

# Dynamic REI equivalents for short circuit and transient stability analyses

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## ARTICLE INFO

### Article history:

Received 18 April 2008

Received in revised form 16 October 2008

Accepted 18 November 2008

Available online 3 January 2009

### Keywords:

Dynamic equivalents

REI (Radial, Equivalent and Independent) equivalents

Model aggregation

Power transfer distribution factors

Short circuit analysis

Transient stability

## ABSTRACT

This paper proposes a systematic approach for dynamic power system equivalents based on power transfer distribution factors. The proposed method divides the original network into an internal interconnected system and an external one. Static equivalents are computed at frontier buses that separate the retained internal system from the external one. The equivalents are formed using REI (Radial, Equivalent and Independent) networks and generator model aggregation. Generator parameters are computed based on power transfer distribution factors of the generated active power. The equivalent models are able to accurately approximate the behavior of the original system for short circuit and transient stability analyses. Two test systems, namely the Kundur's 2-area test system and a 1213-bus network that model a real transmission system are used to illustrate and test the proposed technique.

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## 1. Introduction

### 1.1. Motivation

Power systems all over the world have increased in size and complexity due to the rapid growth of widespread interconnections. Today interconnected power systems cover large geographical areas and comprise thousands of devices. For such large systems, it is neither practical nor necessary to perform studies such as the electromagnetic transient analysis, on-line dynamic security assessment, off-line stability studies and design of controls with the full detailed system model.

While analyzing a large system, the engineers are usually interested in the behavior of a certain part of the system. Such a part of the large system is called *internal* or *study* system and the rest of the system is referred to as *external* system. Static and dynamic reduction or equivalencing is the process of reducing the complexity of external system model while retaining its effect on the study system. The large electric power system models can be reduced significantly with this method while maintaining acceptable accuracy with respect to a specific phenomenon.

### 1.2. Literature review on static equivalents

Classical methods for computing static network equivalents are Ward equivalents and REI (Radial, Equivalent and Independent) equivalents. The interest on static equivalents is demonstrated by the large number of proposals and task forces dedicated to this topic [1–4].

Ward equivalents were initially proposed in [5] and then further discussed in [6–8]. The Ward equivalent is composed of a linear part and a nonlinear one. The issue of this equivalent is that the physical behavior of the internal system (which is accurate) and the behavior of the external system (which is approximated), cannot be simulated by the same algorithm process.

REI stands for Radial, Equivalent and Independent. This method was originally proposed in [9] and has been documented in great details in several publications [10–12]. Generally speaking REI equivalents is a loss-less network representation of a set of base case injections or, in other words, the so called *zero power balance network*. For its flexibility, the basic principle of REI equivalents is used in this paper for the static network reduction technique and is summarized in Section 2.

One important question when dealing with equivalent static networks is the nature of the equivalent buses of the reduced system. Typically, equivalent fictitious buses do not fall into one of the classical power flow buses (e.g. constant admittance, PQ, PV or slack bus), but are actually a composition of several bus types. In [13,14], sensitivities are used as an index of the impact of a change in the retained system. The external network is reduced based on sen-

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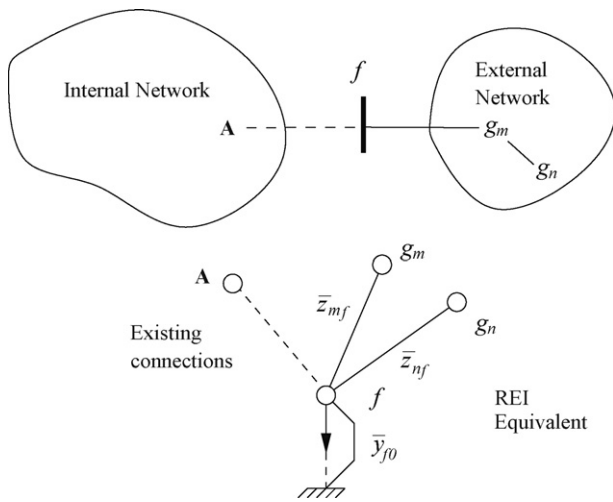


Fig. 1. REI equivalent.

sitivities and the nature of fictitious buses is determined based on sensitivity values. Ref. [15] provides an interesting application of the sensitivity approach similar to the one that is proposed in [13]. In [15], sensitivities are used within the framework of the probabilistic power flow analysis.

Other interesting techniques based on static equivalencing are as follows:

- (1) In [16,17], the topology of the external system is not known. The equivalents are determined based on measurement and state estimation techniques.
- (2) In [18], equivalent are computed based on an expert system. The bigger the data base, the better the estimation of the equivalent at frontier buses.
- (3) Ref. [19] proposes a method to evaluate static equivalents so that the resulting reduced network minimize the error of participation factors with respect to the original system. The application of this kind of equivalents is intended for transmission cost allocations and electricity markets.

### 1.3. Literature review on dynamic equivalents

Like static equivalents, dynamic equivalent methods have also had a key role in power systems research. The typical problem of dynamic equivalents is to define equivalent synchronous machines so that the reduced network transient stability features are as close as possible to the original system [20]. Another topic, although less exploited in the literature, is to determine dynamic equivalents of loads. This problem is typically solved through identification techniques (for example, see [21]).

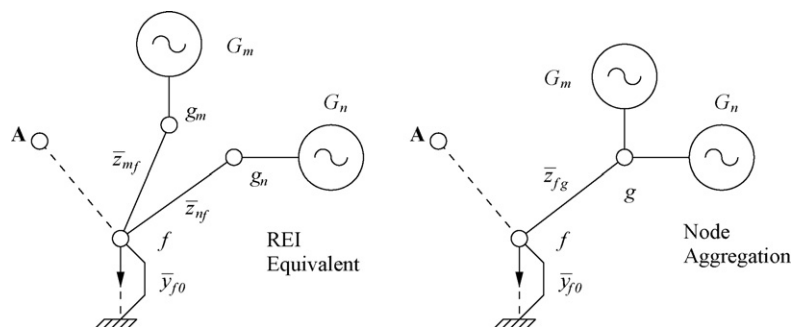


Fig. 2. Node aggregation.

