

# Extraneous Instabilities Arising in Power Systems with Non-Synchronous Distributed Energy Resources

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## Abstract

This short communication describes an extraneous instability that can be observed when solving time domain simulations for power systems with inclusion of non-synchronous distributed energy resources such as those based on voltage source converters and asynchronous generators. The instability object of the paper is caused by the interaction of (i) synchronous machines modeled using a synchronous reference speed and (ii) non-synchronous generators whose controllers depend on a  $d$ - $q$  transformation. The paper also provides two simple solutions able to remove such extraneous instabilities. The New England 39-bus benchmark system is used for testing the proposed solutions.

*Keywords:* Distributed energy resources, center of inertia, phase-locked loop, voltage-source converter, doubly-fed asynchronous generator, limit cycle.

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## 1. Introduction

In recent years, power systems have experimented drastic changes in terms of both operation, i.e., electricity markets, and technology, i.e., distributed and/or renewable generation. The latter has introduced in the electric grid a variety of “unconventional” generating devices [1]. Most of these new devices are asynchronous, e.g., wind turbines with induction generators, or fully decoupled from system frequency through voltage source converters (VSCs). Examples of such non-synchronous devices are wind turbines direct-drive synchronous generator and dc devices (photo-voltaic cells, fuel cells, etc.). The main consequence of these deep changes in the nature of power system devices may lead to unexpected interactions between conventional and unconventional devices. Thus there is the need of carefully revising both modeling and simulation tools in order to avoid such undesired interactions.

This short communication describes an extraneous instability that can occur in power system including both conventional synchronous generators and distributed energy resources. The instability is a consequence of the incorrect interaction between synchronous machines modeled based on the synchronous reference speed and the  $d$ - $q$  axis transformation used for controlling non-synchronous distributed energy resources, which depend on the absolute phase of the voltage of the point of common connection. The oscillatory instability shown during time domain simulations is due to an improper modeling, not to the occurrence of a bifurcation or to a poor regulation. The paper proposes two solutions: (i) the use of the center of inertia during time domain simulations; and (ii) a reformulation of the interface equations of

distributed energy resources. The latter has the advantage of being computationally lighter than the center of inertia.

The paper is organized as follows. Section 2 briefly outlines basic electromechanical equations of synchronous generators as well as the concepts of synchronous reference speed and of center of inertia. Section 3 describes the main equations that couple non-synchronous distributed energy resources to the main grid. Section 4 illustrates the case studies. For the sake of space, the case study only shows an example with wind-turbine doubly-fed asynchronous generators. Finally, Section 5 draws relevant conclusions.

## 2. Synchronous Machines, Synchronous Reference Speed and Center of Inertia

The equations that link synchronous generators  $d$ - $q$  axes with the terminal voltage are as follows:

$$v_d = v_h \sin(\delta - \theta_h) \quad (1)$$

$$v_q = v_h \cos(\delta - \theta_h) \quad (2)$$

where  $v_h$  and  $\theta_h$  indicate the terminal bus voltage magnitude and phase, respectively;  $\delta$  is the machine rotor angle; and  $v_d$  and  $v_q$  are the terminal voltage components on the machine  $d$ - $q$  axes. Active and reactive power injections at the terminal bus are:

$$p_h = v_d i_d + v_q i_q \quad (3)$$

$$q_h = v_q i_d - v_d i_q \quad (4)$$

where  $i_d$  and  $i_q$  are the components of the machine stator current on the  $d$ - $q$  axes. Finally, electromechanical equations:

$$\dot{\delta} = \Omega_b(\omega - \omega_0) \quad (5)$$

$$\dot{\omega} = (\tau_m - \psi_d i_q + \psi_q i_d - D(\omega - \omega_0))/2H \quad (6)$$

where  $\psi_d$  and  $\psi_q$  are the components of the stator fluxes on the  $d$ - $q$  axes;  $\omega$  is the rotor speed;  $\Omega_b$  is the system fundamental frequency in rad/s;  $H$  and  $D$  are the machine inertia constant and damping, respectively;  $\tau_m$  is the mechanical torque; and  $\omega_0$  is the synchronous reference speed. Associated with the reference speed  $\omega_0$ , there is implicitly a constant reference angle  $\delta_0$  to which all machine rotor angles as well as all bus voltage phases refer.

