

# System-wide Dynamic Performance Metric for IBR-based Power Networks

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**Abstract**—In power networks based on Inverter-Based Resources (IBRs), fast controllers cause frequency and voltage dynamics to overlap. Thus, it becomes critical to assess the overall dynamic performance of such networks through a combined system-wide metric. This letter presents a unified metric designed to evaluate dynamic performance in such cases. The proposed metric consists of a weighted sum of local voltage phasor variations at each bus, where the weights are the complex powers injected at the buses. The proposed metric is further decomposed into device-driven and network-driven components, enabling a more comprehensive assessment of grid dynamics. A case study based on a modified version of the IEEE 39-bus system is presented, in which synchronous machines are replaced by inverter-based resources. A sensitivity analysis of the  $R/X$  ratio is utilized to evaluate the metric in conventional grids, as well as in those characterized by strong voltage-frequency coupling with complex power flows.

**Index Terms**—Center of inertia, complex frequency, power system dynamic performance.

## I. INTRODUCTION

Voltage in AC power systems is commonly described as a phasor, defined by two fundamental components: magnitude and frequency. Historically, these components have been assessed through different paradigms. Frequency has traditionally been treated as a global quantity, represented by aggregated metrics such as the frequency of the Center of Inertia (CoI) to evaluate overall dynamic performance [1]–[7]. Nevertheless, recent research has redefined frequency as an inherently local and instantaneous variable [8], which is a key feature in grids with high shares of IBRs, where dynamics become faster and heterogeneously distributed.

To calculate the CoI frequency, techniques for identifying power system's inertia are fundamental, and have been categorized into model-based and measurement-based approaches [2], [3]. Model-based methods rely on dynamic models and parameter estimation, while measurement-based techniques are subdivided into large-disturbance methods, such as estimating inertia from transient Rate of Change of Frequency (RoCoF) and power imbalance, and ambient-data methods, such as extracting inertia from continuous small fluctuations [1], [4]. Some implementations include robust Kalman filtering for online CoI frequency estimation [5] and direct algebraic calculation of the CoI frequency as a weighted sum of bus frequencies [6].

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On the other hand, voltage magnitude has been studied as a local quantity. As IBRs cause the time scales of voltage magnitude and frequency dynamics to overlap, the question arises whether it is relevant to define a system-wide, aggregated metric for voltage magnitude and frequency. While frequency serves as an effective index to assess power imbalances, the literature has expanded the concept of global variables to coordinate multiple objectives beyond frequency regulation. Examples include global voltage quality metrics for distribution networks [9], unified nonlinear controllers for transient stability and voltage regulation [10], [11], and consensus-based regulation of voltage drift in dc microgrids [12].

This letter investigates the value of a unified system-wide metric that captures both voltage magnitude and frequency dynamics. Unlike conventional aggregated metrics that consider only frequency or voltage, the proposed metric integrates local voltage behavior, power injections, and the propagation of voltage perturbations through the network.

## II. NOMENCLATURE

Unless otherwise stated, all variables are time-dependent. Symbols  $\dot{\cdot}$  and  $*$  denote the time derivative and the complex conjugate of a variable, respectively. Complex variables are denoted by  $\bar{x}$ , and complex-valued matrices by  $\bar{Y}$ . In the case of the complex frequency of a complex variable in polar form  $\bar{x} = x e^{j\theta}$ , we will use the nomenclature given by the complex frequency as a time derivative operator of a complex number as  $\bar{\dot{x}} = \bar{x} \bar{\eta}_x$ , where  $\bar{\eta}_x = \varrho_x + j\omega_x$ ,  $\varrho_x = \dot{x}/x$  and  $\omega_x = \dot{\theta}$ .

## III. PROPOSED SYSTEM-WIDE DYNAMIC METRIC

Let the power injection at bus  $h$  of an ac power system be:

$$\bar{s}_h = p_h + jq_h, \quad \forall h \in [1, \dots, n], \quad (1)$$

where  $p_h$  and  $q_h$  indicate the active and reactive powers injected at bus  $h$ , respectively, and  $n$  is the number of network buses. Assuming that loads are modeled as negative power injections, the sum of the powers injected across all buses describes the complex power losses of the system:

$$\bar{s}_l = \sum_h^n \bar{s}_h. \quad (2)$$

This variable is a global quantity that continues to be widely used in power systems for assessing overall system efficiency. Nevertheless, by itself, it does not provide dynamic information on the overall system behavior. Taking its time derivative and normalize it by the instantaneous complex power losses itself, we obtain the proposed metric, namely, complex frequency of system losses:

$$\bar{\eta}_{s_l} = \frac{\dot{\bar{s}}_l}{\bar{s}_l} = \varrho_{s_l} + j\omega_{s_l} \quad (3)$$

where  $\varrho_{s_1}$  denotes the instantaneous relative rate of change of the apparent power loss, useful as a normalized dynamic stress indicator, whereas  $\omega_{s_1}$  reflects the balance between real and reactive losses, both quantities remain independent of system size.  $\bar{\eta}_{s_1}$  provides a compact, instantaneous metric of the scale and composition of the dynamic behavior of network losses.

To further analyze the physical meaning and practical application of the proposed metric, one can derive (3) by expanding (1) for a quasi-steady state model of the transmission system. This leads to the well-known power flow equations, as follows:

$$p_h = v_h \sum_k^n v_k [G_{hk} \cos \theta_{hk} + B_{hk} \sin \theta_{hk}], \quad (4)$$

$$q_h = v_h \sum_k^n v_k [G_{hk} \sin \theta_{hk} - B_{hk} \cos \theta_{hk}], \quad (5)$$

where  $G_{hk}$  and  $B_{hk}$  are the real and imaginary parts of the  $(h, k)$  element  $\bar{Y}_{hk}$  of the network admittance matrix  $\bar{Y}$ ,  $v_h$  and  $v_k$  are the voltage magnitudes at buses  $h$  and  $k$ , respectively, and  $\theta_{hk}$  is the phase difference between  $\theta_h$  and  $\theta_k$ , the voltage phase angles at buses  $h$  and  $k$ , respectively.

Differentiation with respect to time of (4) and (5) gives:

$$\dot{p}_h = \frac{p_h}{v_h} \dot{v}_h - q_h \dot{\theta}_h + \sum_k^n \left( \frac{p_{hk}}{v_k} \dot{v}_k + q_{hk} \dot{\theta}_k \right), \quad (6)$$

$$\dot{q}_h = \frac{q_h}{v_h} \dot{v}_h + p_h \dot{\theta}_h + \sum_k^n \left( \frac{q_{hk}}{v_k} \dot{v}_k + p_{hk} \dot{\theta}_k \right), \quad (7)$$

where:

$$p_{hk} = v_h v_k [G_{hk} \cos \theta_{hk} + B_{hk} \sin \theta_{hk}], \quad (8)$$

$$q_{hk} = v_h v_k [G_{hk} \sin \theta_{hk} - B_{hk} \cos \theta_{hk}].$$

Using the notation of the complex frequency, we obtain the expression [8]:

$$\dot{s}_h = \bar{s}_h \bar{\eta}_{v_h} + \sum_k^n \bar{s}_{hk} \bar{\eta}_{v_k}^*, \quad (9)$$

where  $\bar{\eta}_{v_h}$  and  $\bar{\eta}_{v_k}$  are the complex frequencies of the voltage at buses  $h$  and  $k$ , and  $\bar{s}_{hk} = p_{hk} + j q_{hk}$ . The first term of the right side of equation (9) refers to the total variation of the complex power due to local changes in the bus voltage, which is related to the devices connected to the bus itself; the second term captures the effect of network topology on the power variations at bus  $h$ .

The sum of the currents incident at bus  $h$  is given by:

$$\bar{i}_h = \sum_k^n \bar{Y}_{hk} \bar{v}_k. \quad (10)$$

Derivation with respect to time leads to:

$$\dot{\bar{i}}_h = \sum_k^n \bar{Y}_{hk} \dot{\bar{v}}_k, \quad (11)$$

or equivalently, in terms of complex frequencies:

$$\bar{i}_h \bar{\eta}_{i_h} = \sum_k^n \bar{Y}_{hk} \bar{v}_k \bar{\eta}_{v_k}. \quad (12)$$

