Synthetic Discrete Inertia

Ángel Vaca, *IEEE Student Member* and Federico Milano, *IEEE Fellow*

***Abstract*—This letter demonstrates how synthetic inertia can be obtained with the control of flexible discrete devices to keep the power balance of power systems, even if the system does not include any synchronous generator or conventional grid-forming converter. The letter also discusses solutions to cycling issues, which can arise due to the interaction of uncoordinated discrete inertia controllers. The effectiveness, dynamic performance, and challenges of the proposed approach are validated through simulations using modified versions of the WSCC 9-bus test system and of the all-island Irish transmission system.**

***Index Terms*—Synthetic inertia, discrete control, fast frequency regulation, power system dynamic performance.**

I. Introduction

Synthetic inertia is defined as the controlled response of a generating unit to emulate the rotational energy exchange typical of synchronous machines [1]. Due to the high penetration in power systems of inverter-based resources, synthetic inertia has become essential for frequency stability, as it compensates for the decrease of physical inertia. Foundational work, such as [2] and [3], redefines system inertia and introduces virtual inertia strategies suited to modern grids.

Microgrids, often characterized by low inertia, benefit from synthetic inertia control models like those presented in [4] and [5], which use inverter-based elements and energy storage to stabilize frequency. At a larger scale, hybrid approaches such as distributed virtual inertia from supercapacitors [6] and grid-scale energy storage systems [7] offer innovative solutions for weak grids. Other studies [8], [9] emphasize synthetic inertia's role in maintaining frequency stability in grids with high renewable penetration. Moreover, inertia estimation methods reviewed in [10] and [11] are critical for adapting to fluctuating renewable energy sources while maintaining system stability.

While the literature demonstrates the importance of continuous controllers for frequency stability and resilience, there is limited research on how discrete devices can support a power system independently, i.e., without synchronous generators or conventional grid-forming converters providing slack bus capability.

Motivated by the theory of modeling and simulation, which states that continuous systems can be, in effect, considered a special case of discrete model events [12], this letter demonstrates that discrete devices (DDs) can guarantee the power balance of the overall system through the provision of discrete virtual inertia, as well as significantly improve the dynamic performance of large power systems with the inclusion of conventional generation. The letter also shows the challenges of discrete inertia control in standalone systems and offers potential solutions to these issues.

Á. Vaca and F. Milano are with the School of Electrical and Electronic Engineering, University College Dublin, Belfield Campus, D04V1W8, Ireland. e-mails: angel.vaca1@ucdconnect.ie, federico.milano@ucd.ie

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II. Modeling

We consider the conventional model of power systems as hybrid automata, that is, as a set of differential-algebraic equations with the inclusion of discrete variables that define the behaviors of breakers and faults, as well as the connection status of discrete elements. Formally, one has:

$$\begin{aligned} \boldsymbol{x}' &= \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{u}, \boldsymbol{z}), \\ \boldsymbol{0}_{n_y,1} &= \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{u}, \boldsymbol{z}), \end{aligned} \tag{1}$$

where $\boldsymbol{f}$ are differential equations, $\boldsymbol{g}$ are algebraic equations, $\boldsymbol{x} = \boldsymbol{x}(t) \in \mathbb{R}^{n_x}$ are state variables, e.g., machine rotor speeds, and $\boldsymbol{y} = \boldsymbol{y}(t) \in \mathbb{R}^{n_y}$ denote algebraic variables, e.g., bus voltage angles; $\boldsymbol{u}$, $\boldsymbol{u} \in \mathbb{R}^{n_u}$ are inputs, e.g., load variations; and $\boldsymbol{z}$, $\boldsymbol{z} \in \mathbb{R}^{n_z}$ are discrete variables, e.g., the status of breakers, tap ratios of under-load transformers and discrete devices.

In this work, we assume that the system includes flexible resources that can switch on/off part of their power consumption in discrete packets, say Δp. The rule to switch on/off each is as follows:

$$\begin{aligned} \delta' &= \omega - \omega_{\rm ref}, \\ M\omega' &= p_{\rm ref} - \tilde{p}_e(t, p_e) - D(\omega - \omega_{\rm ref}), \\ p_{\rm ref}' &= \frac{1}{R}{\rm db}(\omega - \omega_{\rm ref}) - p_{\rm ref}, \\ p_e &= K_p v \sin(\delta - \theta), \end{aligned} \tag{2}$$

where $\delta, \omega, p_{\rm ref} \in \boldsymbol{x}$ are the state variables of the device, which emulate the rotor angle and speed of a synchronous machine as well as the droop control of a turbine governor; $p_e \in \boldsymbol{y}$ is a virtual electrical power whereas $\tilde{p}_e \in \boldsymbol{z}$ is the actual power consumption/generation of the device; θ is the phase angle of the terminal bus voltage as determined, for example, with a PLL; and $\omega_{\rm ref} \in \boldsymbol{u}$ is the synchronous reference frequency; M and D are virtual inertia and virtual damping, respectively; R is the droop coefficient; v is the magnitude of the voltage at the device terminal bus; K_p is a coefficient that emulates the ratio between the internal emf and reactance, say e/x, of a synchronous machine; and ${\rm db}$ is a dead-band on the frequency error.