The synchronous reference speed  $\omega_0$  pertains to an ideal infinite inertia machine that provides a reference frame to all other machines. The introduction of such ideal machine is not strictly necessary since the rotor speed of any machine of the system could be used as reference. However, to define an exogenous speed (and angle) reference is a common practice. The main advantage of doing so is that the speed reference is independent on the system, is computationally inexpensive and can be straightforwardly implemented [2]. On the other hand, any deviation of machine rotor speeds with respect to  $\omega_0$  causes rotor angles to move away from the initial value. Consider, for example, that as a consequence of a line outage, transmission losses increase and, thus, the final equilibrium point is characterized by rotor speeds slightly lower than  $\omega_0$ . Then, from (5), rotor angles decrease indefinitely.

To avoid the drift of rotor angles, a common solution is to substitute  $\omega_0$  in (5) and (6) for the speed of the center of inertia (COI), which is a weighted

mean of all generator rotor speeds, as follows:

$$\omega_{\text{COI}} = \frac{\sum_{j=1}^n H_j \omega_j}{\sum_{j=1}^n H_j} \quad (7)$$

where  $n$  is the number of synchronous machines connected to the grid. The COI rotor angle  $\delta_{\text{COI}}$  is defined in same manner as  $\omega_{\text{COI}}$ .

The COI allows avoiding the machine rotor angle drift and, especially in case of long term time domain simulations, allows unequivocally visualizing the trajectories of generator rotor angles. The only issue when using the COI is that it introduces two extra equations and that it reduces the sparsity of the system Jacobian matrix, thus slightly increasing the computational burden of time domain simulations.

### 3. Non-Synchronous Devices

The main difference of non-synchronous devices with respect to synchronous machines is that for the formers there is no unique definition of the  $d$ - $q$  axis, since their equations do not depend on the rotor angle  $\delta$  (in VSC-based systems, there is no such angle at all). Hence, the  $d$ - $q$  axis components of the terminal voltage become:

$$v_d = v_h \sin(-\theta_h) = -v_h \sin \theta_h \quad (8)$$

$$v_q = v_h \cos(-\theta_h) = v_h \cos \theta_h \quad (9)$$

Power injections are the same as (3) and (4). The  $d$ - $q$  axis transformation is required because of the regulators (for example, see [1] for the description of most common wind turbines and VSC controls) and is obtained in practice by means of the phase-locked loop (PLL).

The PLL measures the system frequency and feeds the  $d$ - $q$  transformation block with the phase synchronous angle  $\theta_h$  signal, more precisely  $(\sin \theta_h, \cos \theta_h)$ . In steady-state,  $\theta_h$  is in phase with the fundamental positive sequence of phase  $A$  of the terminal voltage of the point of connection.

During transients, (8) and (9) follows the trajectory of  $\theta_h$ . This behavior is fine if  $\theta_h$  does not move “too much” from its initial value (say,  $\theta_h^0$ ) computed for  $t = 0$ , i.e., at the power flow solution. However, if  $\theta_h(t)$  drifts beyond  $2\pi$ , as it could be the case of using the synchronous speed reference  $\omega_0$ , issues can show up. In particular, wind turbine and VSC controllers can become inconsistent if their internal  $d$ - $q$  axis reference signals are not updated.

One simple way to solve the issue described above is rewrite (8) and (9) as follows:

$$v_d = -v_h \sin \theta_h^0 \tag{10}$$

$$v_q = v_h \cos \theta_h^0 \tag{11}$$

Of course, using a constant angle  $\theta_h^0$  is arbitrary, but  $\theta_h(t)$  is also arbitrary, so using  $\theta_h^0$  does not introduce any simplification. Moreover, (10) and (11) better model the behavior of the PLL than (8) and (9). In fact, the PLL is not locked to the synchronous reference angle  $\delta_0$ . Finally, (10) and (11) are computationally lighter than (8) and (9) since  $\sin \theta_h^0$  and  $\cos \theta_h^0$  are constant.

#### 4. Case Study

This section presents a case study based on the well-known New England 39-bus system. The original data include 10 fourth-order synchronous machines with AVR, turbine governor and PSS. The base case system is

modified in order to include distributed energy resources. In particular, 5 wind parks are added to buses 1, 9, 14, 17 and 22. The total wind power generation is 10% of the base case load and is equally spread among the five buses. In order to maintain the power balance and not to overload existing lines, 5 loads are connected to the wind park buses. The demand of each additional load is exactly the same as the wind park power production at that bus. This choice is made by purpose, so that the transient behavior of the modified New England system is very similar to that of the base case without wind parks. The considered disturbance is a self-clearing fault that occurs at bus 16 at  $t = 0.5$  s and that lasts 0.2 s. All simulations have been carried out using a Python-based version of the software package PSAT [3].