Several methods such as heuristic approach, modal analysis approach, coherency approach have been developed to determine static and dynamic equivalents of power systems.

The heuristic approach dates back to 1950s and has been used with AC network analyzer [22]. The procedure was extended to digital computers in 1969 by Brown et al. [23]. It has been widely used for many years but the practice was not based on any solid theory. This may provide reasonable results when the stability problem is local to the study system with dynamics of external areas having only secondary effect.

The modal analysis approach to dynamic equivalencing was introduced in the seventies by Price et al. [24]. This approach suffers from two major drawbacks: it is very time consuming, and equivalents do not have structural identity.

Off late coherency approach has found favor amongst researchers. It involves coherency identification based on rotor angle swings and aggregating each coherent group. Several methods have been proposed for coherency identification based on linearized models. They include inspection of time responses [25–30], pattern recognition [31], closest unstable equilibrium point [32], Liapounov function [33], weakly coupled subsystems [34], modes of low frequency oscillations [35]. Coherency identification without linearization has also been attempted [36]. Some of the recent developments in this area are discussed in references [37–39]. In [40], the authors propose a method for aggregating not only synchronous machines, but also machine AVRs.

Although coherency methods are recognized as the most reliable for dynamic equivalencing, these have the drawback that it is not always possible to reduce a given part of the network, since the coherency impose the regions into which the network can be divided. Approaches that are able to retain a given part of the network are [41–43]. Other relevant approaches include: dynamic Ward equivalents for transient stability [44], and dynamic identification using artificial neural networks [45].

In this paper we use the main concepts of coherency methods to aggregate several machines into an equivalent one, and to compute the parameters of the equivalent machine.

### 1.4. Overview of existing software tools for network equivalencing

Few production-grade tools that meet requirements of modern power system exist. Some of the requirements of equivalencing software are described below.

- (1) Retention of key system characteristics impacting on specific aspects of stability.
- (2) Validity over the expected range of system operating conditions.
- (3) Adequate modeling capability.
- (4) Compatibility with programs used for analysis of different aspects of stability.

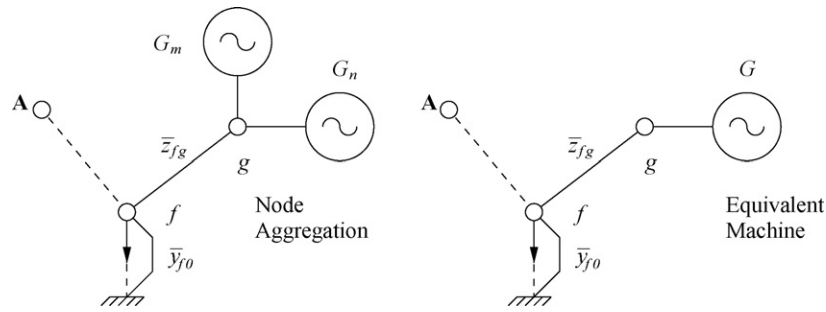


Fig. 3. Determination of the equivalent synchronous machine.

(5) Ease of use requiring minimal user judgment and interaction.

Some of currently solutions are as follows. (i) DigSILENT software is a unified tool for RMS and EMT simulation [46]. It claims to have all the models suitable for RMS and EMT simulations. It is possible to run two instances of DigSILENT, namely one for EMT and the other for RMS and interface them. The advantage is that the dynamic reduction is done away with. (ii) DYNRED is a tool from EPRI for dynamic reduction [47]. It is a coherency-based program and adopts a linearized approach. (iii) Electrical equivalents implemented in PSS/E are basically static REI equivalents. The equivalent procedure is implemented in the module EEQV [48]. The equivalent is only valid for small perturbation around the initial power flow solution.

Refs. [49–51] show that some users and/or utilities have used available software along with external *ad hoc* scripts to obtain accurate and well-behaved network reductions. At this aim, a variety of tools for power system analysis have been developed around Matlab [52,53]. In this paper we propose a solution based on a flexible and extensible Matlab-based software code for computing static and dynamic equivalents. We call this tool NEQUIT, that stands for NNetwork EQUIvalent. NEQUIT is composed of a suite of Matlab and Perl scripts that accomplish the static and dynamic equivalencing procedure through power system analyses based on the proprietary software SIMPOW [54]. For the sake of completeness, a brief description of NEQUIT is given in Appendix A.

1.5. Contributions

In summary, the novel contributions of the paper are as follows:

(1) A general technique for static and dynamic reduction of networks. The reduction focuses mainly on synchronous machines

and makes use of the concepts of REI equivalents, generator participation factors and node aggregation.

(2) The proposed reduction technique does not need the computation of the eigenvalues of the Jacobian power flow matrix nor the state matrix of the system. This fact makes the proposed method suitable for large networks.

1.6. Organization

The paper is organized as follows. Section 2 describes the proposed techniques for computing dynamic equivalents based on REI equivalents, participation factors and node aggregation. Section 3 discusses two examples of the proposed equivalencing techniques based on the Kundur's 2-area system and a 1213-bus network. Finally, in Section 4, conclusions are duly drawn.

2. Proposed equivalencing technique

The proposed equivalencing technique consists in the following three steps:

- (1) Determination of REI equivalents at frontier buses.
- (2) Node aggregation.
- (3) Computation of equivalent machine parameters through participation factors.

