Hybrid Control Model of Under Load Tap Changers

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***Abstract*—This paper proposes a hybrid control model of under load tap changer transformers. The proposed model is based on the well-known discrete and continuous control models of such device, and is designed so that it preserves the discrete behavior of the tap ratio while allowing solving small-signal stability and eigenvalue analysis. The proposed model is tested through the IEEE 14-bus test system and a real-world 1488-bus model of a sub-transmission and distribution system.**

***Index Terms*—Under load tap changer (ULTC), small signal stability, power flow analysis, time domain simulation, dead band, time delay.**

I. Introduction

A. Motivation

This paper focuses on the model of Under Load Tap Changers (ULTCs). Although ULTCs have a long history in power system analysis, there is a surprising dichotomy in the models currently accepted for such device. For time domain simulations, a discrete model is generally preferred as it precisely represents the physical structure of the ULTC. In fact the tap ratio can only jump from one turn to another of the transformer winding, thus yielding to a discrete variable. On the other hand, for small-signal stability analysis, a continuous tap ratio model is preferable, since it allows defining the eigenvalues associated to the ULTC controllers. This paper proposes an hybrid model aimed to unify the discrete and the continuous ULTC control models proposed so far in the literature.

B. Literature Review

ULTC transformers are relatively common devices in sub-transmission and distribution systems. This fact is demonstrated by the constant interest in the improvement of ULTC technology, modelling, and regulation, as well as by several studies on power flow and stability analysis. Relevant literature on each topic is given below.

1) ULTC technology: The oldest and most common technology for changing the tap ratio is based on motors that physically move the brushes over the turns of the coil. This is called the mechanically assisted ULTC. Such motor-based ULTCs are characterized by a slow time response (tens of seconds). A dead band in the voltage control is needed to avoid unnecessary movements of the brushes. More recently, two novel technologies have been object of intensive study and development, namely electronically assisted and solid-state ULTCs. Relevant literature on such device is as follows: [1]–[6] for electronically assisted ULTCs; and [7]–[12] for solid-state tap changers. Both technologies are intrinsically faster than the motor-based one. However, the differences in the model are mainly in the low-level logic of the electronic switches. The overall controller scheme is basically the same as for the motor-driven technology, but for the speed of the time response. Note that electronically assisted ULTCs are also sometimes called "hybrid" ULTC. It is important to note that, in this paper, the proposed hybrid control model refers only to the mathematical formulation and has no relation with the technology used to build the controller itself.

2) Modelling and Control Schemes for Distribution Systems: Classic ULTC discrete control model is given in [13] while a stochastic model has been proposed in [14]. A discrete model fuzzy-logic controller is presented in [15]. A new scheme for thyristor assisted tap changers is presented din [16] while a scheme of local ULTC controllers are described in [17]. Relevant contributions on the discrete models of the tap changers are given by Faiz and Siakolah, e.g., [11], [18]–[22]. Optimal tap selection is proposed in [23] and a quasi-resonant control is discussed in [24]. Multiple line drop compensation based on a dead band discrete control is proposed in [25].

3) Power Flow Analysis: Classic methods for power flow analysis with discrete models are [26]–[28]. In [26], the tap ratio is included as a variable in the power flow equations (a continuous model is used). Other methods using the discrete model in power flow analysis is [29]. A generalized power flow model for tap changer as well as phase shifting transformers is given in [30].

4) Continuous Models for Voltage Stability Analysis: The discrete model reproduces precisely the physical behavior of the ULTC regulator. However, discrete variables complicate the analysis of DAE systems. For this reason, the continuous model is preferred for stability analysis [31]–[35]. In other cases, locking the tap ratio is used to avoid the need of considering discrete variables [36]. Limit cycles that originate from the interactions of the ULTC dead band with the load dynamics are discussed in [37], [38]. Multiple tap position scenarios are considered in [39] in order to assess the best configuration for voltage stability. An interesting stability study that consists in bounding the discrete behavior through an upper and a lower continuous models is proposed in [40]. In [41], the use of the continuous model is criticized for standard transient stability analysis, due to the slowness of mechanical driven ULTCs. However, the continuous model is justified for electronically assisted and solid-state ULTCs.

C. Contributions

The main contributions of the paper are the following.

1) A simple yet detailed model of the ULTC control that

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is both continuous and discrete. This is called *hybrid control model* throughout the paper.