Then, conjugation and multiplication by  $\bar{v}_h$  give:

$$\bar{v}_h \bar{i}_h^* \bar{\eta}_{i_h}^* = \bar{v}_h \sum_k^n \bar{Y}_{hk} \bar{v}_k^* \bar{\eta}_{v_k}^*, \quad (13)$$

or, equivalently, in terms of complex powers:

$$\bar{s}_h \bar{\eta}_{i_h}^* = \sum_k^n \bar{s}_{hk} \bar{\eta}_{v_k}^*, \quad (14)$$

where  $\bar{i}_h$  and  $\bar{\eta}_{i_h}$  are the net current and its complex frequencies at bus  $h$ . By replacing (14) and (9) in (2), normalizing

it with respect to (1) and taking its derivative with respect to time, we obtain the following form for (3):

$$\bar{\eta}_{s_1} = \frac{\sum_h^n \bar{s}_h \bar{\eta}_{v_h}}{\sum_h^n \bar{s}_h} + \frac{\sum_h^n \bar{s}_h \bar{\eta}_{i_h}^*}{\sum_h^n \bar{s}_h}, \quad (15)$$

or, equivalently, in terms of complex frequencies:

$$\boxed{\begin{aligned} \bar{\eta}_{s_1} &= \bar{\eta}_{v_{\text{sys}}} + \bar{\eta}_{i_{\text{sys}}}^* \\ &= \varrho_{v_{\text{sys}}} + \varrho_{i_{\text{sys}}} + j(\omega_{v_{\text{sys}}} - \omega_{i_{\text{sys}}}) \end{aligned}} \quad (16)$$

where  $\bar{\eta}_{v_{\text{sys}}}$  and  $\bar{\eta}_{i_{\text{sys}}}$  refer to the system weighted complex frequency of voltages and net currents across the network buses, for which  $\varrho_{v_{\text{sys}}}$ ,  $\varrho_{i_{\text{sys}}}$ ,  $\omega_{v_{\text{sys}}}$  and  $\omega_{i_{\text{sys}}}$  represent their real and imaginary parts, respectively.

In (16),  $\bar{\eta}_{v_{\text{sys}}}$  is a local device-driven metric. In particular, the term  $\omega_{v_{\text{sys}}}$  can be interpreted as a generalization of the conventional frequency of the CoI, with the property of being independent from device technology and, thus, it does not require information on the inertia constants of the machines. The term  $\varrho_{v_{\text{sys}}}$  reflects voltage variations at the most ‘influential’ buses, capturing the aggregate effect of voltage control. On the other hand,  $\bar{\eta}_{i_{\text{sys}}}$  is a network-driven metric and captures how current magnitudes and phases through transmission lines are responding to the voltage dynamics, which reflects the propagation of disturbances across the grid. As such, the metric may be calculated and monitored by system operators using existing infrastructure, such as Phasor Measurement Unit (PMU) bus power injection data. Finally, note that, in steady-state, the metric  $\bar{\eta}_{s_1}$  tends to zero, which also reflects the inherent local synchronization of devices with the grid [13] through a coherent network response.

It is relevant to note that this metric is valid also for DC grids. However, for this type of networks, only the real part of (16) is not zero as  $q_h = 0$ , and  $\omega_{v_h} = \omega_{i_h} = 0$  for all  $h$ .

#### IV. CASE STUDY

This section illustrates the dynamic behavior of the proposed metric through a modified version of the IEEE 39-bus system, where the original 10 synchronous machines have been replaced with a combination of 50% Grid-Following (GFL) and 50% Grid-Forming (GFM) Virtual-Synchronous Machine (VSM) IBRs with droop frequency and voltage control. A sensitivity analysis with respect to the  $R/X$  ratio is carried out to assess the impact of voltage-frequency coupling. Simulations were performed using Dome software tool [14].

Figure 1 shows the magnitude of the apparent power losses, frequency of the CoI, the real and imaginary parts of the local voltage component, the real and imaginary parts of the network current component, and the complex frequency of the losses, for a load outage at bus 8 and a  $R/X$  ratio of 0.1 and 1, while maintaining the same impedance for each line.

