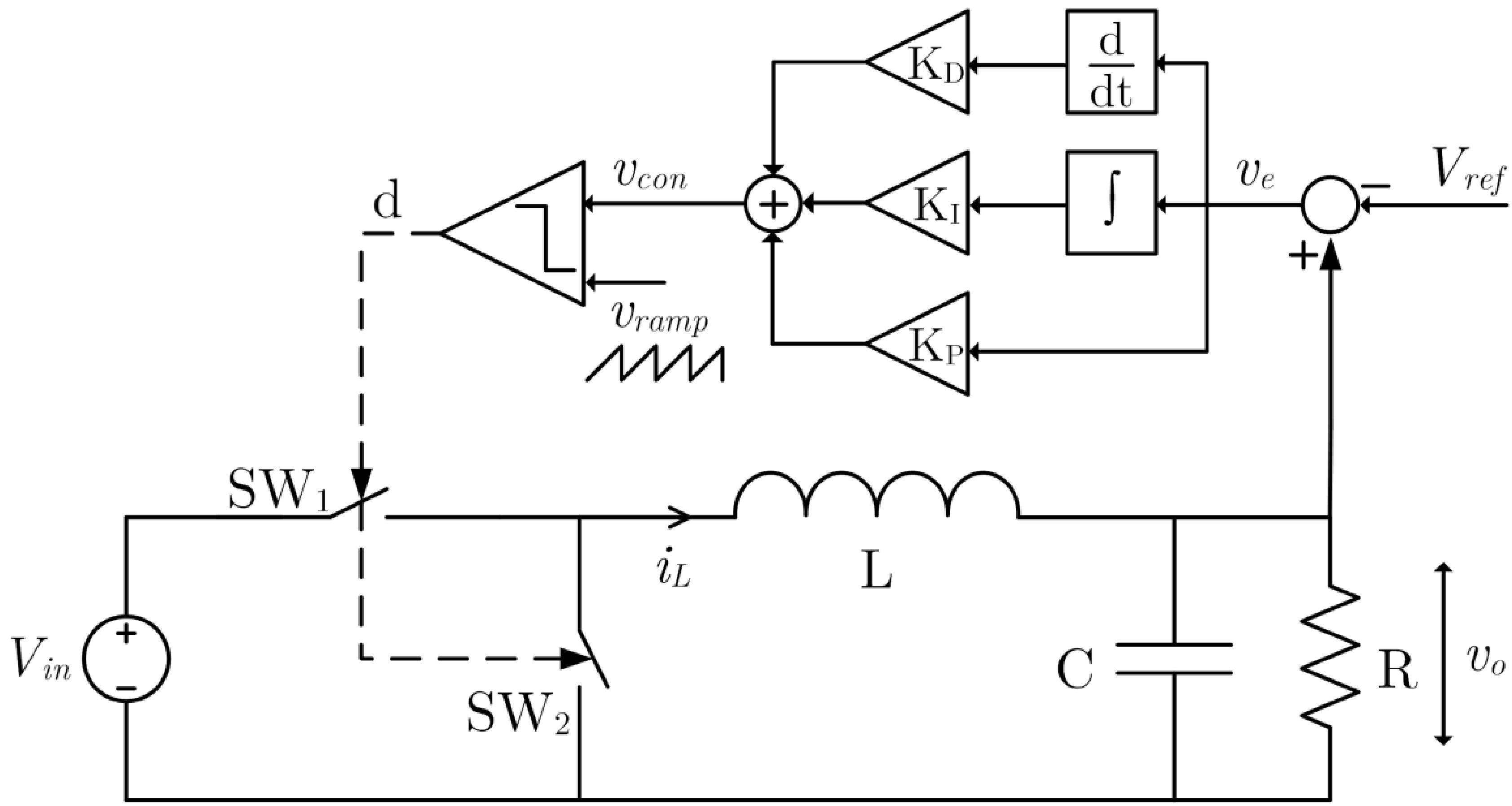
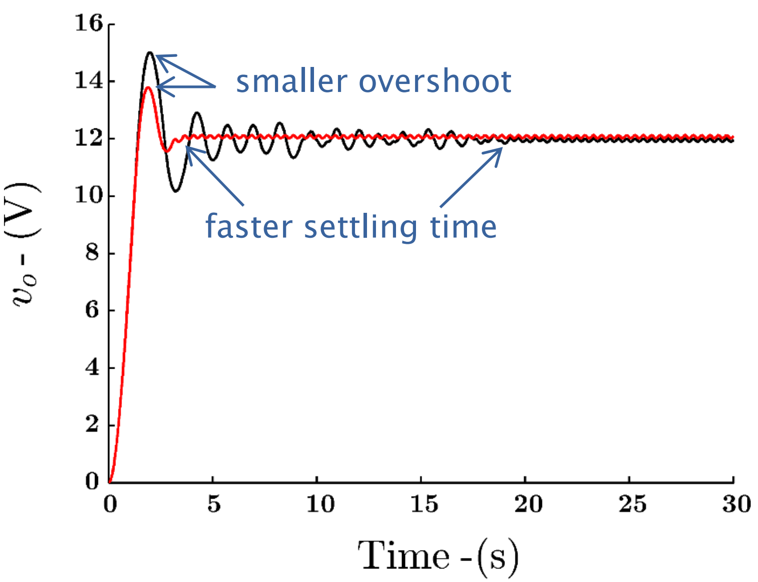
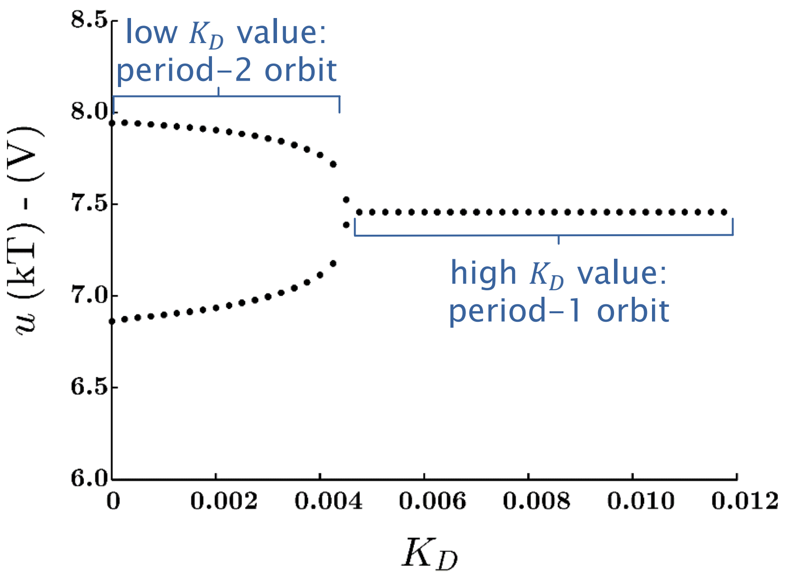
PID controlled buck converter



