Challenges of Decentralized Stochastic Control of Discrete Flexible Resources

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***Abstract*—This work discusses the stochastic control of distributed discrete flexible resources (DFRs), such as loads, small energy sources, and EV chargers. The work shows that demand response strategies must be adapted to the operating conditions and coordinated with the system operator. The size and number of DFRs, as well as the inertia available in the system, are shown to be relevant parameters for tuning stochastic control. The paper also shows that the fairness of decentralized controllers depends on the aforementioned parameters. Fairness is estimated based on the stationarity of the stochastic processes driven by DFR switching. Potential solutions, such as coordination among DFRs, are discussed and validated through simulations carried out with modified versions of the WSCC 9-bus test system and the all-island Irish transmission system.**

***Index Terms*—Power system dynamics, stochastic control, decentralized control, discrete flexible resource (DFR).**

I. Introduction

Over the last decade, flexible resources have become an important support for power systems to provide, among other services, load shaving, energy savings, and demand response [1]. In this work, we focus on the frequency support provided by discrete flexible resources (DFRs).

Studies related to DFRs are primarily divided into two areas: the analysis of individual DFRs' types and their characteristics and the application of DFRs. Regarding the analysis of characteristics of different load types, references [2]–[5] explore the attributes of DFRs and techniques to model small DFRs to predict their behavior, such as the duty cycles, typical responses, limitations, and applications on residential cooling and heating loads and the impact of direct load control on customer comfort. Regarding applying DFRs, [6] reviews international experiences with end-use loads providing ancillary services, suggesting effective market designs for broader customer load participation in wholesale markets. This contribution aligns with [7] and [8], which explore frequency-responsive appliances, the use of bitumen tanks [9], residential, small generators and appliances coordination contributing to load shedding [10].

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The potential of flexible load control as a resource for balancing generation and demand, especially in ancillary services, is discussed [11] and [12]. These studies highlight technical and economic challenges and the untapped potential of DFRs to enhance grid stability and frequency control, reducing reliance on backup generation. Analyses of DFRs in primary frequency control are presented in [13] and [14], which focus in particular on decentralized characteristics. In [15], the stochastic control of DFRs is combined with energy storage systems to smooth the overall system response and, at the same time, reduce the required size of storage systems.

This paper addresses two issues of the stochastic control of discrete resources that have not been discussed so far, and that appear critically important when designing and operating a power system with DFRs. The first issue is that the flapping phenomenon, i.e., undesirable cycling leading to wide power oscillations, cannot be guaranteed to be avoided with stochastic control. Most DFRs' participation studies assume that the total capacity of DFRs is fixed and small compared to the system's size [11], [12], [16], [17]. However, if the number of DFRs is very large or if the overall inertia of the system is low, over-responses of the DFRs, i.e., flapping, can be observed. We propose a solution based on partially giving up the decentralized nature of the stochastic control.

The second issue is the fair participation of a DFR in the frequency support based on the stationarity of its stochastic operation. We show that, for a DFR to reach fair participation compared to other DFRs, it first requires to complete a certain number of operations. We also show that stationarity requires time to be reached [18] and is associated with the *fairness* of DFR's operations.

II. Challenges of Stochastic Control

A simple yet general decentralized stochastic control of DFRs is as follows [15]. Assume the active power consumption/generation of the i-th DFR is:

$$p_i(t) = \kappa_i(t)\, p_{i,o} \,, \tag{1}$$

where $p_{i,o}$ is a fixed amount of power and $\kappa_i(t)$ models the switching logic of the stochastic control, as follows. Let

$$u_i(t) = \frac{\Delta\omega(t) + \Delta\omega_{\rm thr}}{2\Delta\omega_{\rm thr}} \,, \tag{2}$$

where $\Delta\omega$ is the measured frequency deviation of the system, and $\Delta\omega_{\rm thr}$ is the frequency deviation threshold such that above/below, all DFRs reserve will be switched on/off. Once the value of u_i is determined, each DFR independently generates a random number between 1 and 0 using a uniform distribution, say $X \sim \mathcal{U}(0,1)$. Then the flexible load will switch on or off if:

$$\kappa_i(t) = \begin{cases} 1 & X \leq u_i(t)\,, \\ 0 & \text{otherwise}\,. \end{cases} \tag{3}$$

The evaluation of the frequency in (2) is repeated at fixed time intervals, say Δt. A minimum time up or minimum time down can be enforced to prevent the DFR from switching on/off too often. Different time operations of DFRs can be considered without the loss of generalization of the methodology.