2) An improved continuous ULTC model as well as a discussion on how to set up the parameters of the proposed hybrid model to properly emulate the commonly used discrete model.
3) A review and a comparison, within a unique framework, of existing static and dynamic ULTC continuous and discrete models and control schemes.
4) A discussion on the ability of the proposed hybrid model to properly behave in power flow analysis. The discussion shows that the proposed model intrinsically optimizes tap ratios in order to get the power solution.
5) A small signal stability analysis that shows that the proposed hybrid model provides accurate eigenvalues associated with the ULTC control.
6) A variety of time domain simulations that show the accuracy of the hybrid model with respect to both the continuous and the discrete ones.

D. Paper Organization

The paper is organized as follows. Section II briefly recalls existing discrete and continuous ULTC models and presents the proposed ULTC hybrid model. In particular, subsections II-A, II-B and II-C describe the ULTC circuit and the discrete and the continuous models, respectively. The reader expert on ULTC models can skip these sections. Subsection II-D discusses the proposed hybrid model. Section III discusses in details a case study based on the IEEE 14-bus system and as well as on a real-world 1488-bus model of a sub-transmission and distribution system. The behavior of the proposed hybrid control model steady-state, small-signal stability analysis and time domain simulations are discussed in this section. Finally, Section IV draws relevant conclusions.

II. Modelling the Under Load Tap Changer

This section describes the existing ULTC circuit and control models, namely the discrete and the continuous models, as well as the proposed hybrid control model. All models are discussed regardless the specific ULTC technology. Furthermore, the scheme and the logic with which the physical switches are operated are not taken into account. In other words, since the object is to study the interactions of the ULTC with the power system to which it is connected, the section focuses on the *macroscopic* model of the ULTC.

A. Circuit Model

The circuit model of the ULTC is well-known, e.g., [42], [43], but it is included in this section for the sake of completeness and because it helps understanding the case studies discussed in Section III.

Assuming that the tap is on the primary side, the complete equivalent circuit of a generic two-winding transformer is depicted in Figure 1. The π-circuit that depends on the series admittance $\boldsymbol{y}_T$ and on the off-nominal tap ratio m can be obtained from the circuit depicted in Figure 2 [42]. The currents $\boldsymbol{i}'_h$ and $\boldsymbol{i}_k$ can be written as:

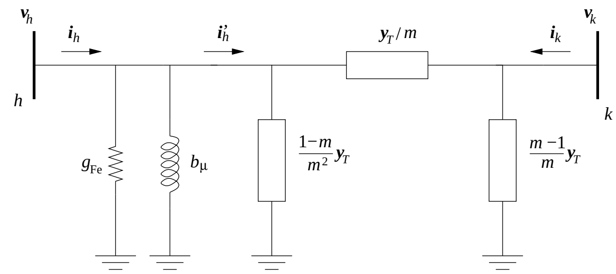

Fig. 1. Transformer equivalent circuit.

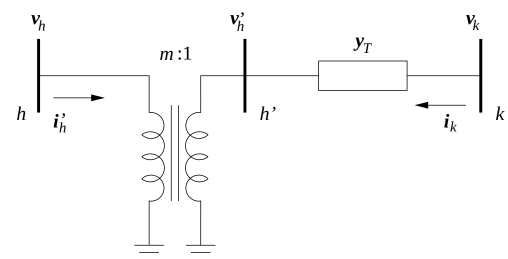

Fig. 2. Equivalent circuit of the tap ratio module and series impedance.

$$
\begin{aligned}
\boldsymbol{i}'_h &= \frac{1}{m}\boldsymbol{y}_T(\boldsymbol{v}'_h - \boldsymbol{v}_k) = \frac{1}{m}\boldsymbol{y}_T(\boldsymbol{v}_h/m - \boldsymbol{v}_k) \\
\boldsymbol{i}_k &= \boldsymbol{y}_T(\boldsymbol{v}_k - \boldsymbol{v}'_h) = \boldsymbol{y}_T(\boldsymbol{v}_k - \boldsymbol{v}_h/m)\,,
\end{aligned}
\tag{1}
$$

where $\boldsymbol{v}'_h = \boldsymbol{v}_h/m$. Equations (1) in vectorial form become:

$$
\begin{bmatrix} \boldsymbol{i}'_h \\ \boldsymbol{i}_k \end{bmatrix} = \boldsymbol{y}_T \begin{bmatrix} \frac{1}{m^2} & -\frac{1}{m} \\ -\frac{1}{m} & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{v}_h \\ \boldsymbol{v}_k \end{bmatrix} . \tag{2}
$$

Considering the physical buses h and k, one obtains:

$$
\begin{bmatrix} \boldsymbol{i}_h \\ \boldsymbol{i}_k \end{bmatrix} = \begin{bmatrix} g_{\rm Fe} + jb_\mu + \boldsymbol{y}_T\frac{1}{m^2} & -\boldsymbol{y}_T\frac{1}{m} \\ -\boldsymbol{y}_T\frac{1}{m} & \boldsymbol{y}_T \end{bmatrix} \begin{bmatrix} \boldsymbol{v}_h \\ \boldsymbol{v}_k \end{bmatrix} . \tag{3}
$$

