Synthesis of an Equivalent Dynamic Model for Load Areas with LTC Transformers

G.B. Denegri, M. Invernizzi, and F. Milano

Abstract -- **This paper proposes a procedure for determining an equivalent dynamic model of a sub-transmission network. The dynamic behaviour of the area to be reduced is provided by regulators of LTC transformers feeding a static load. First it is assumed a suitable topology of the 132 kV area and a procedure for reporting static load expressions at the ideal primary winding of the transformers is defined. With the hypothesis of a complete parameter knowledge of the sub-transmission area, an approximated nodal reduction is actuated. Then a modal equivalent is also determined by assuming a dynamic coherence of the tap changer regulator dynamics. The complete procedure is finally validated and criticised by some simulations on this kind of sub-transmission configuration area typical of the Italian network.**

Index Terms -- **Sub-transmission area, load modelling, LTC transformer, nodal and modal reduction.**

I. INTRODUCTION

SYNTHESIS of dynamic equivalents is always an actual problem in studying and simulating large power system behaviours. While equivalents of generation areas are fully explored and a common methodology is currently accepted, there is no general approach concerning load areas since several components and very different dynamic phenomena, including also self-generation, can be encountered.

By the way, open access and deregulation of power systems, makes more and more useful defining good equivalents of some typical areas of the network. In the ENEL case, for example , the original monopolistic society has been split into several disjointed groups, leaving at the Italian ISO the responsibility in exploiting the HV-EHV network. Simulation studies of this network usually do not require details for the sub-transmission areas and stop network implementation at interface buses. To include also sub-transmission areas, normally at 132 kV, could lead to a considerable computational effort, although contributions of sub-transmission areas should help for a more reliable comprehension of dynamic phenomena.[1]

This paper develops some aspects presented in [1] and concerns medium-long term voltage phenomena caused by LTC transformers. To this aim, Section II describes a reference network and a sub-transmission 132 kV area that will be subjected to a synthesis of a dynamic equivalent Assuming a complete knowledge of sub-transmission area and load parameters, the proposed methodology results in two distinct processes, namely a nodal and a modal reduction.

Nodal reduction utilizes a down-top strategy, i.e. starting from load nodes and arriving to interface ones. This approach is finalized in deducing the simplest approximated structure of the sub-transmission area, with the goal of preserving only interface buses with the HV remaining system.

To this aim, in Section III, voltage dependent static loads, corresponding to active and reactive power demand, are reported at the primary winding of LTC transformers, then Section IV describes how the network is structurally reduced in an equivalent star connection. This kind of reduction cannot be exact and has been widely investigated in [2]; anyway the particular structure of the sub-transmission network suggest to apply an easier approach based on pseudo-inverse matrix theory [3].

At this step, one new, not physical “barycentric” bus is created instead of all internal nodes. The resulting network holds consequently only one load bus, whose power injection expression is built by adding all contributions pertaining the series connected structure LTC-static load.

In Section V, power injection at the remaining equivalent load bus is split to the interface nodes, leading to a compact description of the whole sub-transmission network. Assuming a particular structure for static asynchronous loads, namely constant current for the active power request, and constant susceptance for the reactive power absorption, it is possible to simplify further on the resulting group of equations leading to an equivalent network reduced to its lowest terms.

Aiming to reduce also dynamic order of the equivalent model, in Section VI, a dynamic coherence for medium/long term time frame studies has been assumed for applying clustering methodology applied in a similar way as done in transient stability analysis [4].

Finally, Section VII provides a comparison between the full and the equivalent model behaviour, on a typical sample power system by mean of some test simulations.

II. STRUCTURE OF THE REFERENCE NETWORK

Reference structure of a typical network presenting a sub-transmission area is reported in Fig. 1. A and B are the interface nodes, whereas the network below these two nodes is the area to which the equivalenting procedure has to be applied. Each load of the MV network is fed by an LTC transformer whose voltage regulator constitutes the only dynamic component of the branch.

