A General Expression to Determine the Rotor Field Current of Synchronous Machines

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***Abstract*— This letter focuses on the dynamic modeling of synchronous machines and proposes a general exact equation to compute the field current of the rotor winding for transient stability analysis. The proposed approach is based on the semi-implicit formulation of differential-algebraic equations, which allows expressing the filed current in terms of the times derivatives of machine state variables, rather than in terms of the variables themselves. This leads to an expression which is compact, linear and model-independent and, thus, easily implementable in power system software tools. The letter illustrates the derivation of the equation leading to the field current for different machine models and shows, through a 21,177 bus model of the ENTSO-E transmission grid, how the derived equation reduces the computational burden of power system models.**

***Index Terms*— Field current, synchronous machine, differential-algebraic equations, semi-implicit formulation.**

I. Introduction

STANDARD models of synchronous machines for transient stability analysis do not include explicitly the expressions of the currents of rotor windings. These currents, in fact, can be expressed as a function of fluxes, voltages and stator currents and, hence, are eliminated from machine equations [1].

While most rotor currents are actually flowing in short-circuited d- and q-axis damper windings and, thus, their magnitudes are actually not needed, the field current (i_f) is an important quantity that is required in several controllers of the machine. Some examples are the over-excitation limiter (OEL) and several models of automatic voltage regulators (AVRs) – see, for example, models in [2].

The explicit expression of the field current is model dependent. For each machine model, one has thus to include such an expression and calculate i_f even if it is not expressly required by OEL and AVRs connected to the machine. This letter proposes an implicit expression of the field current which is simple and independent from the machine model. Such an expression is compact, exact and linear and appears to be particularly useful for the implementation in power system software tools for transient stability analysis.

II. General Formula of the Field Current

This section determines a model-independent expression of the field current of a synchronous machine. To illustrate the procedure, the specific model of the machine as described in [1] is considered first. This machine has a salient-pole rotor with one field (excitation) winding (subscript f) and a three-phase system of stator windings. The effect of induced currents in the rotor core is modeled as a lumped winding (subscript $1d$) in quadrature with the field winding. Finally, damping effects are modeled as two fictitious lumped windings (subscripts $1q$ and $2q$, respectively) in the rotor. In this context, only the d-axis magnetic circuit dynamics are of interest:

$$T'_{d0}\dot{e}'_q = -e'_q - (x_d - x'_d)(i_d + \gamma_{d2}T''_{d0}\dot{\psi}_{1d}) + v_f \quad (1)$$

$$T''_{d0}\dot{\psi}_{1d} = -\psi_{1d} + e'_q - (x'_d - x_\ell)i_d \ , \quad (2)$$

where v_f is the field voltage; e'_q is the transient emf proportional to the field magnetic flux:

$$e'_q = \frac{x_d - x_\ell}{x_f}\psi_f \ ;$$

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TABLE I

Synchronous machine parameters

Parameters	Description
x_ℓ	leakage reactance
x_f	reactance of the rotor field winding
x_d	d-axis synchronous reactance
x'_d	d-axis transient reactance
x''_d	d-axis sub-transient reactance
T'_{d0}	d-axis open circuit transient time constant
T''_{d0}	d-axis open circuit sub-transient time constant

ψ_{1d} is the magnetic flux of the d-axis rotor damper winding; i_d is the d-axis component of the stator current; machine parameters are defined in Table I; and

$$\gamma_{d2} = \frac{1-\gamma_{d1}}{x'_d - x_\ell} \ , \quad \text{and} \quad \gamma_{d1} = \frac{x''_d - x_\ell}{x'_d - x_\ell} \ .$$