Finally, the algebraic equations of the power injections are as follows:

$$
\begin{aligned}
p_h &= v_h^2(g_{\rm Fe} + g_T/m^2) \\
&\quad -v_h v_k(g_T\cos\theta_{hk} + b_T\sin\theta_{hk})/m \\
q_h &= -v_h^2(b_\mu + b_T/m^2) \\
&\quad -v_h v_k(g_T\sin\theta_{hk} - b_T\cos\theta_{hk})/m \\
p_k &= v_k^2 g_T - v_h v_k(g_T\cos\theta_{hk} - b_T\sin\theta_{hk})/m \\
q_k &= -v_k^2 b_T + v_h v_k(g_T\sin\theta_{hk} + b_T\cos\theta_{hk})/m\,,
\end{aligned}
\tag{4}
$$

where $g_T + jb_T = \boldsymbol{y}_T$.

B. Discrete Control Model

The discrete model consists in representing the tap ratio as a discrete variable, which can vary between a minimum and a maximum values $m^{\max}$ and $m^{\min}$, respectively, by a fixed step Δm. A complete model of the ULTC control is shown in Fig. 3. It includes a dead band, a time delay and the switching tap ratio logic itself.

The model of the switching logic depends on the analysis that has to be solved. For power flow analysis, it is not relevant to track the trajectory followed by the tap ratio, but just the final position. So one can neglect the effect of time delays and focusing only on the dead band and the tap switching [26], [28], [29]. The switching logic consists in moving up or down by one step Δm the tap ratio if the deviation of the regulated quantity v_k (e.g., the voltage on the secondary winding) with respect to the reference $v^{\rm ref}$ exceeds a given dead band db_v. At a generic i-th iteration of the power flow analysis, the switching logic equations are as follows:

$$
\begin{aligned}
&\alpha^{(i)} = \alpha(\Delta v^{(i)}, m^{(i-1)}, db_v, m^{\max}, m^{\min}) \qquad (5)\\
&= \begin{cases} 1, & \text{if } \Delta v^{(i)} > db_v \text{ and } m^{(i-1)} < m^{\max} \\ -1, & \text{if } \Delta v^{(i)} < -db_v \text{ and } m^{(i-1)} > m^{\min} \\ 0, & \text{otherwise}\,, \end{cases}
\end{aligned}
$$

where $\Delta v^{(i)} = v_k^{(i)} - v^{\rm ref}$. Finally the tap ratio is updated using the value of $\alpha^{(i)}$:

$$
m^{(i)} = m^{(i-1)} + \alpha^{(i)} \Delta m\,. \qquad (6)
$$

The previous model can show numerical issues. For example, if there are several ULTCs in the network, the result may be non-optimal. Moreover, due to the jumps of the tap ratio, the numerical routine can require several steps to complete [27]. Both issues can be easily solved by using a continuous model, which, by the way, provides only an approximated value of the tap ratios. If one wants to preserve the discrete model, the determination of the optimal tap positions is not straightforward since it leads to a mixed integer non linear programming (MINLP) problem. With this aim, a computationally feasible approach has been proposed in [23].

For time domain simulations, the time delay cannot be neglected as it plays an important role in ULTC behavior [37], [38]. A relatively well-accepted model is as follows [13], [25]:

$$
\begin{aligned}
&e(\Delta v(t), m(t-\Delta t), db_v, m^{\max}, m^{\min}) \qquad (7)\\
&= \begin{cases} 1, \text{ if } \Delta v(t) > db_v \text{ and } m(t-\Delta t) < m^{\max} \\ -1, \text{ if } \Delta v(t) < -db_v \text{ and } m(t-\Delta t) > m^{\min} \\ 0, \text{ otherwise}\,, \end{cases}\\
&c(e(t), c(t-\Delta t)) \qquad (8)\\
&= \begin{cases} c(t-\Delta t) + \Delta t, \text{ if } e(t) = 1 \text{ and } c(t-\Delta t) \geq 0 \\ c(t-\Delta t) - \Delta t, \text{ if } e(t) = -1 \text{ and } c(t-\Delta t) \leq 0 \\ 0, \text{ otherwise}\,, \end{cases}\\
&f(e(t), c(t), \tau(t)) \qquad (9)\\
&= \begin{cases} 1, \text{ if } e(t) = 1 \text{ and } c(t) > \tau(t) \\ -1, \text{ if } e(t) = -1 \text{ and } c(t) < \tau(t) \\ 0, \text{ otherwise}\,, \end{cases}
\end{aligned}
$$