Starting from this configuration, it is our aim to describe the entire sub-transmission area by a set of algebraic and differential equations in the form:

$$\dot{x} = f(x, v_A, v_B, u) \tag{1}$$

$$P_A = g_{PA}(x, v_A, v_B, u) \tag{2}$$

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$$Q_A = g_{QA}(x, v_A, v_B, u) \tag{3}$$

$$P_B = g_{PB}(x, v_A, v_B, u) \tag{4}$$

$$Q_B = g_{QB}(x, v_A, v_B, u) \tag{5}$$

where x represents a single state variable and u a vector containing load and line parameters.

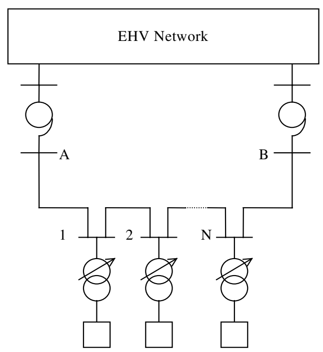

Fig. 1. Reference structure of the network. A and B represent the interface nodes.

III. Simplified Model of LTC and Load

Fig. 2 shows the simplified model of an LTC transformer and its corresponding load. The transformer is assumed to have tap ratio m, being x_T its leakage reactance computed for $m = 1$.

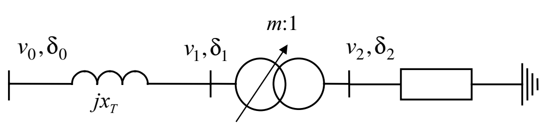

Fig. 2. Scheme of LTC transformer and corresponding load.

To assume a constant leakage reactance permits to define a static network and will appear to be an useful simplification for the subsequent nodal reduction, without significantly compromising result accuracy.

Voltage regulator of the LTC is considered to be a first order low pass, whereas load is static and assimilated to a steady-state asynchronous motor behaviour, i.e. a constant current for the active power rate and a pure reactance for the reactive one.

The following equations set the model as described above:

$$\dot{m} = -hm + k\left(\frac{v_1}{m} - v_{rif}\right) \tag{6}$$

$$P_1 = \beta_P \frac{v_1}{m} \tag{7}$$

$$Q_1 = \alpha_Q \frac{v_1^2}{m^2} \tag{8}$$

$$P_0 = \frac{v_1 v_0}{x_T} \mathrm{sen}(\delta_{01}) \tag{9}$$

$$Q_0 = \frac{v_0^2}{x_T} - \frac{v_1 v_0}{x_T} \cos(\delta_{01}) \tag{10}$$

$$-P_1 = \frac{v_1 v_0}{x_T} \mathrm{sen}(\delta_{10}) \tag{11}$$

$$-Q_1 = \frac{v_1^2}{x_T} - \frac{v_1 v_0}{x_T} \cos(\delta_{10}) \tag{12}$$

A preliminary step consists on describing the group of LTC and static load only by power absorptions at bus 0, and thus eliminating nodes 1 and 2. By an algebraic manipulation of equations (7,8,11,12), it can be derived the expression of voltage amplitude v_1:

$$v_1 = \sqrt{\frac{v_0^2}{x_T^2} - \frac{\beta_P^2}{m^2}} \Bigg/ \left(\frac{1}{x_T} + \frac{\alpha_Q}{m^2}\right) \tag{13}$$

and subsequently (6,9,10) can be rewritten eliminating dependence by v_1. Furthermore, assuming that:

$$v_0^2 / x_T^2 >> \beta_P^2 / m^2 \tag{14}$$

it is also possible recovering a formal expression of power injections in the form of a steady-state asynchronous motor; in other words, (14) allows obtaining active and reactive powers respectively linear and quadratic with regards to v_0.

$$\dot{m} = -hm + k\left(\frac{v_0}{mx_T} \Bigg/ \left(\frac{1}{x_T} + \frac{\alpha_Q}{m^2}\right) - v_{rif}\right) \tag{15}$$

$$P_0 = \frac{\beta_P v_0}{mx_T} \Bigg/ \left(\frac{1}{x_T} + \frac{\alpha_Q}{m^2}\right) = k_P(m, \alpha_Q, \beta_P, u) \cdot v_0 \tag{16}$$

$$Q_0 = \frac{\alpha_Q v_0^2}{m^2 x_T} \Bigg/ \left(\frac{1}{x_T} + \frac{\alpha_Q}{m^2}\right) = k_Q(m, \alpha_Q, u) v_0^2 \tag{17}$$

IV. Nodal Reduction

Nodal reduction proposed for our intents is a transformation of the sub-transmission network topology from the actual one to an approximated star connection.