The pictorial representation of the procedure that implements the REI equivalents is shown in Fig. 1, where *f* is the node where the REI equivalent is built; **A** represents the set of nodes of the internal network (the equivalencing procedure does not modify the existing connections between *f* and **A**); *g<sub>m</sub>* and *g<sub>n</sub>* are the nodes of the external network (*f* can be directly connected to external buses or indirectly, i.e. through other external buses);  $\bar{z}_{mf}$ ,  $\bar{z}_{nf}$  and  $\bar{y}_{f0}$  are

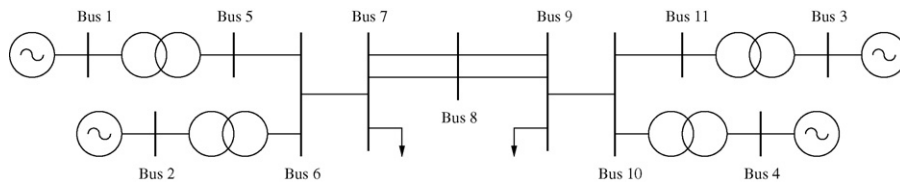


Fig. 4. Kundur's 2-area test system.

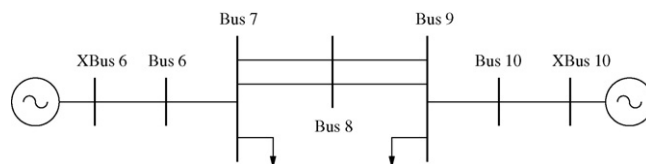


Fig. 5. Reduced Kundur's 2-area system.

**Table 1**  
Comparison of power flow results for the Kundur's 2-area system.

Impedance (bus #)	Original system	STAPOW equivalent	Proposed equivalent
$p_{6-7}$ (MW)	1387.64	1398.61	1391.97
$p_{10-9}$ (MW)	1406.02	1416.80	1410.18
$v_6$ (kV)	224.986	250.950	232.507
$v_{10}$ (kV)	226.221	249.460	232.426

the equivalent impedances and the shunt admittance, respectively, that are determined by means of the REI procedure, as described later on in this section.

The first step is to compute the admittance matrix of the network, linearize all PQ loads and add the load equivalent admittances to the diagonal elements of the network admittance matrix. These steps are the same as the ones needed by standard Thevenin equivalents, except for the fact the REI equivalents does not include the internal synchronous machine impedances in the network admittance matrix.

Before further describing the REI equivalencing technique, let us define the following indices:

- (1)  $k$ : vector of indices of all generators contained in a given external network.
- (2)  $f$ : index of the border bus at which one wants to compute the REI equivalent.
- (3)  $c$ : vector of indices of load buses contained in the external network.
- (4)  $r$ : vector obtained from the union of  $k$  and  $f$ .

Then, the following relation between currents and voltages applies:

$$\begin{bmatrix} \bar{i}_r \\ 0 \end{bmatrix} = \begin{bmatrix} \bar{Y}_{rr} & \bar{Y}_{rc} \\ \bar{Y}_{cr} & \bar{Y}_{cc} \end{bmatrix} \begin{bmatrix} \bar{v}_r \\ \bar{v}_c \end{bmatrix} \quad (1)$$

The currents relative to loads are 0 since the admittance matrices already contains the load equivalent admittances. Thus, one can eliminate  $\bar{v}_c$  from (1):

$$\bar{i}_r = \bar{Y}_{rr} - \bar{Y}_{rc}\bar{Y}_{cc}^{-1}\bar{Y}_{cr}\bar{v}_r = \bar{Y}_r\bar{v}_r \quad (2)$$

Remembering the meaning of  $r$ , one obtains:

$$\begin{bmatrix} \bar{i}_k \\ \bar{i}_f \end{bmatrix} = \begin{bmatrix} \bar{Y}_{kk} & \bar{Y}_{kf} \\ \bar{Y}_{fk} & \bar{Y}_{ff} \end{bmatrix} \begin{bmatrix} \bar{v}_k \\ \bar{v}_f \end{bmatrix} \quad (3)$$

where

$$\bar{Y}_r = \begin{bmatrix} \bar{Y}_{kk} & \bar{Y}_{kf} \\ \bar{Y}_{fk} & \bar{Y}_{ff} \end{bmatrix} \quad (4)$$

**Table 2**  
Comparison of positive sequence impedances for the Kundur's 2-area system.

Impedance (bus #)	Original system	STAPOW equivalent	Proposed equivalent
$\bar{z}_{p6}$ (p.u.)	2.9804 + j11.4081	2.9845 + j11.4099	2.6982 + j11.0144
$\bar{z}_{p7}$ (p.u.)	5.6299 + j13.9043	5.6375 + j13.9072	5.3093 + j13.6281
$\bar{z}_{p8}$ (p.u.)	6.3066 + j22.3675	6.3190 + j22.4141	6.0455 + j22.2972
$\bar{z}_{p9}$ (p.u.)	7.9611 + j11.6859	7.9699 + j11.6806	7.5944 + j11.5688
$\bar{z}_{p10}$ (p.u.)	4.2737 + j10.2801	4.2786 + j10.2775	3.9341 + j10.0046

**Table 3**  
Comparison of zero sequence impedances for the Kundur's 2-area system.

Impedance (bus #)	Original system	STAPOW equivalent	Proposed equivalent
$\bar{z}_{06}$ (p.u.)	24.3593 + j53.7164	24.3884 + j54.0042	26.3271 + j53.2180
$\bar{z}_{07}$ (p.u.)	22.6659 + j37.7226	22.7763 + j38.0998	24.6223 + j37.3975
$\bar{z}_{08}$ (p.u.)	5.0743 - j38.8160	5.0300 - j39.3575	6.0729 - j39.4843
$\bar{z}_{09}$ (p.u.)	8.9702 - j36.2225	8.7087 - j37.3192	10.6120 - j37.8401
$\bar{z}_{010}$ (p.u.)	10.5663 - j20.4495	10.2995 - j21.4743	12.1291 - j22.0062