where t is the current simulation time and $t - \Delta t$ the previous simulation step, which is fully known, e models the dead band, f the time delay and c is a memory function that stores the time elapsed since the last tap switch. Finally, the tap ratio is updated as follows:

$$
m(t) = m(t-\Delta t) + f(e(t), c(t), \tau(t)) \Delta m\,. \qquad (10)
$$

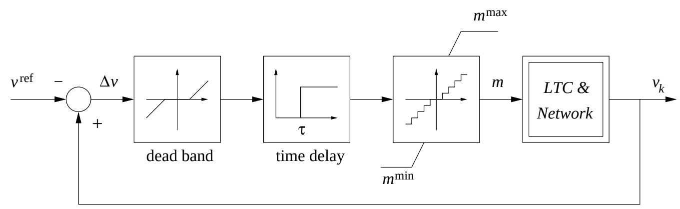

Fig. 3. Diagram of the discrete model of the ULTC voltage control.

In some models of the ULTC [28], [35], [41], the time delay τ is constant, hence:

$$
\tau(t) = \tau_0\,. \qquad (11)
$$

However, most references recognize that the higher the voltage error Δv, the faster the tap ratio changes [19], [38]. The most accepted model is as follows:

$$
\tau(t) = \tau_0 \frac{db_v}{|\Delta v|}\,. \qquad (12)
$$

In order to avoid numerical issues in case $\Delta v = 0$, in this paper the following equation is used:

$$
\tau(t) = \begin{cases} \tau_0 \frac{db_v}{|\Delta v|} & \text{if } |\Delta v| > db_v \\ \tau_0 & \text{otherwise}\,. \end{cases} \qquad (13)
$$

Observe that in case $|\Delta v| < db_v$, the ULTC control does not modify the tap ratio, hence the value of the time delay is actually not relevant in such case.

An alternative discrete control model regulates the tap ratio in order to maintain the voltage within a certain range, say $v^{\max} \leq v_k \leq v^{\min}$. Equation (5) can be rewritten as follows:

$$
\begin{aligned}
&\hat{\alpha}^{(i)} = \hat{\alpha}(\Delta v^{(i)}, m^{(i-1)}, db_v, v^{\max}, v^{\min}) \qquad (14)\\
&= \begin{cases} 1, & \text{if } v_k - v^{\max} > db_v \text{ and } m^{(i-1)} < m^{\max} \\ -1, & \text{if } v_k - v^{\min} < -db_v \text{ and } m^{(i-1)} > m^{\min} \\ 0, & \text{otherwise}\,, \end{cases}
\end{aligned}
$$

Equations (7)-(9) can be rewritten in a similar way. This model is cited here for the sake of completeness but it is not further considered in the remainder of the paper.

C. Continuous Control Model

The continuous model was firstly proposed in [31] and assumes that the tap ratio step Δm is small so that discrete switches can be approximated with a continuous variation of the tap ratio m. The time delay is approximated as a lag transfer function (see Figure 4). Hence, the tap ratio differential equation is:

$$
\dot{\tilde{m}} = -K_d(\tilde{m} - 1) + K_i \Delta v\,, \qquad (15)
$$

where all parameters are defined in the Notation section at the beginning of the paper. The tap $\tilde{m}$ undergoes an anti-windup limiter and the sign of the error Δv is due to the fact that the regulator stable equilibrium point occurs for a negative tangent slope of the ULTC-load characteristic [43].

The function of the term $K_d(\tilde{m} - 1)$, which is contribution of this paper, is twofold: (i) to introduce a static voltage error

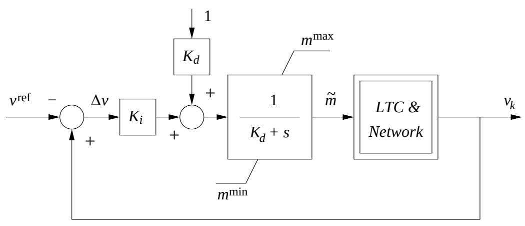

Fig. 4. Diagram of the continuous model of the ULTC voltage control.