In order to obtain this result, let us consider the general reference structure of the sub-transmission network, as represented in Fig. 3, where N load branches are reported for sake of generality. In particular, each branch i'-i represents the leakage reactance of an LTC transformer.

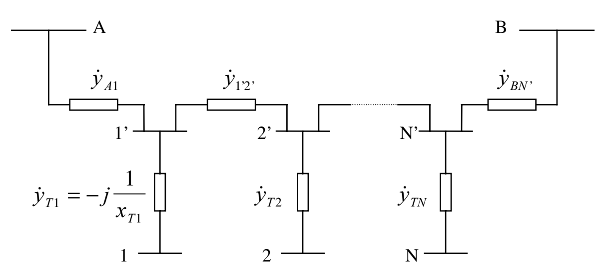

Fig. 3. Scheme of the sub-transmission island with N load branches.

Our aim is to define a new equivalent network with a star topology, as defined in Fig. 4, where only one "barycentric" bus 0 substitutes 1',2',…N' nodes.

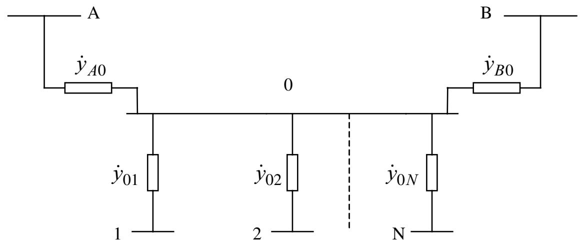

Fig. 4. Scheme of the equivalent star topology network.

We have to point out that such topological transformation can not be exactly determined because only a star-polygon transformation is always possible, as stated by Rosen-Kennelly theorem. Anyway, an approximated identification can be applied, deriving an over-determined linear system to be solved by a last mean square technique [2].

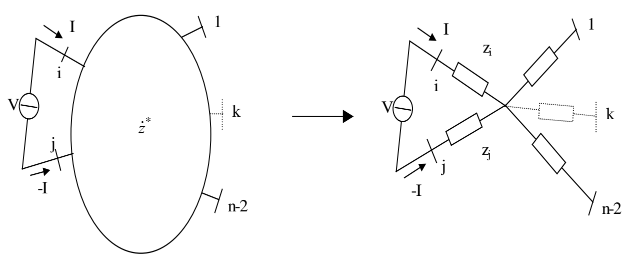

Fig. 5. Identification of a star from a generic polygonal network.

Fig. 5 shows graphically the identification procedure, that consists on applying a voltage V at two generic nodes i and j, being the other ones open circuited, and subsequently imposing an equality between the impedances of the original and the star connections. This procedure leads to define an algebraic system:

$$\dot{z}_{ij}^* = \dot{z}_i + \dot{z}_j \quad \text{for } i = 1, 2, ..., n\text{-}1 \text{ e } j\ (>i) = i+1,...,n \tag{18}$$

where n is the total number of nodes and $\dot{z}_{ij}^*$ is obtained by the following expression, assuming that the network does not have shunts:

$$\dot{z}_{ij} = \left[\dot{Y}_{ii} - \dot{Y}^T_{(n-ij)(i)} \dot{Y}^{-1}_{(n-ij)(n-ij)} \dot{Y}_{(n-ij)(i)}\right]^{-1} \tag{19}$$

In (19), $\dot{Y}_{ii}$ is the i^{th} diagonal element of the admittance matrix $\dot{Y}$, $\dot{Y}_{(i)(n-ij)}$ is the i^{th} row of $\dot{Y}$ without the i and j elements and $\dot{Y}_{(n-ij)(n-ij)}$ is the $\dot{Y}$ itself where the i and j rows and columns have been eliminated.

Equation (18) defines an $n(n - 1)/2$ order system that is typically not directly invertible, apart from the well-known case $n = 3$. Applying (18) to the network delimited by nodes A, B, 1, 2, …, N, we obtain 10 equations and 5 unknown quantities, namely the reactance of the equivalent star network. The problem can be solved by applying a least mean square technique, and a network in the form of Fig. 4 is finally determined.