In conventional systems ( $R/X \approx 0.1$ ), the frequency of the CoI  $\omega_{\text{CoI}}$ , as defined in [7] for systems with GFM-VSM converters, coincides with the steady-state frequency and is comparable to  $\omega_{v_{\text{sys}}}$  during transients. However, as the  $R/X$  ratio increases, bus voltage control increasingly distorts the frequency indices as the influence of coupled voltage-frequency terms with complex power is amplified,

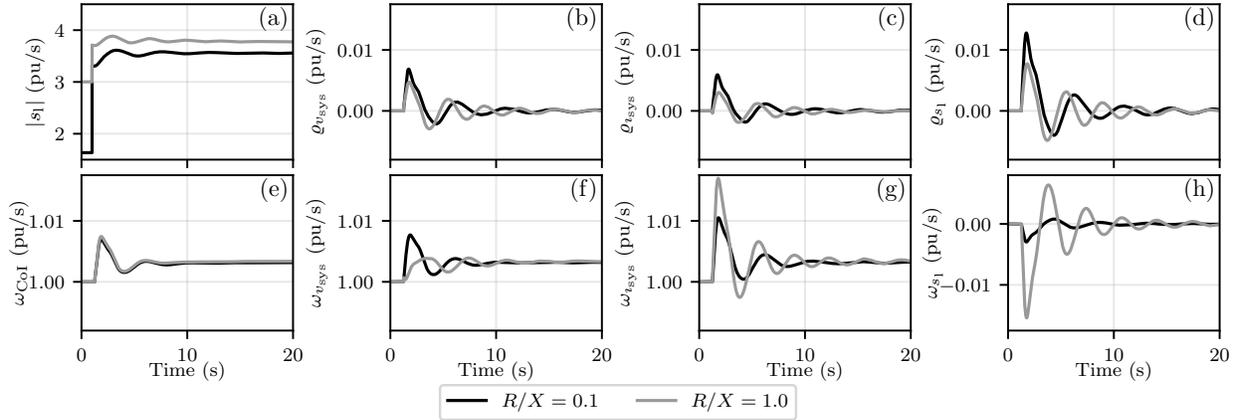


Fig. 1. 39-bus system: load outage at bus 8 for  $R/X \in \{0.1, 1.0\}$ . Panels (a) and (e): magnitude of the apparent system losses and frequency of the CoI. Panels (b) and (f): real and imaginary parts of the CF of the local components. Panels (c) and (g): real and imaginary parts of the CF of the network components. Panels (d) and (h): real and imaginary parts of the CF of the losses.

while not affecting the conventional frequency of the CoI nor the dynamic behavior of system losses, which are substantially identical in both scenarios, except for an offset of system losses due to different value of line resistances.

Conversely, the proposed metrics exhibit a different response and capture dynamics not provided by the frequency of the CoI, in particular, the amplitude of  $\omega_{v_{\text{sys}}}$  decreases as voltage control at the bus level gains influence, while  $\omega_{t_{\text{sys}}}$  increases and its damping is reduced. In terms of RoCoF at 500 ms after the perturbation with increasing  $R/X$  ratio, as expected, the value for  $\omega_{\text{CoI}}$  remains unchanged around 0.009 pu/s, whereas that for  $\omega_{v_{\text{sys}}}$  decreases significantly from 0.009 to 0.001 pu/s. In contrast, the value for  $\omega_{t_{\text{sys}}}$  rises from 0.014 to 0.023 pu/s. Consequently, the network-driven component becomes dominant in the overall  $\omega_{s_1}$  metric, suggesting that synchronism, according to the definition given in [13], becomes more difficult to achieve.

With regard to the real part of the proposed metric, a small decrease in the first-swing amplitude is observed for  $\rho_{v_{\text{sys}}}$ ,  $\rho_{t_{\text{sys}}}$ , and their sum  $\rho_{s_1}$ . This is due to the normalizing effect of increased total active and apparent losses relative to the load outage. Once steady state is reached, the real part of the proposed metrics returns to zero.

## V. CONCLUSIONS

This work proposes a system-wide dynamic metric for assessing power system performance. By considering all devices that influence power flows, the proposed metric is inherently technology-agnostic and may be calculated and monitored by system operators using existing infrastructure, such as bus-level PMU power injection data. The metric is defined as the complex frequency of system losses, where its real and imaginary parts relate to stress variation and synchronization, respectively. The metric can be decomposed into device-driven and network-driven components, enabling a more comprehensive dynamic assessment. Simulation results show that the imaginary part of the device-driven component closely resembles the conventional frequency of the CoI in conventional systems. In contrast, for grids with strong voltage-frequency

coupling, the proposed metrics capture dynamic information beyond the frequency of the CoI. Future work will consider applications of the proposed metric to DC and hybrid AC-DC systems as well as discuss with system operators its use as a frequency quality metric.

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