From (3), one can extract the current injected at the frontier bus  $i$ :

$$\bar{i}_f = \sum_k \bar{Y}_{fk}\bar{v}_k + \bar{Y}_{ff}\bar{v}_f \quad (5)$$

from where one obtains the value of the fictitious shunt admittance at the frontier bus  $f$  (see Fig. 1):

$$\bar{y}_{f0} = -\sum_k \bar{Y}_{fk} - \bar{Y}_{ff} \quad (6)$$

Furthermore the elements of  $\bar{Y}_{fk}$  are the series admittances of the fictitious lines that connects the frontier bus  $f$  with the retained generator buses  $k$  (i.e. buses  $g_m$  and  $g_n$  in Fig. 1). In other words:

$$\bar{Y}_{fk} = \left[ \frac{1}{\bar{z}_{f1}}, \frac{1}{\bar{z}_{f2}}, \dots, \frac{1}{\bar{z}_{fk}} \right] \quad (7)$$

The pictorial representation of the procedure that implements the node aggregation is shown in Fig. 2. Once obtained the REI equivalent, node aggregation is straightforward:

$$\bar{z}_{fg} = \frac{1}{\sum_k \bar{Y}_{fk}} \quad (8)$$

The last step is to aggregate also the synchronous machine at the fictitious bus  $g$ . The pictorial representation of the procedure that implements the determination of the equivalent synchronous generator is shown in Fig. 3. The first step is to compute the power that flows in each radial fictitious line of the REI equivalent. This is actually a straightforward computation, since the voltages at the bus terminal of each line of the REI equivalent are known from the power flow solution. Thus, one has:

$$\bar{s}_{if} = \bar{v}_j \frac{(\bar{v}_j - \bar{v}_f)^*}{\bar{z}_{jf}^*} \quad \forall j \in k \quad (9)$$

The distribution participation factor of each generator  $j$  to the frontier bus  $f$  is then computed as

$$PF_{jf} = \frac{\Re\{\bar{s}_{jf}\}}{P_j} \quad (10)$$

where  $P_j$  is the total active power generator  $i$  as for the power flow base case. Observe that this participation factor definition is similar to the one given in [55].

Finally, one has to compute the parameters of the equivalent machine at bus  $g$ . The equivalent power base  $S_{Ng}$  of the machine is

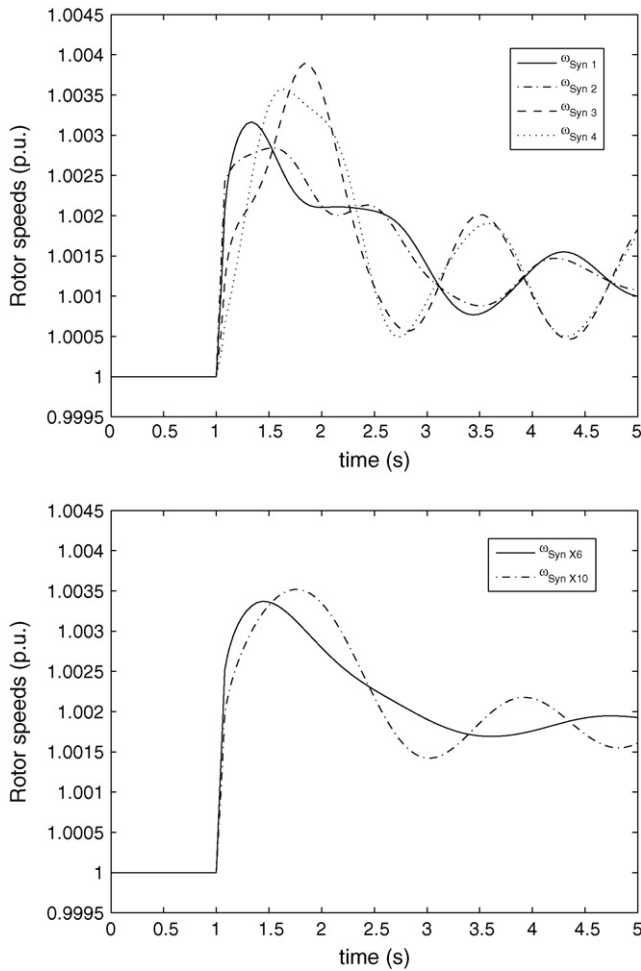


Fig. 6. Kundur's system: rotor speeds following an 80 ms fault at bus 8 for the original and the reduced networks.

computed as

$$S_{N_g} = \sum_j PF_{jf} S_{N_j} \tag{11}$$

where  $S_{N_j}$  is the power base of each machine of the external network. For simplicity, the voltage base  $V_{N_g}$  of the equivalent machine can be set as the voltage rating of the frontier bus  $f$ . Then, the inertia  $H_g$  of the equivalent machines is

$$H_g = \frac{1}{S_{N_g}} \sum_j PF_{jf} H_j S_{N_j} \tag{12}$$

A similar expression holds for computing the damping of the equivalent machine. Resistances, reactances, and time constants are evaluated using an inertia weighted mean. For example, for the sub-transient direct axis reactance, one has:

$$x''_{dg} = \frac{1}{H_g} \sum_j H_j x''_{dj} \tag{13}$$

The rationale for internal node aggregations is based on the theory of singular perturbation [56]. The internal node aggregation of synchronous machines implicitly assumes that all machines that are aggregated are coherent. This assumption is typically verified in the practice, since the intervention of modern protection is fast enough (three to four periods of the fundamental frequency) to avoid the loss of synchronism of the generators. Thus, the determination of coherent machine group is more a theoretical problem