V. Power Injections at Interface Nodes

Fig. 6 represents the final topology of the sub-transmission network after the polygon-star identification and the utilization of simplified load model, being P_0 and Q_0 the summation extended to power injections of all the LTC branches:

$$P_0 = \sum_{i=1}^{N} P_{0i} = v_0 \sum_{i=1}^{N} \frac{\beta_{Pi}}{m_i x_{0i}} \Bigg/ \left(\frac{1}{x_{0i}} + \frac{\alpha_{Qi}}{m_i^2}\right) \tag{20}$$

$$Q_0 = \sum_{i=1}^{N} Q_{0i} = v_0^2 \sum_{i=1}^{N} \frac{\alpha_{Qi}}{m_i^2 x_{0i}} \Bigg/ \left(\frac{1}{x_{0i}} + \frac{\alpha_{Qi}}{m_i^2}\right) \tag{21}$$

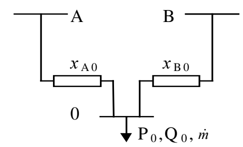

Fig. 6. Scheme of sub-transmission network after the nodal reduction and the utilization of simplified model for LTC transformer and corresponding load.

It remains to evaluate the unknown v_0; the load flow equations of the system are:

$$P_A = \frac{v_A v_0}{x_{A0}} \text{sen}(\delta_{A0}) \tag{22}$$

$$Q_A = \frac{v_A^2}{x_{A0}} - \frac{v_A v_0}{x_{A0}} \cos(\delta_{A0}) \tag{23}$$

$$P_B = \frac{v_B v_0}{x_{B0}} \text{sen}(\delta_{B0}) \tag{24}$$

$$Q_B = \frac{v_B^2}{x_{B0}} - \frac{v_B v_0}{x_{B0}} \cos(\delta_{B0}) \tag{25}$$

$$-P_0(m, v_0, u) = \frac{v_A v_0}{x_{A0}} \text{sen}(\delta_{0A}) + \frac{v_B v_0}{x_{B0}} \text{sen}(\delta_{0B}) \tag{26}$$

$$-Q_0(m, v_0, u) = \frac{v_0^2}{x_{A0}} - \frac{v_A v_0}{x_{A0}} \cos(\delta_{0A}) + \frac{v_0^2}{x_{B0}} - \frac{v_B v_0}{x_{B0}} \cos(\delta_{0B}) \tag{27}$$

Substituting (20,21) in (26,27), it can be derived the formal expression of voltage magnitude v_0:

$$v_0 = \frac{\sqrt{\frac{v_A^2}{x_{A0}^2} + \frac{v_B^2}{x_{B0}^2} + 2\frac{v_A v_B}{x_{A0} x_{B0}} \cos(\delta_A - \delta_B) - \left(\sum_{i=1}^{N} k_{Pi}\right)^2}}{\left(\frac{1}{x_{A0}} + \frac{1}{x_{B0}} + \sum_{i=1}^{N} k_{Qi}\right)} \tag{28}$$

where it has been assumed:

$$v_0^2 \big/ x_{0i}^2 >> \beta_{Pi}^2 \big/ m_i^2 \qquad \text{for } i = 1,2,...,N \tag{30}$$

in accordance with (14), and the variables k_{Pi} and k_{Qi} are defined as follow:

$$k_{Pi} = \frac{\beta_{Pi}}{m_i x_{0i}} \Bigg/ \left(\frac{1}{x_{0i}} + \frac{\alpha_{Qi}}{m_i^2} \right) \quad \text{for } i = 1,2,...,N \quad (31)$$

$$k_{Qi} = \frac{1}{x_{0i}} - \frac{1}{x_{0i}^2} \Bigg/ \left(\frac{1}{x_{0i}} + \frac{\alpha_{Qi}}{m_i^2} \right) \quad \text{for } i = 1,2,...,N \quad (32)$$

With no further hypotheses, (22-25) can be rewritten by substituting v_0 with (28). Formally, node 0 is thus eliminated from the set of load flow equations.