In (1) and (2), rotor currents do not appear explicitly. d-axis currents satisfy the following expressions:

$$0 = e'_q + (x_d - x'_d)(i_d - i_{1d}) - \hat{x}_d i_f \quad (3)$$

$$0 = \gamma_{d2}[\psi_{1d} + (x'_d - x_\ell)i_d - e'_q] - i_{1d} \ , \quad (4)$$

where $\hat{x}_d = x_d - x_\ell$ and i_{1d} is the current in the d-axis damper winding. Substituting (2) into (1), one obtains:

$$\begin{aligned} T'_{d0}\dot{e}'_q = &-(x_d - x'_d)[i_d - \gamma_{d2}\psi_{1d} - (1-\gamma_{d1})i_d + \gamma_{d2}e'_q] \\ &+ v_f - e'_q \ , \end{aligned} \quad (5)$$

then, substituting (4) into (5):

$$T'_{d0}\dot{e}'_q = -e'_q - (x_d - x'_d)(i_d - i_{1d}) + v_f \ . \quad (6)$$

And, substituting (3) in (6):

$$\boxed{T'_{d0}\dot{e}'_q = v_f - \hat{x}_d i_f} \quad (7)$$

Equation (7) is the sought expression for the field current i_f. It only depends on the field voltage v_f and the time derivative of e'_q, which are always available, regardless the order and the approximations of the synchronous machine, except for, of course, the classical electro-mechanical model.

Equation (7) is written in a semi-implicit form. This formulation, which is thoroughly discussed in [3], consists in writing differential-algebraic equations utilizing both state variables and their first time derivatives. The general expression of semi-implicit DAEs is:

$$\begin{bmatrix} \boldsymbol{T} & \boldsymbol{0} \\ \boldsymbol{R} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \dot{\boldsymbol{x}} \\ \boldsymbol{0} \end{bmatrix} = \begin{bmatrix} \boldsymbol{f}(\boldsymbol{x},\boldsymbol{y}) \\ \boldsymbol{g}(\boldsymbol{x},\boldsymbol{y}) \end{bmatrix} \quad (8)$$

where $\boldsymbol{x}$, ($\boldsymbol{x} \in \mathbb{R}^n$) are the state variables; $\boldsymbol{y}$, ($\boldsymbol{y} \in \mathbb{R}^m$) are the algebraic variables; $\boldsymbol{f}$, ($\boldsymbol{f} : \mathbb{R}^{n+m} \mapsto \mathbb{R}^n$) are the differential equations; $\boldsymbol{g}$, ($\boldsymbol{g} : \mathbb{R}^{n+m} \mapsto \mathbb{R}^m$) are the algebraic variables; and $\boldsymbol{T}$ and $\boldsymbol{R}$ are, respectively, $n \times n$ and $m \times n$ matrices. According to the notation of (8), the general expression of the field current given in (7) is an element of $\boldsymbol{R}\dot{\boldsymbol{x}}$ and $\boldsymbol{g}$ vectors.

Noteworthy, expressing i_f as a function of $T'_{d0}\dot{e}'_q$, i.e., $\boldsymbol{R}\dot{\boldsymbol{x}}$, allows eliminating the model-dependency from (7), as illustrated in the remainder of this section. Note also that model-independent expressions similar to (7) can be obtained for all other rotor currents of the synchronous machine.

A. Non-linear Magnetic Circuit

Equation (7) is also valid if magnetic circuit saturation are taken into account. Let us consider the non-linear magnetic circuit of the synchronous machine defined in [1]:

$$T'_{d0}\dot{e}'_q = -e'_q - (x_d - x'_d)(i_d + \gamma_{d2}T''_{d0}\dot{e}''_d) - S_f(\boldsymbol{u}) + v_f \quad (9)$$

$$T''_{d0}\dot{\psi}_{1d} = -\psi_{1d} + e'_q - (x'_d - x_\ell)i_d - S_{1d}(\boldsymbol{u}) \quad (10)$$

$$0 = e'_q + (x_d - x'_d)(i_d - i_{1d}) + S_f(\boldsymbol{u}) - \hat{x}_d i_f \quad (11)$$

$$0 = \gamma_{d2}(\psi_{1d} + (x'_d - x_\ell)i_d - e'_q + S_{1d}(\boldsymbol{u})) - i_{1d} \ , \quad (12)$$