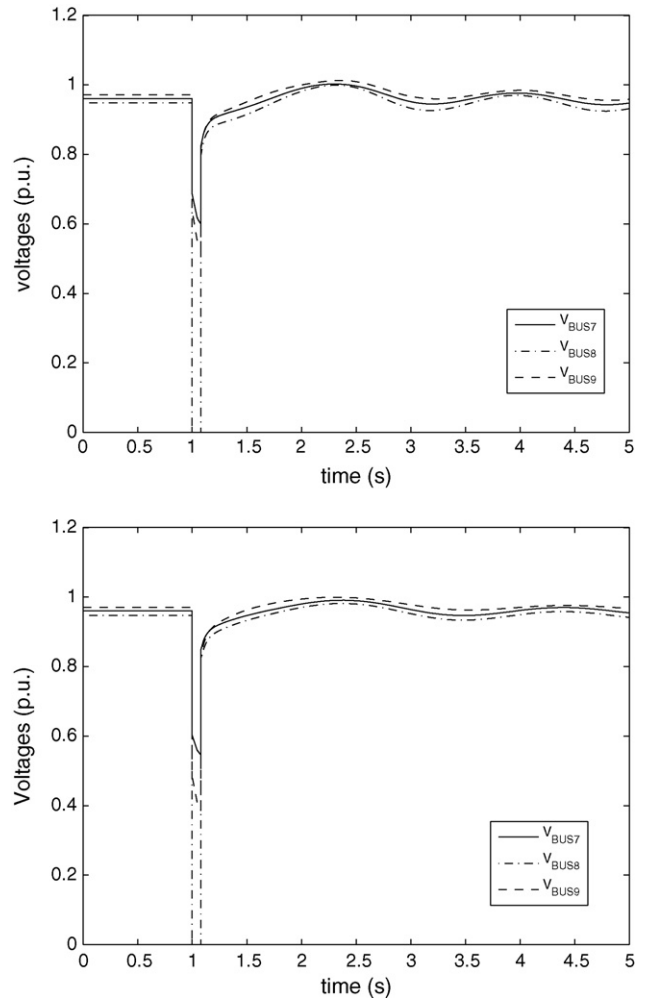


Fig. 7. Kundur's system: voltages at buses 7, 8 and 9 following an 80 ms fault at bus 8 for the original and the reduced networks.

than a real practical need. Observe also that the internal node aggregation allows reducing the *stiffness effect*, which is typical of bus aggregation.<sup>1</sup>

### 3. Cases studies

The proposed technique for dynamic equivalents is applied to the Kundur's 2-area test system and to a 1213-bus model of a real transmission system. The equivalencing procedure was solved using the NEQUIT toolbox (see Appendix A), while time domain simulations have been obtained using PSAT [53]. Results of the proposed procedure are compared with results obtained with the original network and, if possible, with the results obtained using the STAPOW equivalencing procedure, which is the default short circuit analysis and static equivalencing module provided by SIMPOW. Appendix B briefly describes the STAPOW equivalencing method. Finally, for the interested reader, a detailed description of all procedures and static and dynamic models used in the case studies can be found in [54].

<sup>1</sup> An aggregate network shows the stiffness effect if the frequencies of its inter-area modes are higher than those of the original network.



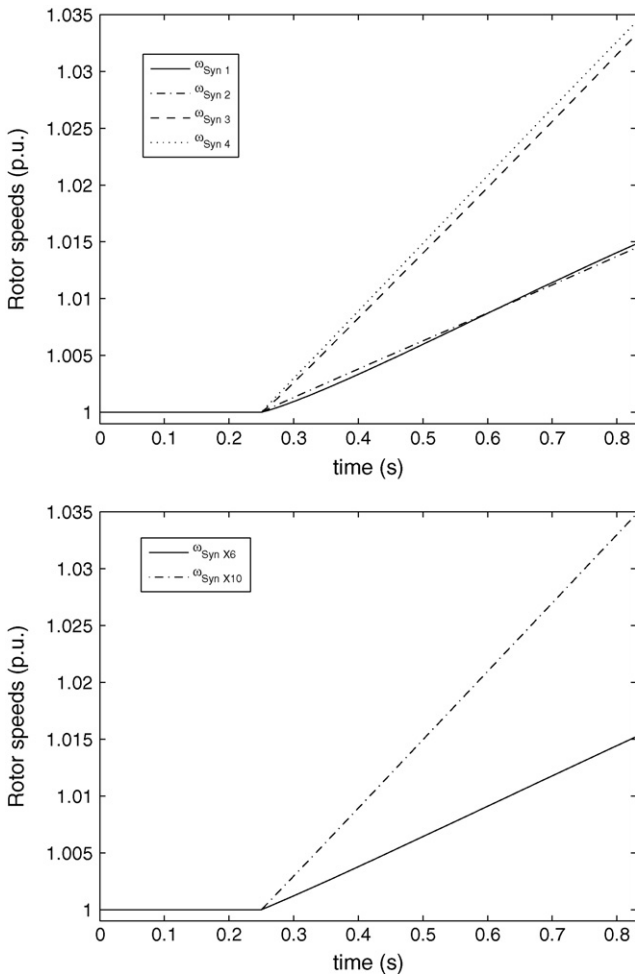


Fig. 8. Kundur's system: rotor speeds following a 400 ms fault at bus 9 for the original and the reduced networks.

3.1. Kundur's 2-area test case

The original Kundur's 2-area test system [57] is depicted in Fig. 4, while Fig. 5 shows the reduced Kundur's 2-area system used in the case study. XBus 6 and XBus 10 are the external equivalent buses of buses 1, 2 and 5, and 3, 4 and 11, respectively. For the power flow analysis, we assume that fictitious PV generators are connected at these external buses. Lines Bus 6–XBus 6 and Bus 10–XBus 10 are fictitious lines created by the network reduction procedure. Synchronous machines at buses XBus 6 and XBus 10 are also computed by the equivalencing procedure.