Anyway, final expressions of power injections result too complicated and of no practical utilization. It could be a good task to recover a formal expression of power demands applied to buses A and B in accordance with the original nature of static loads. In other words, active powers rate should be proportional to voltages v_A and v_B and reactive ones proportional to v_A^2 and v_B^2. This goal can be reached applying the following simplification, that is generally verified:

$$\left(\frac{v_A}{x_{A0}} + \frac{v_B}{x_{B0}} \right)^2 >> \left(\sum_{i=1}^{N} k_{Pi} \right)^2 \quad (33)$$

and assuming in some convenient cases that $\cos(\delta_A - \delta_B) \cong 1$ or $\text{sen}(\delta_A - \delta_B) \cong 0$. In this way, final expressions of power injections at buses A and B become:

$$P_A = \frac{x^*}{x_{A0}} \left(\sum_{i=1}^{N} k_{Pi} \right) v_A + \frac{x^*}{x_{A0} x_{B0}} v_A v_B \,\text{sen}(\delta_A - \delta_B) \quad (34)$$

$$Q_A = \left(\frac{1}{x_{A0}} - \frac{x^*}{x_{A0}^2} \right) v_A^2 - \frac{x^*}{x_{A0} x_{B0}} v_A v_B \cos(\delta_A - \delta_B) \quad (35)$$

$$P_B = \frac{x^*}{x_{B0}} \left(\sum_{i=1}^{N} k_{Pi} \right) v_B + \frac{x^*}{x_{A0} x_{B0}} v_A v_B \,\text{sen}(\delta_B - \delta_A) \quad (36)$$

$$Q_B = \left(\frac{1}{x_{B0}} - \frac{x^*}{x_{B0}^2} \right) v_B^2 - \frac{x^*}{x_{A0} x_{B0}} v_A v_B \cos(\delta_B - \delta_A) \quad (37)$$

where:

$$x^* = \left(\frac{1}{x_{A0}} + \frac{1}{x_{B0}} + \sum_{i=1}^{N} k_{Qi} \right)^{-1} \quad (38)$$

Equations (34-37) can also be interpreted as an equivalent circuit, as shown in Fig. 7, where:

$$b_A = -\frac{1}{x_{A0}} \left[1 - x^* \left(\frac{1}{x_{A0}} + \frac{1}{x_{B0}} \right) \right] \quad (39)$$

$$b_B = -\frac{1}{x_{B0}} \left[1 - x^* \left(\frac{1}{x_{B0}} + \frac{1}{x_{A0}} \right) \right] \quad (40)$$

$$b_{AB} = -\frac{x^*}{x_{B0} x_{A0}} \quad (41)$$

$$I_A = \frac{x^*}{x_{A0}} \sum_{i=1}^{N} k_{Pi} \qquad I_B = \frac{x^*}{x_{B0}} \sum_{i=1}^{N} k_{Pi} \quad (42)$$

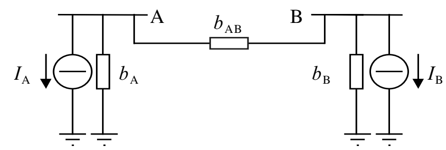

Fig. 7. Equivalent circuital scheme of the sub-transmission area.

VI. Modal Reduction

Applying also to differential equations the substitution of v_0 with its simplified expression, we can deduce:

$$\begin{aligned} \dot{m}_i &= -h_i m_i + k_i \left(x^* \left(\frac{v_A}{x_{A0}} + \frac{v_B}{x_{B0}} \right) \Bigg/ x_{Ti} m_i \left(\frac{1}{x_{Ti}} + \frac{\alpha_{Qi}}{m_i^2} \right) - v_{(rif)i} \right) \\ &= f_i(m_i) \qquad \text{for } i = 1,2,...,N \end{aligned} \quad (43)$$

Anyway the mere nodal reduction procedure does not affect dynamic order of the sub-transmission area.

For reducing the dynamic order, let us consider the summation of all differential equations:

$$\frac{1}{N} \sum_{i=1}^{N} \dot{m}_i = \frac{1}{N} \sum_{i=1}^{N} f_i(m_i) \quad (44)$$

Practical considerations about dynamic term of tap changer regulators suggests to consider a dynamic coherence of LTCs, as well it is done for equivalents in transient stability studies. In addiction to dynamic coherence, the limited m_i variation ranges suggest to assume also an unification of the state variables. Consequently, a common state variable m can be defined and an unique differential equation is obtained:

$$\dot{m} = \frac{1}{N} \sum_{i=1}^{N} f_i(m) = f(m) \quad (45)$$

It has to be noted that such procedure allows defining a final differential equation dependent on system parameters but not on a particular initial state of the original dynamic equations. For this reason, load flow studies for setting the initial value of m can be performed by using directly equations of the reduced model.