where $S_f(\boldsymbol{u})$ and $S_{1d}(\boldsymbol{u})$ are non-linear function modeling the magnetic-current saturation of the field and d-axis windings, respectively, and

$$\boldsymbol{u} = [i_d, e'_d, \psi_{1d}, i_q, e'_q, \psi_{2q}, i_o]^T \ ,$$

where e'_d is the transient emf proportional to the magnetic flux on 1st q-axis damper winding; ψ_{2q} is the sub-transient magnetic flux on 2nd q-axis damper winding; and i_q and i_o are, respectively, the q-axis and homopolar components of the stator current. Substituting (10) into (9), (12) into (11), and merging the resulting expressions lead again to (7), which, hence, does not depend on $S_f(\boldsymbol{u})$ and $S_{1d}(\boldsymbol{u})$.

B. Alternative Models

A variety of different 6th-order models of the synchronous machine have been proposed. A common one is given in [4]. According to this model, the field current is given by:

$$0 = -e'_q - (x_d - x'_d)i_d + \hat{x}_d i_f \quad (13)$$

While the first derivative of e'_q is:

$$T'_{d0}\dot{e}'_q = -e'_q - (x_d - x'_d)i_d + v_f \quad (14)$$

It is straightforward to observe that substituting (13) into (14) leads to (7).

Another 6th order model that can be found in the literature is given in [5]. In this case the filed current and the dynamic of e'_q are expressed by:

$$0 = -e'_q - (x_d - x'_d - \gamma_d)i_d - \gamma_A v_f + \hat{x}_d i_f \quad (15)$$

$$T'_{d0}\dot{e}'_q = -e'_q - (x_d - x'_d - \gamma_d)i_d + (1 - \gamma_A)v_f \quad (16)$$

where $\gamma_A = \frac{T_{AA}}{T'_{d0}}$, $\gamma_d = \frac{T''_{d0}x''_d}{T'_{d0}x'_d}(x_d - x'_d)$, and T_{AA} ($T_{AA} \ll T'_{d0}$) is a rotor leakage time constant that accounts for the fact that the field voltage is weakly coupled with the d-axis damper winding. Substituting (15) into (16) leads again to (7).

C. Reduced Order Models

This subsection considers the machine reduced-order models discussed in [4]–[6]. These models are obtained by imposing conditions on machine time constants and parameters. Reduced-order models with two d-axis dynamic equations are the same as those discussed above, and have thus same expression of the field current. Models with one d-axis dynamic equation show same expressions for i_f and e'_d as in (13) and (14). These include the one d- and one q-axis model (4th order) as well as the one d-axis model (3rd order), which are commonly utilized in transient stability analysis.

III. Case Study

This section discusses the advantages offered by the proposed semi-implicit expression of the field current with respect to the computational burden of power system models. With this aim, the ENTSO-E transmission system is considered. The model includes 21,177 buses, 30,968 transmission lines and transformers, 1,144 zero impedance branches, 15,756 loads, and 4,828 generators.[1] 2,592 generators are modeled as 6th-order synchronous machine as in [1].

[1]The data of the system has been licensed to the author by ENTSO-E. Data can be requested through an on-line application at `www.entsoe.eu`.

TABLE II

STATISTICS FOR THE ENTSO-E SYSTEM

	Standard Expression (17)		Approx. Expression (18)		Proposed Expression (7)	
Matrix	NNZ	NNZ%	NNZ	NNZ%	NNZ	NNZ%
$\boldsymbol{g_y}$	$400,830$	0.00406	$405,406$	0.00411	$400,846$	0.00406
$\boldsymbol{g_x}$	$39,952$	0.00077	$34,768$	0.00067	$34,768$	0.00067
$\boldsymbol{R}$	844	0.00002	844	0.00002	$3,436$	0.00007
$\boldsymbol{R} + \boldsymbol{g_x}$	$39,952$	0.00077	$34,768$	0.00067	$37,360$	0.00072
CPU Time	29.86 s		30.03 s	$+0.57\%$	23.74 s	-20.50%

These machines include AVR models that requires the field current as input signal, namely, IEEE Type ST1A, Type 3 and Type AC1 excitation systems; as well as a simple over-excitation limiter. All results shown in this section are obtained using Dome [7].