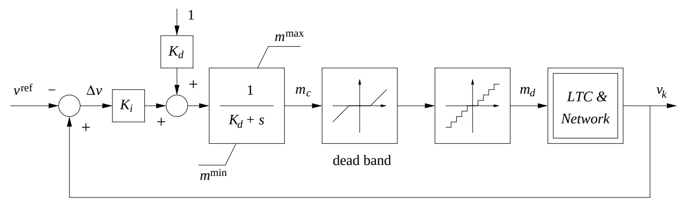

Fig. 5. Diagram of the proposed hybrid model of the ULTC control.

and, hence, a deviation from the integral deviation of the ULTC controller; and (ii) to allow that more than one ULTC controls the voltage at a given bus k. The latter is a common case in practice, e.g., two or more ULTC transformers connected in parallel. Moreover, the value of the coefficient K_d defines the participation of each ULTC connected in parallel to the voltage regulation. Clearly, the smaller K_d, the higher the participation of the transformer. If a ULTC has $K_d = 0$, then that ULTC alone will regulate the voltage v_k, while all other transformers connected in parallel will behave as fixed tap ratio devices with $\tilde{m} = 1$. Finally, it is relevant to observe that $K_d \neq 0$ allows avoiding singular Jacobian matrices in case of multiple ULTC regulating the voltage at the sane bus k.

Observe that $\tilde{m}$ is a state variable. While $\tilde{m}$ can be directly used into (3) and (4), according to the notation used in the paper, the following constraints holds:

$$m = \tilde{m}\,. \tag{16}$$

The continuous model does not require an explicit representation of the time delay block because the transfer function models such delay. Moreover, the time response of the continuous model approximates (13), since its response is faster if Δv increases, and *vice versa*.

The consistency of the continuous control model depends on the time scale of the time domain simulation and on the technology used in the ULTC. In fact, as observed in [41], for slow-dynamic mechanically-driven ULTCs, the continuous model only makes sense for long term simulation. In this case, also turbine governors and other slow control dynamics have to be modelled. Electronically-driven and solid-state ULTCs are characterized by fast time response and justify the continuous control model for transient, e.g., short term, stability analysis. In any case, from the modelling viewpoint, the technology only modifies the value of the parameter K_i. So, the continuous model is acceptable as long as all other device models are consistent with the simulation time frame.

D. Hybrid Control Model

Figure 5 shows the proposed hybrid control model. The control is composed of the transfer function of the continuous model, which implements the time delay and of the dead band block inherited from the discrete model.

The resulting equations are as follows. The differential equation associated with the continuous tap ratio m_c is the same as (15):

$$\dot{m}_c = -K_d(m_c - 1) + K_i \Delta v\,. \tag{17}$$

The switching logic for the discrete tap ratio m_d is:

$$\begin{aligned}\beta(t) &= \beta(m_c(t), m_d(t-\Delta t), db_m) \\ &= \begin{cases} 1, & \text{if } m_c(t) - m_d(t-\Delta t) > db_m \\ -1, & \text{if } m_c(t) - m_d(t-\Delta t) < -db_m \\ 0, & \text{otherwise}\,, \end{cases}\end{aligned} \tag{18}$$

and

$$m_d(t) = m_d(t-\Delta t) + \beta(t)\Delta m\,. \tag{19}$$

Finally, the tap ratio that actually modifies the transformer current injections (3) and power injections (4) is m_d, hence one has:

$$m = m_d\,. \tag{20}$$

The following remarks are relevant.

1) The differential equation (17) implicitly accounts for the time delay of the ULTC control, hence the switching logic (18) is much simpler than (7)-(9).
2) Equations (18) and (19) work for time domain simulations as well as for power flow analysis. In the latter case, one has to substitute (t) and $(t-\Delta t)$ for the superindexes (i) and $(i-1)$, respectively.
3) The dead band db_m in (18) has a different meaning than db_v in (7). In fact, db_m is the dead band that allows mapping m_c into m_d. The voltage error Δv only affects the dynamic of the state variable m_c.
4) The state variable m_c is an auxiliary variable in the proposed control but is not used *outside* the ULTC control. The advantages are twofold: (i) the system *sees* the ULTC as discrete, and (ii) it is possible to define a dynamic (and, hence, an eigenvalue) to the proposed hybrid ULTC control.

E. Conversion of Parameters among Different ULTC Models

An important issue is how to pass from one ULTC model to another preserving the main dynamic behavior of the controller. Converting from the discrete model into the continuous one means finding the correct value for K_i that mimics the transient behavior of the discrete model. Taking into account that the settling time of step response of the continuous model can be evaluated as 3 times the inverse of K_i (e.g., 3 times the time constant of the transfer function), then one can set:

$$\tau_0 \approx \frac{3}{K_i} \tag{21}$$

The values of the dead bands db_v and db_m must be chosen so that they have similar effect on the system. Assuming that the dead bands generally take small values, db_m can