The initial power flow solution is a stable equilibrium point. Moreover, since the fault is cleared without topological changes, the final post-disturbance equilibrium point is the same as the initial one (assuming that no machine goes out step, which is not the case in this example). So, the transient behavior of the system is expected to be stable. Figure 1 shows the trajectory of the rotor speed of synchronous generator 1 connected to bus 39. Surprisingly enough, the time domain simulation obtained using the synchronous speed reference  $\omega_0$  and (8) and (9) ends up in a limit cycle. Such oscillatory instability is a consequence of the interaction of wind turbine controllers that follow the time evolution of  $\theta_h(t)$ . During the first seconds after the fault,  $\theta_h(t)$  moves far away from the initial value  $\theta_h^0$ . Wind turbines controls attempt to counteract to such variation but, while doing so, increase the overall system losses. Synchronous machines respond to such loss increase by decreasing their rotor speeds, and this counter action causes rotor angles

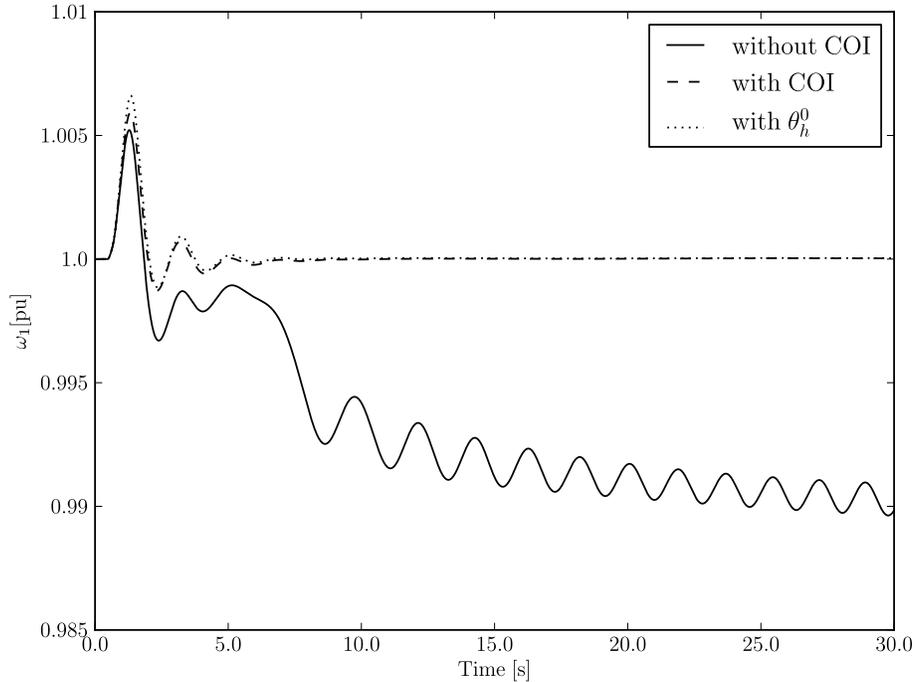


Figure 1: Rotor speed of the machine 1 connected to bus 39 for the modified New England 39-bus system with inclusion of wind turbines.

and thus bus voltage phases to further drift apart. However, if bus voltage phases do not drift, the system trajectory is stable, as expected. As a matter of fact, this is the case if using the COI or, alternatively, (10) and (11). Stable trajectories are also shown in Fig. 1. Observe that the simulation obtained using the COI and that obtained using (10) and (11) provide practically the same solution. However, (10) and (11) lead to a model that is independent of the speed reference used by synchronous machines. Moreover, using (10) and (11) avoids the extra computational effort required by the COI.

## 5. Conclusions

This short communication focuses on a peculiar extraneous instability that may occur in power systems with inclusion of generators that are either asynchronous or decoupled from the system frequency. The paper proposes two solutions: (i) the use of the well-known center of inertia; and (ii) a reformulation of the  $d$ - $q$  axis transformation of non-synchronous generators. The latter has the advantage of being more general and computationally lighter than the center of inertia. Simulation results discussed in the case study show the effectiveness of the proposed modeling solutions.

## References

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