Effect of the D–term

- Can cause the amplification of noise
- Improves the transient response of the system





Filippov Terms

- Converter starts ON then turns OFF

$$f_+ = \begin{bmatrix} -\sigma v_\Sigma + \omega u_\Sigma \\ -\omega v_\Sigma - \sigma u_\Sigma \\ K_I - \left(v_\Sigma - V_{ref}\right) \end{bmatrix}, \quad f_- = \begin{bmatrix} -\sigma v_\Sigma + \omega u_\Sigma \\ -\omega v_\Sigma - \sigma u_\Sigma + \frac{\sigma^2+\omega^2}{\omega} V_{in} \\ K_I - \left(v_\Sigma - V_{ref}\right) \end{bmatrix}$$

- Switching manifold given by:

$$h(x,t) = K_P\left(v_o - V_{ref}\right) + K_D(-\sigma v_o + \omega u) + v_i - V_L - (V_u - V_L)t\bmod T$$

$$n = \begin{bmatrix} \frac{\partial h}{\partial v_o} \\ \frac{\partial h}{\partial u} \\ \frac{\partial h}{\partial v_i} \end{bmatrix} = \begin{bmatrix} K_P - \sigma K_D \\ \omega K_D \\ 1 \end{bmatrix}, \quad \frac{\partial h}{\partial t} = -\frac{1}{T}(V_U - V_L)$$

Characteristic equation

- Monodromy matrix

$$\Phi_M = e^{A(1-d)T} S e^{AdT}$$

- Characteristic equation

$$-\lambda^3 + \mathrm{tr}(\Phi_M)\lambda^2 - \left(\mathrm{tr}(\Phi_M)^2 - \mathrm{tr}(\Phi_M^2)\right)\lambda + \det(\Phi_M)$$

- $\lambda = -1$ for a period-doubling bifurcation:

$$1 + \mathrm{tr}(\Phi_M) + \left(\mathrm{tr}(\Phi_M)^2 - \mathrm{tr}(\Phi_M^2)\right) + \det(\Phi_M)$$

Critical k_D value

$$K_{D_{crit}} = \frac{-K_P P - K_I I - C(V_U - V_L)}{D}$$

where:

$$C = -\frac{1}{T} c_1$$

$$P = 2\delta_{in} V_{in} e^{-\sigma T} \sin(\omega T) + (-\sigma v_\Sigma + \omega u_\Sigma) c_1$$

$$I = \delta_{in} V_{in} e^{-\sigma T} (Y_2 + e^{-\sigma T} Y_2 \cos(\omega T) - e^{-\sigma T} Y_1 \sin(\omega T)) + \left(v_\Sigma - V_{ref}\right) c$$

$$D = \delta_{in} V_{in} e^{-\sigma T} (2\omega e^{-\sigma T} + 2\omega \cos(\omega T) - 2\sigma \sin(\omega T)) + \left((\sigma^2 + \omega^2) v_\Sigma - 2\sigma\omega u_\Sigma\right) c_1$$

Tuning Method

- Tune K_p and K_I terms using linear control theory
- Check if better transient required or if converter exhibits fast-scale instabilities
  - If not, D-term not required
  - If yes, D-term is required and proceed to next step
- Tune the D term using:

$$K_{D_{crit}} = \frac{-K_p P - K_I I + K_D D + C V_U}{C}$$

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Conclusions – Analog Control

- Analog PID controlled buck converter studies
  - Period-doubling bifurcations present
- Including the D-term:
  - Faster transient response
  - Eliminated period-2 orbit
- Derived stability criteria to tune D-term
  - Demonstrated inclusion in adaptive controller

Other Works – Completed

- Discontinuous switching manifold – $h(x, t)$
- Effect of digital controller
  - Quantization not considered
- Effect of noise perturbing the input voltage
- Effect of switching delays
- Effect of quantization on nonlinear dynamics
  - Saltation matrix **not** derived

Thank you for listening!