TABLE I
SUMMARY OF ULTC MODELS

Model Type	Power Flow	Time Domain Sim.
Discrete (fixed τ)	(4), (5), (6)	(3), (7)-(9), (10), (11)
Discrete (variable τ)	(4), (5), (6)	(3), (7)-(9), (10), (13)
Continuous	(4), (15), (16)	(3), (15), (16)
Hybrid	(4), (17)-(20)	(3), (17)-(20)

be approximated given db_v and the sensitivity $d\tilde{m}/dv^{\rm ref}$, as follows:

$$db_m \approx \left(\frac{d\tilde{m}}{dv^{\rm ref}}\right)^{-1} db_v \qquad (22)$$

Finally, in most references that consider a continuous model (e.g., [31], [32]), $K_d = 0$ that leads to a voltage error $\Delta v = 0$ in steady-state. However, $K_d \neq 0$ allows avoiding singularity in the system Jacobian matrix. Hence, a proper choice for K_d is a small value, say $K_d \ll 1$. $K_d = 10^{-3}$ is used in the case studies.

F. Summary of ULTC Models

Table I summarizes the ULTC models considered and proposed in this paper. Observe that the commonly accepted formulation for time domain simulation is the so-called current-injection model [44]. However, the power-injection model is also perfectly consistent with time domain simulation, although slightly computationally heavier than the current-injection one [45]. Hence, (3) can be substituted for (4) in the third column of Table I.

III. CASE STUDIES

In this section, we consider two systems, namely the IEEE 14-bus system and a 1488-bus sub-transmission and distribution system. Due to its reduced size, the 14-bus system is particularly well suited for illustrating and comparing the transient behavior of the proposed hybrid model with existing ULTC models. The 1488-bus system is used for testing the robustness and accuracy of the proposed model on a real-world system.

All simulations have been carried out using a Python-based version of PSAT [46] on a Linux platform running on an Intel i7 processor with 8 MB of RAM.

A. IEEE 14-bus System

The IEEE 14-bus benchmark system consists of 2 generators, 3 synchronous compensators, 2 two-winding and 1 three-winding transformers, 15 transmission lines, 11 loads and 1 fixed shunt capacitor. The system also includes generator controllers, such as the primary voltage and frequency regulators, transmission line and transformer protections and breakers, etc. A full description of this system as well as the base case data can be found in [43].

The base case data are modified in order to include a ULTC transformer connecting buses 4 and 9 and regulating the voltage at bus 9. The reference voltage $v^{\rm ref} = 1.0563$ pu, i.e., the same value obtained for the base case solution. A mechanically-driven ULTC control is considered for this case study. The following parameters are chosen for the discrete, continuous and hybrid models: $\Delta m = 0.0125$, $K_i = 0.1$ Hz, $K_d = 10^{-3}$, $\tau_0 = 30$ s, and $db_v = 0.25\%$, and $db_m = 1.25\%$ ($d\tilde{m}/dv^{\rm ref} \approx 5$ at the base case operating point).

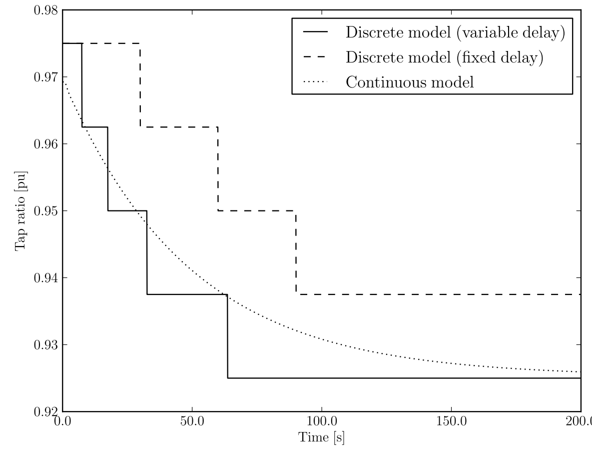

Fig. 6. Transient following line 2-4 outage for the IEEE 14-bus system. Comparison of the tap ratio trajectory for the discrete and the continuous models.

Figure 6 shows the tap ratio during the time domain simulation using the continuous and the discrete ULTC models. The simulations show the dynamics of the dynamic IEEE 14-bus system following line 2-4 outage at $t = 0.5$ s. The discrete model with fixed time delay is also shown in Figure 6 for the sake of comparison.

Figure 7 shows the trajectory of the continuous and discrete tap ratios for the proposed hybrid ULTC model. Observe that the jumps of the discrete variable m_d follows the dynamic of the state variable m_c. On the other way round, the discrete variable m_d modifies the ULTC power injections and, in turn, affects the dynamic of the state variable m_c. The resulting trajectory of the state variable m_c better follows m_d than $\tilde{m}$ of the pure continuous model.