The first step in the equivalencing procedure is to compute the impedances of the equivalent lines Bus 6–XBus 6 and Bus 10–XBus 10. In this case, due to the radial structure of the network, the fictitious lines connected to buses 6 and 10 coincide with the original connections. The proposed procedure gives:

$$\bar{z}_{fg} = 0.000204 + j0.011913 \text{ p.u.}$$

while the STAPOW procedure gives:

$$\bar{z}_{fg} = 0.000527 + j0.027130 \text{ p.u.}$$

The impedance obtained with the proposed method differs from the one obtained with the STAPOW technique because the REI equivalents do not contain the internal transient impedance of synchronous generators. By adding this impedance, the two techniques provide same results. The REI equivalencing procedure gives two radial connections for each external network, since each network contains two generators. Then the node aggregation reduces the two radial connections into a single fictitious line.

The active powers and the voltages at the frontier buses 6 and 10 for the original and the reduced network are shown in Table 1. For the STAPOW and the proposed REI-based equivalencing techniques, it is assumed that equivalent PV generators are connected the fictitious buses XBus 6 and XBus 10. STAPOW PV equivalents take into account losses of the external network, while voltages take into account the voltage drop in the fictitious lines so that power flow results of the original and the reduced networks coincide. In the case of the proposed technique, PV equivalents do not contain the internal generator impedances, thus power injections and voltage values are closer to the original network than the values obtained with the STAPOW procedure.

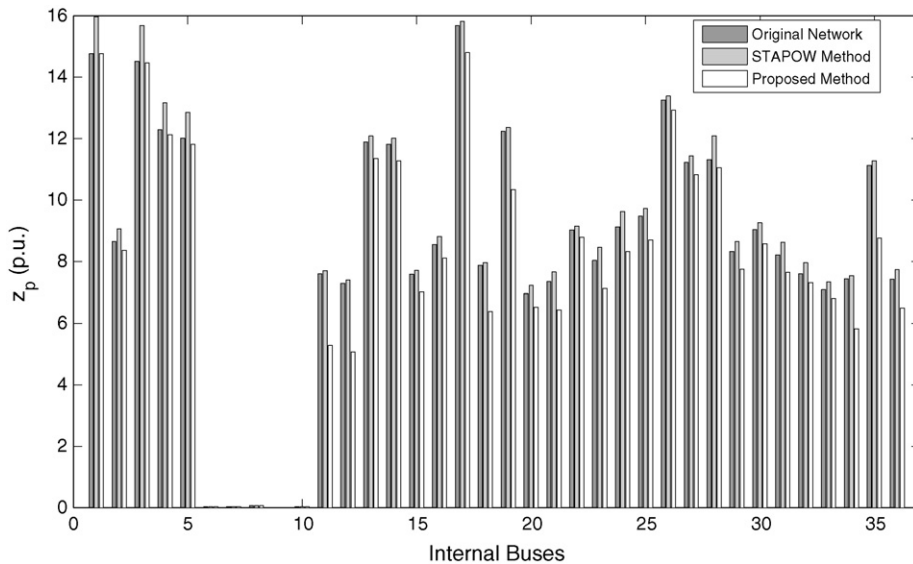


Fig. 9. Real-size system: positive sequence impedances at the retained buses for the original and the reduced networks.

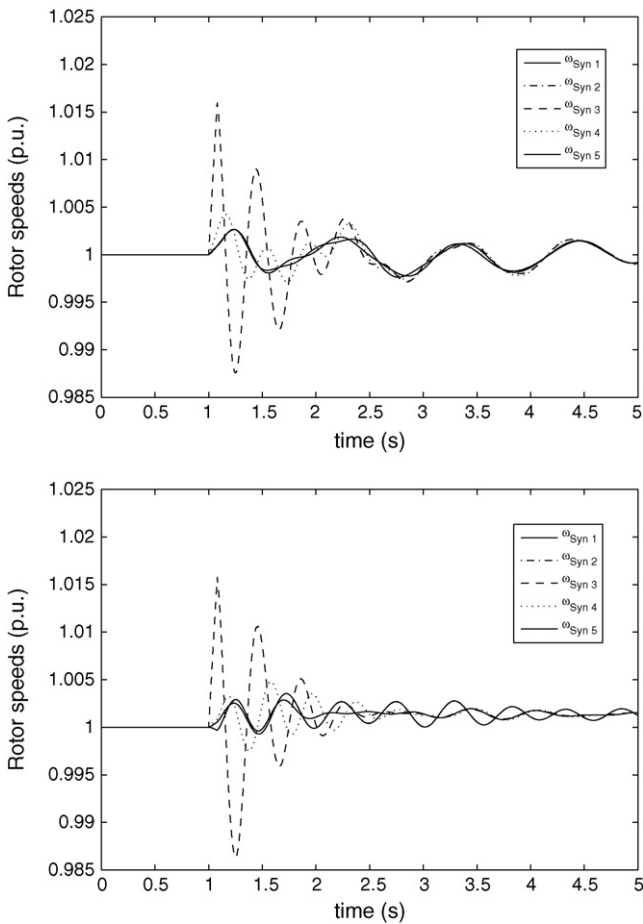


Fig. 10. Real-size system: internal generator rotor speeds following a 80 ms fault at an internal bus for the original and the reduced networks.

The results of the short circuit analysis of the full and the reduced network are shown in Tables 2 and 3. These tables show the positive and the zero-sequence short circuit impedances at the retained buses. Results of the STAPOW and the proposed equivalencing techniques match very well the short circuit impedances of the original network, being the STAPOW method gives slightly better results than the proposed one. However, the advantage of the proposed REI-based approach is that the resulting reduced network contains generator and controller dynamic data of the equivalent machines connected to the frontier buses. In this simple case study, since all AVR have the same model and same parameters, AVRs are also included in the equivalent network.