Finally, limits on the equivalent tap changer ratio may be defined as a mean value of the original regulator limits.

VII. Validation on Test Case

Fig. 8 shows a configuration, with the EHV transmission network feeding a typical Italian sub-transmission area at 132 kV by means of a couple of auto-transformers supplying buses A and B. This area is subjected to the complete nodal and modal reduction procedures and some simulations are

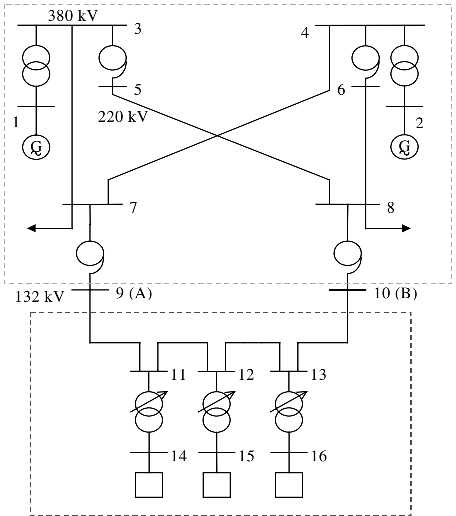

Fig. 8. Test network utilized for validating the equivalenting procedure.

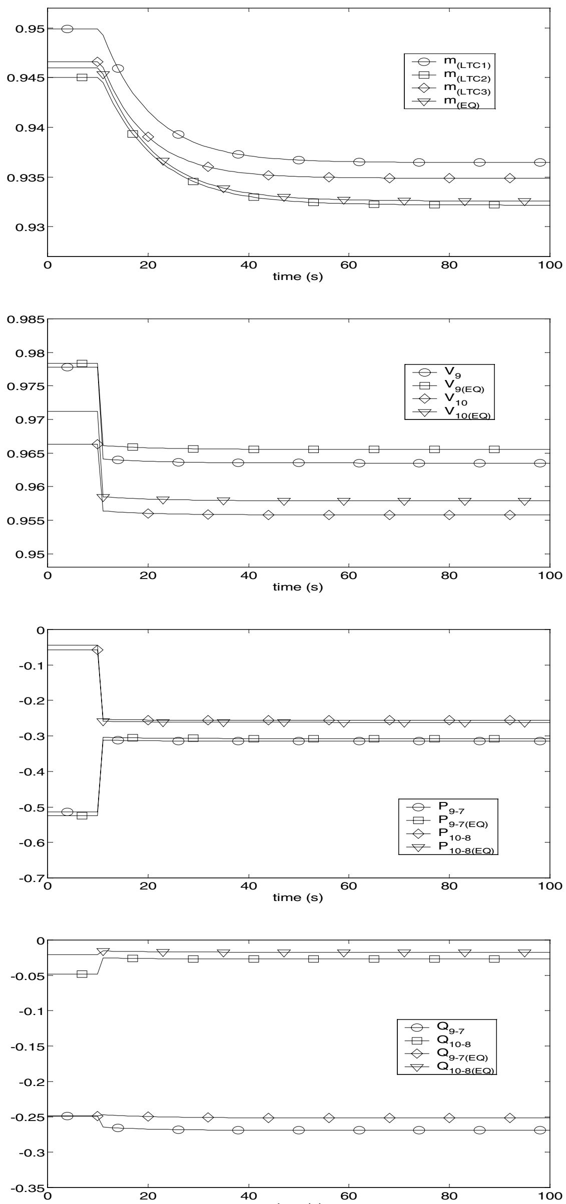

Fig. 9. Loss of line 3-7.