Finally, Fig. 8 compares the trajectories of the discrete model with variable time delay and the discrete variable of the proposed hybrid model. As expected, the two trajectories coincides almost completely.

B. 1488-bus Sub-transmission and Distribution System

In this section a real-world 1488-bus model of a subs-transmission and distribution system is considered for testing the robustness and reliability of the proposed hybrid control model as well as comparing the results obtained for the three models considered in this paper, namely the discrete, the continuous and the hybrid ones. The voltage levels of the system are 400, 220, 132, 66, 45, 30, 24, 18, 15, 14.2, 13.8, 12.6, 11, 10.5, 7.2, 6.6, and 1 kV. The system includes 1758 transmission lines and underground cables and 164 ULTC transformers. Finally, the system contains constant PQ as well as voltage dependent loads.

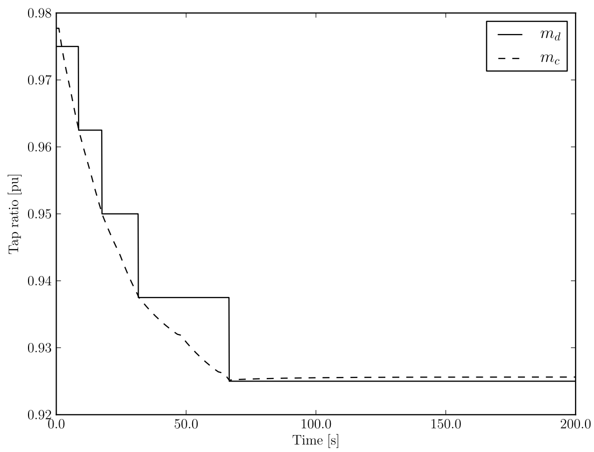

Fig. 7. Transient following line 2-4 outage for the IEEE 14-bus system. Trajectories of the state (m_c) and algebraic (m_d) tap ratios for the proposed hybrid model.

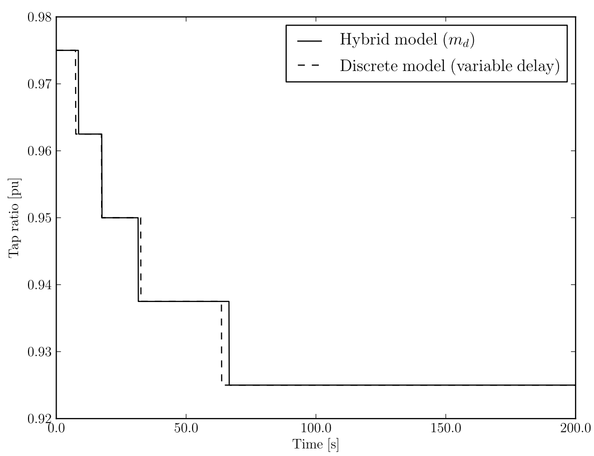

Fig. 8. Transient following line 2-4 outage for the IEEE 14-bus system. Comparison between the trajectories of the discrete variable m_d of the hybrid model and the tap ratio of the discrete model with variable time delay.

TABLE II
TOTAL POWER GENERATION, CONSUMPTION AND LOSSES FOR THE 1488-BUS SYSTEM

Model Type	Active Power [pu]			
	Gen.	Load	Losses	ULTC Losses
Discrete	234.27	228.41	5.86	0.48
Continuous	232.30	227.25	5.05	0.38
Hybrid	231.68	226.99	4.69	0.47

Model Type	Reactive Power [pu]			
	Gen.	Load	Losses	ULTC Losses
Discrete	19.74	63.42	-43.68	13.01
Continuous	7.25	62.90	-55.64	17.68
Hybrid	7.40	62.94	-55.54	16.57

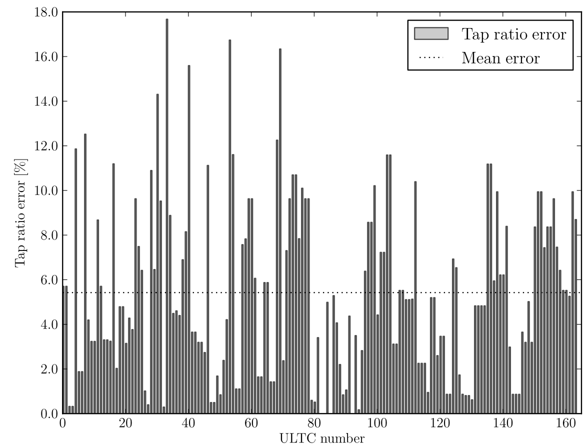

Fig. 9. Tap ratio relative and mean errors resulting from the comparison of the ULTC discrete and hybrid control models for the 1488-bus system. The mean error is 5.42%.