Figs. 6–8 show the results of time domain simulations for the original and the reduced Kundur’s system. In particular Figs. 6 and 7 depict the generator rotor speeds and the bus voltage magnitudes, respectively, that follow an 80 ms three-phase fault at bus 8. The rotor speeds of the equivalent Kundur’s system are an acceptable mean value of the rotor speeds of the original network. Voltage magnitudes of the equivalent system reliably reproduce the behavior of the voltage of the original system.

Fig. 8 shows the generator rotor speeds that follow a 400 ms three-phase fault at bus 9. Thus, the original systems loses the synchronism. The reduced system is able to reproduce the instability of the original system.

3.2. Real-size network

This section presents and discusses the results obtained with the proposed equivalencing procedure for a real transmission system.

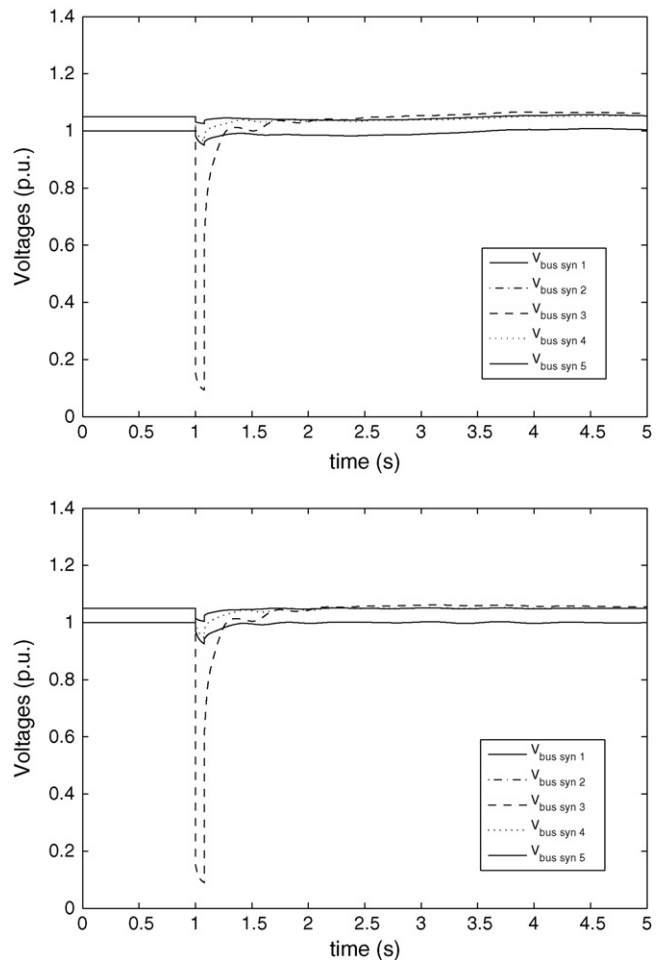


Fig. 11. Real-size system: bus voltage magnitudes following a 80 ms fault at an internal bus for the original and the reduced networks.

The system contains 1213 buses, 1691 transmission lines and transformers, and 140 generators. The reason for including the 1213-bus system in the paper is to prove that the proposed equivalencing method can be efficiently applied to real-size power systems.

The reduced model contains 36 buses, 16 of which are internal buses and 20 frontier buses, 51 lines and 5 generators. The equivalent procedure adds seven equivalent synchronous generators at the frontier buses.

Fig. 9 depicts the positive sequence impedance for the 36 retained buses of the reduced network. The results of the proposed technique are compared with the results obtained for the original full 1213-bus network and the STAPOW equivalencing technique (see Appendix B). The proposed equivalencing technique provides overall good results.

Figs. 10 and 11 depict the rotor speeds and the bus voltage magnitudes, respectively, of the five internal generators of the original system and the equivalent one. The time domain simulation shows the effect of a three phase fault at bus 3. Fig. 10 shows a good correspondence in the first 500 ms after the fault. After that, the frequency of the rotor speeds of the original system is lower than the frequency of the machines of the equivalent system. This is in part due to the

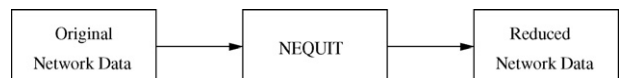


Fig. A.1. Scheme that illustrates the basic NEQUIT functioning.

stiffness effect (see Section 2), in part to the fact that the equivalent system does not include equivalent AVRs and turbine governor of the equivalent generators. However, for transient stability studies, only the first hundreds of milliseconds after the faults are relevant. Fig. 11 shows that the correspondence of the voltage of the original and the equivalent system is fairly acceptable during the whole simulation. This result suggests that for long term studies (e.g. voltage stability studies) modeling AVRs and turbine governor is not really relevant, given that the main dynamics of the internal system and at border buses are preserved.

#### 4. Conclusions

This paper proposes a systematic approach for dynamic power system equivalents based on REI approach and power transfer distribution factors and node aggregation. Test results show that the proposed technique is robust and provide a good approximation of the original network. A relevant contribution of the paper is that the proposed technique does not need the computation of eigenvalues of the state matrix of the system. This fact makes the proposed tool suitable for large networks. A byproduct of the proposed technique is a flexible and extensible software code (NEQUIT) for computing static and dynamic equivalents.

The proposed equivalencing procedure for further development. For example, the node aggregation of AVRs, turbine governors, PSS and other synchronous machine controllers is worth of further investigation. Furthermore, from the literature review, it is clear that most efforts in synthesizing dynamic equivalents have been devoted to transient stability studies. It could be interesting to search suitable equivalencing techniques for voltage stability (e.g. saddle-node bifurcations, voltage collapse) and angle stability analysis (e.g. Hopf bifurcations).

#### Acknowledgment

Federico Milano wishes to thank ABB AB Corporate Research, SE-721 78, Västerås, Sweden, that has supported this work.