A. Dependency of the Stochastic Control on Operating Conditions

While the implementation described above is the one commonly found in the literature, we argue that this implementation is incomplete and might not work correctly depending on the operating condition of the system and the total capacity provided by the DFRs. Let us consider a simple qualitative example to illustrate this point. Consider a frequency deviation that, according to the logic above, will lead to a switch of 100% of the available DFRs. If their total capacity is comparable with the power unbalance that occurred in the system, the action of the DFRs is acceptable, but if their total capacity is double the power unbalance, then the stochastic control will lead to a flapping phenomenon as the switching of the DFRs creates a frequency variation of the same size caused by the initial power unbalance but with opposite sign. Moreover, the amplitude of a frequency variation depends not only on the power unbalance but also on the inertia and the fast frequency control available in the system. If the capacity of the DFRs is large enough, thus, the operating condition of the system has to be taken into account to adjust the amount of power that is switched following an event.

The issues above can be solved, as we propose in this paper, through a proper calibration of the switching logic (2). Let us assume that there are n DFRs in total and that the switching logic (3) is adjusted to respond properly for this number of DFRs. Then, at any given time, the number of available DFRs is $k \leq n$. Let us also assume that the switching logic (3) is calibrated based on reference inertia available in the system, say $m_{\rm ref}$, and m is the inertia available at the time of the measurement of the frequency deviation $\Delta\omega$, then (3) is adjusted as follows:

$$\kappa_i(t) = \begin{cases} 1 & X \leq \alpha(t)\, u_i(t)\,, \\ 0 & \text{otherwise}\,, \end{cases} \tag{4}$$

where

$$\alpha(t) = \frac{n}{k(t)}\,\frac{m(t)}{m_{\rm ref}}\,. \tag{5}$$

The expression (5) takes into account that if the system inertia is, say $m < m_{\rm ref}$, fewer DFRs are required to switch for a given frequency variation, which is reflected by the lower probability to switch on/off in (4). While certain parameters are shared across the system, the calculated individual thresholds do not need to be identical for all DFRs as long as they are properly calibrated.

From the practical point of view, the implementation of (2) involves a series of issues. If the number of DFRs is high, e.g., hundreds of thousands or millions, it is likely impossible to have an exact real-time estimation of k or of n. Then, even if the system operator can estimate fairly accurately the inertia available in the system — for example, see [19] — it is not viable for the system operator to communicate the value of m to all DFRs. However, we observe that it is not necessary to update these parameters in real time. Moreover, according to the simulation results presented in the next section, it is not necessary to have precise estimations. We can thus assume that the coefficient α_i is updated every hour or half an hour and made available by the system operator, e.g., through a webpage on the Internet, to all DFRs.

B. Fairness of the Stochastic Control

The statistical properties of a stationary stochastic process remain constant over time. A process requires a period of time to reach stationarity, as ruled by the Fokker-Planck equation. For a linear process, such time is proportional to the autocorrelation coefficient of the process, whose functions are similar to a time constant for a linear differential equation [18].

The stochastic control of DFRs is a stochastic process and, as such, requires a certain time to become stationary. This is important because if the stochastic control is not stationary, the DFRs participating in it cannot be guaranteed to participate fairly in the frequency support. In fact, if the stochastic control is not stationary, some DFRs have switched more times than others, thus taking most of the burden of the frequency support. Reaching the stationarity of the stochastic control of DFRs implies that the participants of this control, namely, the DFRs, are treated and rewarded fairly.

This point is relevant as the time required for each stochastic DFR to reach stationarity is a function of the operating condition and, in particular, on the number of DFRs available in the grid. In particular, the higher k, the higher the time the stochastic control requires to be "fair" for each DFR. This result can be qualitatively explained as follows. Let us define ideal stationarity as the condition for which each DFR has completed the same number of on/off switchings as all other DFRs. This situation is certainly satisfied if all possible ways to choose k elements from an n-element set have occurred. The number of combinations increases rapidly as n increases, as given by the well-known binomial coefficient:

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!}\,. \tag{6}$$

On the other hand, a minimum number of operations can be estimated through the Coupon collector's problem [20], [21], where the trials $E(T_n)$ required to collect n different coupons, is defined as:

$$E(n) \approx n \log n + n\gamma + 0.5\,, \tag{7}$$

where $\log n$ is the natural logarithm of n and γ is the Euler-Mascheroni constant, approximately $\gamma \approx 0.5772$. The number of operations required by the stochastic control of DFRs to reach stationarity is anywhere in between the $E(n)$ and $\binom{n}{k}$. However, from the test that we have carried out, we have observed that the control becomes stationary approximately close to the number of operations estimated with (7).