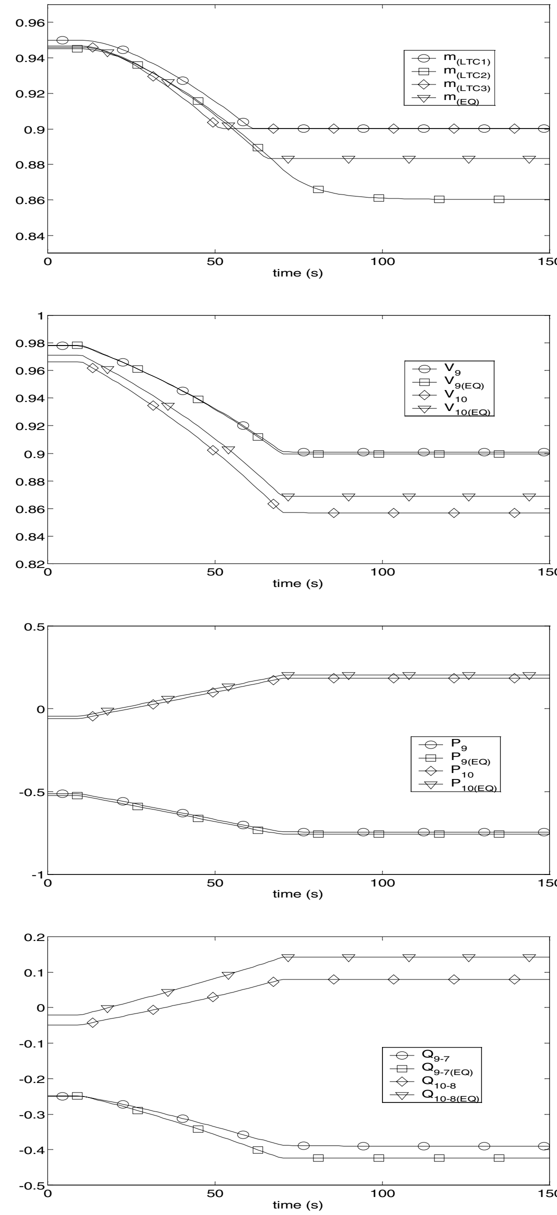

Fig. 10. Increasing in load demands at buses 7 and 9.

performed to test reliability of the resulting equivalent model. To this aim a comparison with results provided by simulating the complete network without any reduction is also presented.

Test cases are performed perturbing the system by a loss of the transmission line between buses 7 and 3 (Fig. 9), a doubling in the external power demands at buses 7 and 8 (Fig. 10), and finally a doubling in the power demands of loads connected to the LTC transformers within the sub-transmission area (Fig. 11). From simulation results, it can be note that the equivalent state variable shows an average motion with respect to the original ones. Also voltage tracking is performed with a good accuracy, within 2%. Besides, the reactive power demands is followed with minor reliability with respect to the active flows. This fact is mainly due to the nature of the quadratic voltage dependent law used for describing the reactive absorption rates and to the approximated star-polygon procedure which mainly affects reactive parameters of the network.