#### Appendix A. NEQUIT Software Tool

The NETwork EQUIvalenT toolbox (NEQUIT) is a set of Matlab scripts and functions aimed at computing static and dynamic equivalents of power system network. The input and output network data are in SIMPOW format (see Fig. A.1). NEQUIT is written in Matlab 7 and Perl script languages. Matlab is used for computing equivalents, while Perl scripts are used for reading SIMPOW data files.

Roughly speaking, NEQUIT converts an input file in SIMPOW format into another one. The conversion can be done interactively through a graphical user interface (GUI) or off-line, through a command line. The user interface is completely independent from the computation core functions. The GUI is shown in Fig. A.2.

NEQUIT offers several options for the selection of the internal network, based on the voltage level, area, voltage threshold or custom bus list. It is also possible to define a bus “depth”, so that the given bus list is expanded spanning border buses up to a given distance. Both the GUI and the command line version are highly customizable.

On the basis of the results discussed in Section 3 and of previous experience of existing power system software firms, it appears reliable to design Matlab and Perl scripts that import data from SIMPOW, process the information in the desired manner and export the information back into the SIMPOW. The result is a plain text SIMPOW data file, that can be exported to any other power system software. The NEQUIT project proves that this strategy is not only cost effective but also allows quickly implementing, improv-

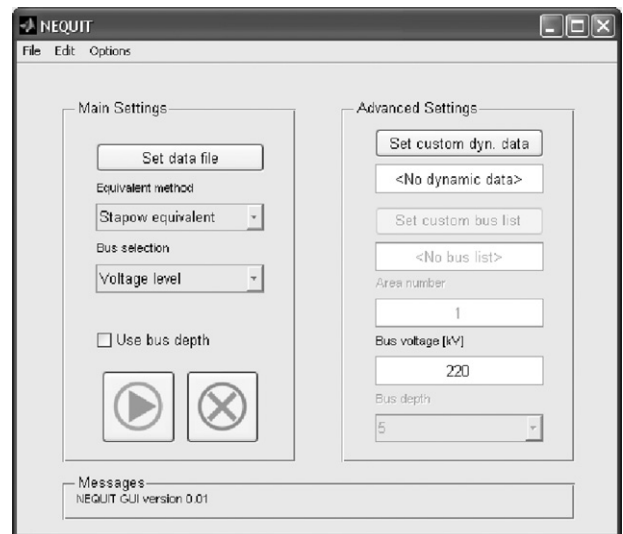


Fig. A.2. Main NEQUIT graphical user interface.

ing and extending the procedures that are currently available in the literature.

#### Appendix B. Outlines of the STAPOW equivalencing procedure

The SIMPOW module that computes short circuit analysis and network equivalents is STAPOW. The equivalent of the network is calculated by a sub-program from positive, negative and zero sequence data and from information from the STAPOW file such as removed elements, internal nodes and the output functions.

Internal nodes are the nodes which are retained in the positive, negative and zero sequence network, i.e. nodes at which faults and other symmetrical or unsymmetrical elements can be connected.

Elements can be removed from the network before the calculation of the equivalent, e.g. a line can be removed in the equivalent and a line with new data can be inserted before the calculation of the static unsymmetrical state with or without faults.

An output function is the positive and zero sequence ac-voltages on a node in the equivalent which is not an internal node or injected positive and zero sequence currents in an element in the equivalent. Observe that the negative sequence data can be defined only for generators. In the output, the negative sequence is assumed to be equal to the positive one.

The pictorial representation of the procedure that implements the STAPOW equivalents is shown in Fig. B.1. Each equivalent system is composed of a fictitious external node  $g$  connected to the frontier bus  $f$  by a positive sequence impedance  $\bar{z}_{fg}$ . Furthermore, the equivalent is typically also composed by some fictitious impedances that connect the frontier bus  $f$  with other internal buses, represented by  $A_1, A_2$  and  $A_n$  in Fig. B.1.

In order to set up equivalent network data, the fictitious buses  $g$  are modeled as PV generators. The power injected by each PV generator is determined by the total net complex power  $\bar{s}_{gf}$  that flows from node  $g$  to  $f$ . This power is computed as the difference of the power balance at node  $f$  as results from the base case power flow solution and the power flows in the fictitious lines  $f-1, f-2, \dots, f-n$  determined by STAPOW:

$$\bar{s}_{gf} = -\sum_h \bar{s}_{fh} - \sum_{i=1, \dots, n} \bar{s}_{fi} \quad (\text{B.1})$$



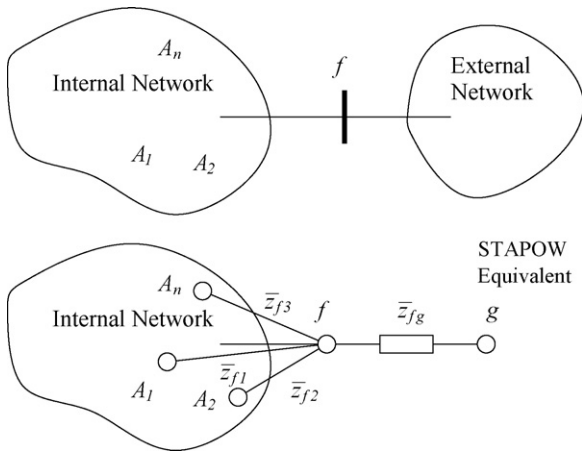


Fig. B.1. STAPOW equivalent.

where  $h$  is the set of internal nodes that are connected to node  $f$ . Finally, the voltage  $\bar{v}_g$  at the fictitious bus  $g$  is

$$\bar{v}_g = \bar{v}_f + \frac{\bar{S}_{gf}^*}{\bar{v}_f^*} \quad (\text{B.2})$$

and the fictitious injected power  $p_g$  at bus  $g$  is

$$p_g = \Re \left\{ \bar{v}_g \frac{\bar{S}_{gf}^*}{\bar{v}_f^*} \right\} \quad (\text{B.3})$$

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