A precise estimation of the time required to reach stationarity is not crucial. The key point is that if the required number of operations increases too much and spans multiple events in the grid, the stochastic control is never fair (stationary) and, hence, might not be appealing for loads and distributed resources to adopt it.

It is possible to decrease the time to reach stationarity in various ways. An obvious one is to reduce the evaluation time Δt and the minimum on/off times of the DFRs. However, this approach might not be feasible in practice. In this work, we consider the option to coordinate DFRs into clusters that act together, that is, the decision logic to switch on or off is common to the whole cluster. The synchronization of a cluster of DFRs can be obtained, for example, through an aggregator. This approach, in turn, is equivalent to reducing the equivalent number n of DFRs but requires a centralized control within the cluster. Once again, thus, the solution to avoid the inconveniences of stochastic control is to give up complete decentralization.

III. Case Study

In this section, the adapted demand response and the fairness of DFRs as described in Section II are evaluated using modified versions of WSCC 9-bus test system and the all-island Irish transmission system. All simulations are performed using the power system analysis software tool Dome [22].

A. *WSCC 9-Bus test system*

The WSCC 9-bus system includes 3 sets of PQ loads, connected to buses 5, 6, and 8, of 2, 0.9, and 1 pu(MW) respectively. For our purposes, loads at bus 5 are divided into three smaller groups of 1.1, 0.3, and 0.6 pu(MW). Then, $n = 20$ DFRs are connected along the system buses; their individual size is 0.01 pu(MW), representing around 6% of the total load. The operation of each DFR is evaluated every $\Delta t = 5$ s. The activation or deactivation of each DFR is not synchronized, that is, DFRs do not switch simultaneously. We use local frequency measurements, properly filtered and averaged, based on PLLs. The system's inertia estimation is calculated as the sum of the synchronous machines' inertias.

1) Frequency Support from DFRs: In this scenario, the disturbance is the disconnection of the 1.1. pu(MW) PQ load connected to bus 5 at time $t = 5$ s. As shown in Fig. 1, the system receives proper support from DFRs when a frequency deviation appears for $n = 20$ DFRs. However, if the number of DFRs triplicates, their random activation under a disturbance leads to flapping. This undesirable oscillation is also observed when the inertia of the system is 30% smaller, as shown in Fig. 2. These changing conditions of the system, either the number of operating DFRs or the inertia, cause an over-response of the demand if DFRs have the same response, which justifies the adaptive method described in Section II.

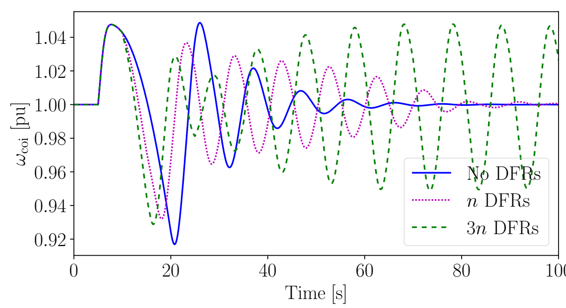

Fig. 1: WSCC 9-bus system – Sensitivity of the stochastic control to the number of DFRs.

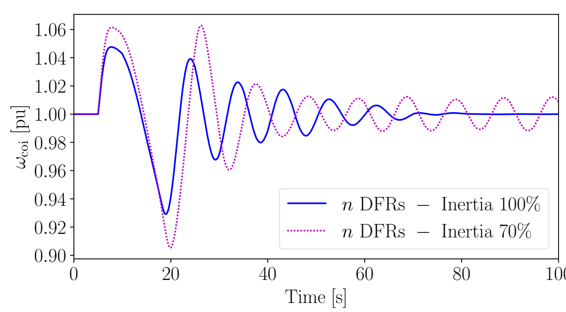

Fig. 2: WSCC 9-bus system – Sensitivity of the stochastic control to the inertia of the system.

Figure 3 shows the response of the system for the proposed adaptive stochastic control of DFRs. In this case, despite the increased number of DFRs ($3n$) and the reduced inertia (70% of the base case), the demand response adapts and returns to a scenario in which DFR supports the frequency effectively.

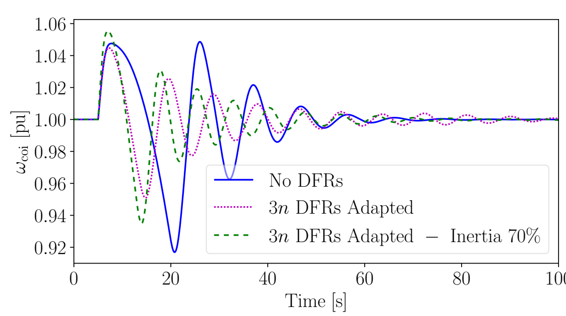

Fig. 3: WSCC 9-bus system – Adaptive stochastic control responding to the change of the number of DFRs and inertia of the system.