The computational burden of three expressions of the field current are compared, as follows: (i) the proposed general formula (7); (ii) the expression obtained derived from (1)-(4), namely:

$$0 = e'_q + (x_d - x'_d)[\gamma_{d1}i_d - \gamma_{d2}(\psi_{1d} - e'_d)] - \hat{x}_d i_f \ ; \quad (17)$$

and (iii) the following approximated, nonlinear expression [6]:

$$0 = \sqrt{(v + \kappa_q)^2 + p^2} + \left(\frac{x_d}{x_q} + 1\right)\frac{\kappa_q(v + \kappa_q) + \kappa_p^2}{\sqrt{(v + \kappa_q)^2 + p^2}} - i_f \quad (18)$$

where $\kappa_p = x_q p/v$, $\kappa_q = x_q q/v$; v is the voltage magnitude at the generator terminal bus; p and q are the active and the reactive power of the generator, respectively; and x_d and x_q are the machine d- and q-axis synchronous reactances, respectively.

The full DAE system includes $n = 51,988$ state variables and $m = 99,360$ algebraic variables. The total number of variables of the system is the same independently from the expression used for i_f. The sparsity of the Jacobian matrices of the system, however, changes depending on the expression utilized to compute i_f. In the case of (18) the computational burden also increases due to the nonlinearity of the expression and the fact that its Jacobian elements are non-constant.

Table II shows the effect of using (17), (18), and (7) on system Jacobian matrices $\boldsymbol{g_y}$, $\boldsymbol{g_x}$ and $\boldsymbol{R}$. Equation (7) increases the number of non-zero (NNZ) elements in $\boldsymbol{R}$, but decreases those of $\boldsymbol{g_y}$ and $\boldsymbol{g_x}$ and $\boldsymbol{R} + \boldsymbol{g_x}$ with respect to (17) and (18). Note that only $\boldsymbol{g_y}$ and $\boldsymbol{R} + \boldsymbol{g_x}$ are actually used during factorization [3].

A time domain simulation for the ENTSO-E system considering a shortcircuit at the bus with code 129218, occurring at $t = 1$ s and cleared after 200 ms is carried out to compare the computational burden of the three expressions (7), (17) and (18). The simulation is solved using a standard Very DisHonest Newton (VDHN) method [6] coupled with an implicit trapezoidal scheme with fixed step of 0.2 s, and Linux OS (Fedora 23) running on an Intel Xeon 3.50 GHz with 12 GB of RAM. The total simulation time is 5 s. Results are shown in the last row of Table II. It is interesting to note that even though the increase in the sparsity of the system Jacobian matrices is small, the proposed formulation based on (7) consistently helps reducing the overall computational burden of the system. This unexpected speed up has to be ascribed to the highly nonlinear process involving numerical factorization of sparse matrices.

References

[1] P. W. Sauer and M. A. Pai, *Power System Dynamics and Stability*. Upper Saddle River, NJ: Prentice Hall, 1998.

[2] Siemens Power Technologies International, *PSS®E 33.5 – Program Application Guide – Volume 2*, Siemens Industry, Inc., Schenectady, NY, Oct. 2013.

[3] F. Milano, “Semi-Implicit Formulation of Differential-Algebraic Equations for Transient Stability Analysis,” *IEEE Transactions on Power Systems*, vol. 31, no. 6, pp. 4534–4543, Nov. 2016.

[4] P. M. Anderson and A. A. Fouad, *Power System Control and Stability*. New York, NY: Wiley-IEEE Press, 2002.

[5] R. Marconato, *Electric Power Systems*. Milano, Italy: CEI, Italian Electrotechnical Committee, 2002, vol. 2.

[6] F. Milano, *Power System Modelling and Scripting*. London: Springer, 2010.

[7] ——, “A Python-based Software Tool for Power System Analysis,” in *Procs. of the IEEE PES General Meeting*, Vancouver, BC, July 2013.