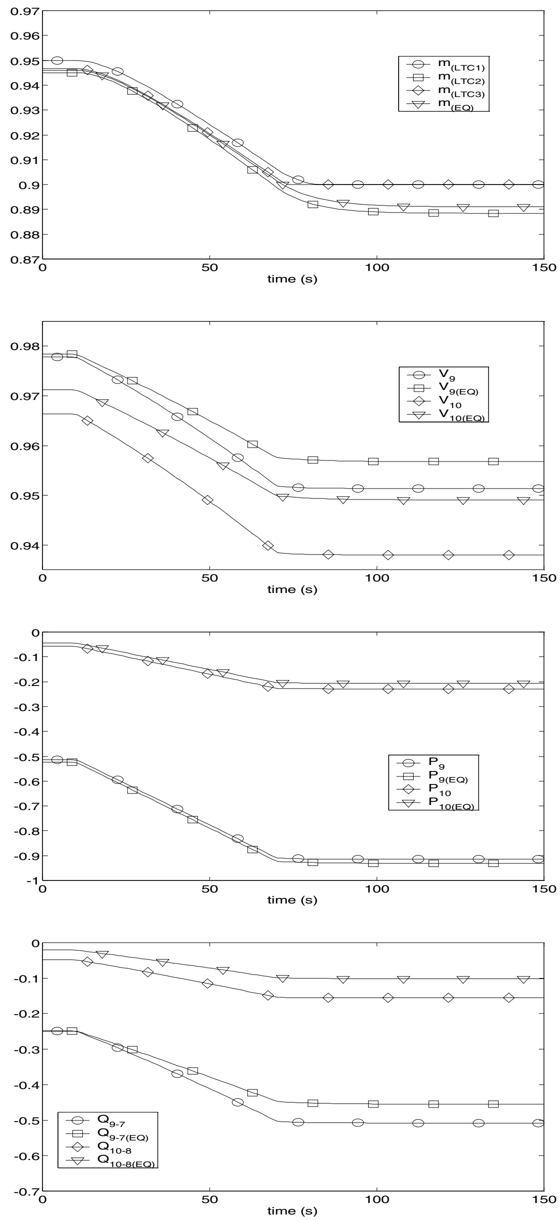

Fig. 11. Increasing in the power absorptions of loads connected to the LTC transformers within the sub-transmission area.

VIII. Conclusions

Equivalenting procedures for power system networks are going to become more and more useful techniques in view of deregulation processes that are involving the electric market. Although computational means could now easily manage dynamic simulations even of extremely large networks, power companies are often owners of a delimited system that has to be interconnected in an affordable way to other networks. The presented paper focuses on a possible equivalenting procedure of a sub-transmission island, fed by a transmission network, supposed managed by a separate company.

Attention has been paid to medium/long term voltage dynamics, whereas asynchronous motor loads are approximated to their static behaviour.

Results of theoretical procedure are validated by some test simulations on a simple but typical sample transmission network.

Future development of the proposed method appears to be an extension to other dynamic loads behaviours taken into account, including possible self-generation contributes.

It also seams a good task to combine the presented circuital model with some identification techniques, with the intent of bypassing an eventual lack of information about parameters and operating states of the area subjected to the equivalenting procedure.

IX. References

[1] R. Aresi, B. Delfino, G. B. Denegri, M. Invernizzi, S. Massucco, “Dynamic equivalents of tap-changing-under-load transformers in a transmission system”, *IEE Proc.-Gener. Transm. Distrib.*, vol. 146, no. 1, pp. 45-52, Jan. 1999.

[2] H. Hager, H. Glavitsch, “GRANEQ – A radial, internal network equivalent for power system security calculations”, *IEEE Transaction on Power Apparatus and Systems*, vol. PAS-102, no. 2, pp. 452-458, Feb. 1983.

[3] G.T. Heydt, *Computer Analysis Methods for Power Systems*, Stars in a Circle Pubblications, 2nd edition, 1996.

[4] U. Di Caprio, R. Marconato, “A novel criterion for the development of multi-area simplified models oriented to the on-line evaluation of power system dynamic security”, *5th Power System Computation Conference*, Cambridge, UK, 1-5 Sep. 1974.

X. Biographies

Gio Battista Denegri was born in Rocca Grimalda, Italy, on October 9, 1946. He received the degree in electrical engineering from the University of Genoa, Italy, in 1970. Until 1972, he worked as an R&D Engineer at ELSAG Corporation. He joined the Electrical Engineering Department of the University of Genoa in 1973, where he is now Full Professor of Electrical Machines and Drives. At present, his research interests are modeling of electrical machines, drive dynamics, computer analysis of electric energy systems and power system harmonics. He is also a member of the Italian Electrical Society (AEI) and of the Italian National Research Council (CNR) – Electric Power System Group.

Marco Invernizzi was born in Genoa, Italy, on December 27, 1959. He received the degree in electrical engineering from the local University in 1984, and a PhD in power systems in 1989, with a dissertation on expert system application to power system emergency control. Since 1990, he has been working at the Electrical Engineering Department of the University of Genoa, where he is now Associate Professor of Power System Analysis. He is presently engaged in modeling and simulation of FACTS devices, management and control of electric energy system in deregulated environments and power system stability assessment. He is a member of the Italian Electrical Society (AEI) and of the Italian National Research Council (CNR) – Electric Power System Group.

Federico Milano was born in Genoa, Italy, on March 24, 1975. He received the degree cum laude in electrical engineering from the University of Genoa in 1999. He is currently attending Ph.D. course at the Electrical Engineering Department, University of Genoa, in the field of power system control and operation.