For the sake of illustration, we show that the transient behavior of the system can be also improved by preserving a fully decentralized stochastic control by decreasing the operating time Δt of the DFRs, creating a faster and more precise response, as seen in Fig. 4, when the DFRs operating time is changed from 5 to 1 s. However, in real applications, it is difficult to have this level of responsiveness from the demand side, making this solution highly impractical.

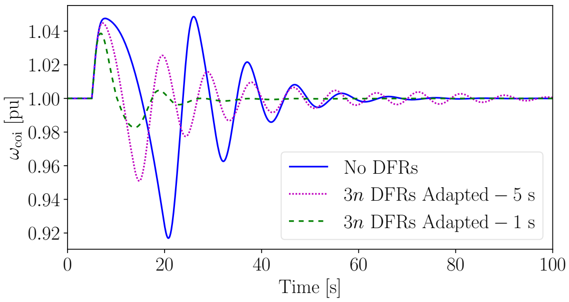

Fig. 4: WSCC 9-bus system – Sensitivity of the stochastic control of DFRs to the operation time interval.

2) Evaluation of Stationarity: We solve Monte Carlo simulations to discuss the stochastic behavior of DFRs. To perturb the system's frequency and induce a response from DFR, load consumption includes Gaussian noise modeled as Ornstein-Uhlenbeck's Processes (OUPs) [18]. OUP is a stochastic mean-reverting process that follows Gaussian distribution and has a bounded standard deviation. The OUPs are modeled as independent processes and have mean the base case value of the active and reactive powers of the loads and a standard deviation of 6% of the base case value.

To observe the dynamic behavior of the statistical properties of a stochastic process, we use the autocorrelation of the process determined by the comparison of the switching of a given DFR with respect to the whole population of DFRs and assume that the process is stationary when the autocorrelation, computed starting at a given time, approaches zero.

Figure 5 shows the autocorrelation of the operation of a single DFR for $n \in \{20, 80, 160\}$ DFRs participating in the decentralized stochastic control of the 9-bus System. As expected, the times with which the autocorrelations of the various scenarios reach zero and, hence, a "fair" participation of each DFR is attained, increases as the number of DFRs available in the system does.

B. All-island Irish transmission system

In this section, we use a dynamic model of the all-island Irish transmission system to demonstrate the performance of the stochastic control with and without the proposed solutions in a real-world grid. The model includes 1479 buses, 1851 lines, 5 conventional power plants and 302 wind power plants with a base case load of 1.8 GW. Note that this operating condition and the following dynamic response, while realistic, do not represent those of the real system. In all cases, DFRs evaluate their participation every $\Delta t = 5$ s, and the evaluation time is randomized in this interval.

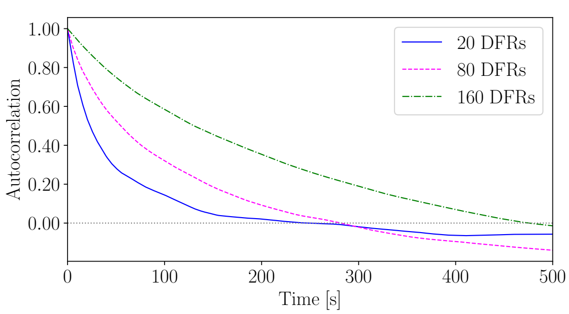

Fig. 5: WSCC 9-bus system – Sensitivity of the autocorrelation of the stochastic behaviour of a DFR to the number of DFRs present in the system.

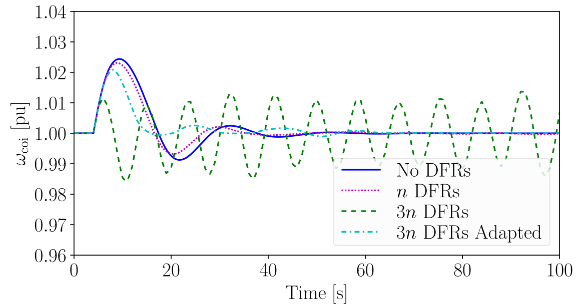

Fig. 6: Frequency response of the all-island Irish system under different DFR setups.