The power flow results for the 1488-bus system and for the three ULTC control models are summarized in Table II (data are in pu with respect of a 100 MVA base). The discrete model is the one that leads to higher total active losses. Observe that overall reactive losses are negative due to the capacitive effect of underground cables. Moreover, including discrete variables within a Newton-Raphson method always creates issues on both convergence [27], [29] and on the set of tap ratio values [23]. With this regard, the continuous and the hybrid ULTC models provide solutions with lower losses then the discrete one thanks to the possibility of continuously varying the tap ratio. Furthermore, the continuous and the hybrid models require less iterations (i.e., 11) that the discrete one (e.g., 38), thus confirming the results discussed in [27]. It is worth noting that the discrete model can also show cycling if the dead band db_v is too small.

Figure 9 shows a comparison between the values of the tap ratios obtained using the discrete and the hybrid ULTC models. While the maximum relative error is about 18%, the mean error is about 5.42%, i.e., one or two tap steps Δm. It is important to note that the discrete model does not actually guarantee an optimal solution. On the other hand, the hybrid model provides a solution closer to the one obtained using the continuous model (see Fig. 10), which can be considered as a virtually "exact" theoretical solution. Thus, the tap ratio errors shown in Fig. 9 can be roughly interpreted as the deviation of the solution obtained with the discrete ULTC model from the optimal one. In this context, "optimal" means a solution that minimizes the vector of voltage errors Δv. In fact, if $K_d = 0$, the voltage error of the ULTC control is $\Delta v = 0$. Hence, if K_d is a small value (say 10^{-3}), the voltage error is also small.

Finally, Fig. 11 shows the comparison between the eigenvalues obtained using the continuous model and those obtained using the hybrid one. The maximum relative error is about 10%. However, in the vast majority of the cases the error is below 2% and the mean relative error is about 1%. The results shown in Fig. 11 confirm that the proposed hybrid model

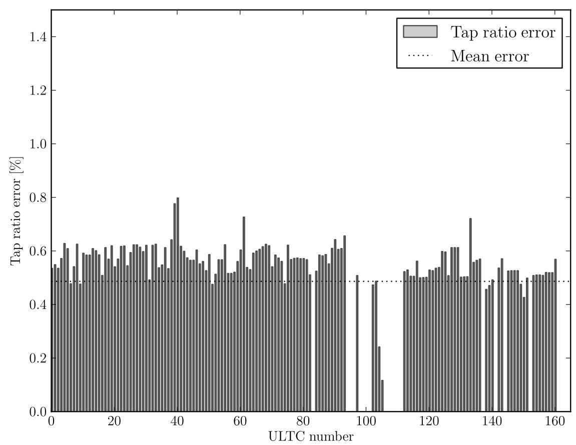

Fig. 10. Tap ratio relative and mean errors resulting from the comparison of the ULTC continuous and hybrid control models for the 1488-bus system. The mean error is 0.49%.

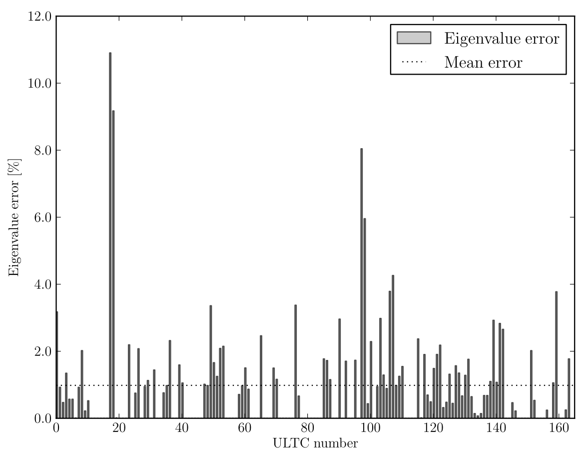

Fig. 11. Eigenvalue relative and mean errors resulting from the comparison of the ULTC continuous and hybrid control models for the 1488-bus system. The mean error is 0.98%.

is able to provide an adequate small-signal stability analysis while preserving the discrete behavior of ULTC devices.

IV. Conclusions

This paper presents a hybrid control model of ULTC devices. This model shows relevant advantages with respect to existing models, namely the discrete and the continuous ones. Compared to the discrete model, the proposed model allows defining eigenvalues and, hence, allows solving a small-signal stability analysis based on the computation of the state matrix eigenvalues. With respect to the continuous model, the proposed hybrid ULTC model preserves the discrete behavior of the tap ratio and allows minimizing the voltage error. The case studies show that the proposed model is robust, precise and has a computational burden comparable with the continuous model.

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