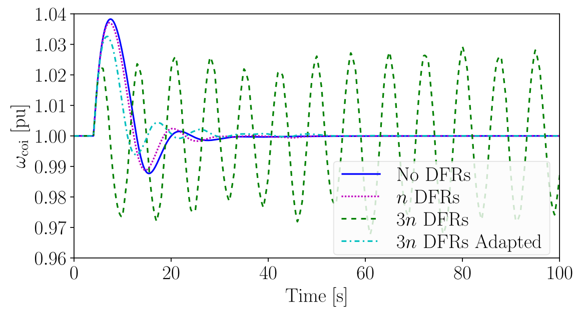

Fig. 7: Frequency response of the all-island Irish system for an operating condition with 30% less inertia than in the base case scenario.

1) Frequency Support from DFRs: We consider four scenarios: a base-case system without DFR frequency support, a scenario with 1500 0.001 pu(MW) DFRs, and two scenarios with conventional and adapted 4500 0.001 pu(MW) DFRs, which account for around 16% of the system's load. The contingency is a disconnection of an equivalent load of 7% of the system. Figure 6 shows that the dynamic performance of the model of the Irish system with the frequency support provided by the 1500 DFRs improves that of the base case. However, if the number of DFRs increases to 4500, the DFRs cause flapping. Flapping is removed using the proposed adjustment of the logic of the stochastic control, which, however, has to be coordinated by the system operator.

Figure 7 shows the results obtained for a system with 30% less system inertia with respect to the base case and various DFR setups. As expected, if the inertia decreases, a fully decentralized stochastic control of DFRs might not work well, whereas the proposed adjustment leads to an acceptable dynamic performance.

2) Evaluation of Stationarity: Following the same approach as for the WSCC 9-bus system, we execute Monte Carlo simulations. The perturbations consist of Gaussian noise applied

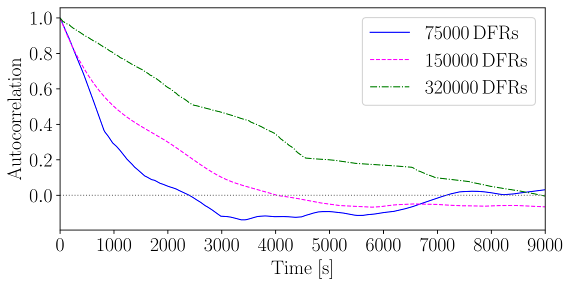

Fig. 8: Effect of number of DFRs on the autocorrelation of a single DFR.

to load consumption. The noise is modeled as OUPs, with a standard deviation of 6% of the base case value.

Figure 8 shows the autocorrelation of the operation of DFR_k for different numbers of DFR participating in the decentralized stochastic control of the Irish power system. Again, the times with which the autocorrelations for the various scenarios reach fairness of each DFR increases as the number of DFRs available in the system increases.

Finally, in Fig. 9, we consider the following three scenarios for the operation of DFRs included in the Irish system: (i) $n = 50000$ DFRs and $\Delta t = 5$ s; (ii) $n = 500$ DFRs clusters distributed along 500 busses of the system, each cluster composed of 100 DFRs, and $\Delta t = 5$ s; and (iii) $n = 50000$ DFRs and $\Delta t = 1$ s. For simplicity, we assume that all DFRs have an equal size equal to $p_i = 10^{-5}$ pu(MW).

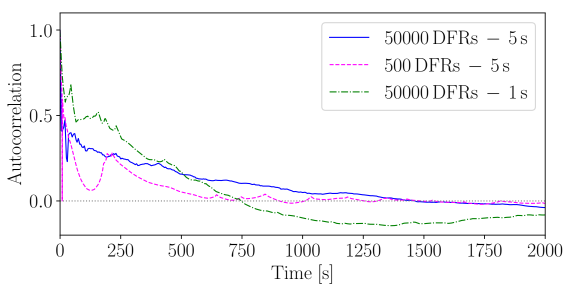

Fig. 9: Effect of clustering on the autocorrelation of a single DFR.

Comparing scenarios (i) and (ii), we observe that the coordinated operation improves the stationarity time from 1750 s to 1200 s. Similarly, scenario (iii) shows a significant decrease in the time to reach stationarity. The decrease is directly proportional to Δt, as expected. Both scenarios (ii) and (iii) allow effectively decreasing the time to stationarity and the two approaches can be combined to improve the fairness of the stochastic control further. However, as stated in Section II, using aggregators might be a more realistic and viable solution than reducing the on/off time of the DFRs.

IV. Conclusions

This paper demonstrates that stochastic control must be carefully calibrated and dynamically adapted in coordination with the system operator to ensure DFRs' efficient contributions to power system stability. Furthermore, the paper shows that with high penetration of DFRs, stochastic control might not be fair due to the long time it requires to be stationary. The main conclusion of this work is that complete decentralization of discrete DFRs is not possible for power systems with high DFR penetration.

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