Equations (2) represent a virtual oscillator that behaves similarly to a simple second-order model of a synchronous machine. The signal p_e is utilized to decide the switching of the various elements of the flexible load. The actual output power of the device is thus discrete, as follows:

$$\tilde{p}_e = d_P(t, p_e) = h(t, p_e)\Delta p, \quad h = \{0, 1, \dots, n\}, \tag{3}$$

where $d_P(\cdot)$ is a discretization function of time and p_e implementing the switching of the blocks Δp of the device. In practice, $d_P(\cdot)$ determines the integer h that approximates p_e to the nearest multiple of discrete power packets Δp.

In the following, we do not distinguish between DD generators and loads. If the DD is a load, which is the most common case, then we assume that $\tilde{p}_e > 0$ is, in effect, implemented as a reduction of the load power consumption. Finally, we assume that the power factor of DDs is constant, hence at a variation of $\tilde{p}_e$ corresponds a proportional variation of the reactive power. Finally, each device evaluates (2) and (3) at fixed intervals Δt and, if necessary, changes its power injection $\tilde{p}_e$ only at the beginning of these intervals.

In this work we consider two scenarios: (i) standalone discrete devices (SDD), where discrete devices providing virtual inertia capability are the only devices in the system that are able to re-establish the power balance after a contingency; and (ii) complementary discrete devices (CDD), where discrete devices complement synchronous machines and or conventional gird-forming converters. CDD scenario is the most likely scenario to happen in large transmission systems. Yet, we show as a proof of concept that SDD is, in principle, at least, feasible and can be considered for small systems such as AC microgrids.

In the SDD scenario, properly calibrating the DDs sizes and operation times is essential. These parameters must be carefully coordinated to ensure that the DDs have enough capacity to supply non-flexible power demand (adequacy) and be stable after a perturbation.

The stability aspect requires further discussion. Let the power balance equation of a system be:

$$\left|\sum p_{\mathrm{gen},i} - \sum p_{\mathrm{DD},k} - \sum p_{\mathrm{load},j} - p_{\mathrm{losses}}\right| \leq \varepsilon\,, \qquad (4)$$

where $p_{\mathrm{gen},i}$ is the active power injection of conventional generation (i.e., with continuous control); $p_{\mathrm{load},j}$ is the active power consumption of every non-flexible load; $p_{\mathrm{DD},k}$ is the combined power of DDs providing discrete inertia support; and ε is a tolerance below which no switching of the discrete devices is activated. If (4) is not satisfied, it is possible that the switching logic of the DDs can lead to cycling, that is, an unnecessary periodic switching on/off of DDs. The cycling can be triggered if the size of available DDs is too big to satisfy (4) but also due to a combination of Δt and Δp. These aspects are illustrated in the case study.

III. Case Study

This section evaluates the DDs described in Section II in two study cases: modified versions of the WSCC 9-bus test system and the Irish Power System. All the simulations are performed using the power system analysis software tool Dome [13].

A. SDD Scenario: WSCC 9-Bus test system

The WSCC 9-bus system includes 3 sets of PQ loads, connected to buses 5, 6, and 8, of 2, 0.9, and 1 pu(MW), respectively. Loads at bus 5 are divided into three smaller groups of 1.1, 0.3, and 0.6 pu(MW). Then, $n = 300{,}000$ DDs are connected along the system buses; the size of the DDs are uniformly distributed in the following set of values $\{10^{-2}, 10^{-3}, 10^{-4}, 10^{-5}\}$ pu(MW). The operation of each DD is evaluated every $\Delta t = 1$ s. The activation or deactivation of each DD is not synchronized, that is, DDs do not switch simultaneously. Finally, to impose that only DDs support the power unbalance, the synchronous machines are modeled as constant PQ injections.

We test the dynamic performance of the system following 3 large perturbations: (i) disconnection of the 1.1 pu(MW) PQ load of bus 5 at time $t = 15$ s; (ii) a 3-phase fault at $t = 20$ s, clearance time at $t = 22$ s, with a fault resistance of 10^{-3} pu(Ω); and (iii) a reconnection of the 1.1 pu(MW) PQ load of bus 5 at time $t = 35$ s. In this scenario, the dead-band on the frequency error is set to zero.

Figure 1 shows that, using $\Delta t = 1$ s, the system's power balance is fully sustained by DDs.

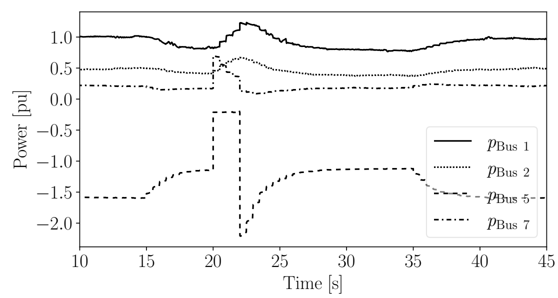

Fig. 1: Power balance on the busses – 9-bus WSCC System.

The effective combination of high device granularity and asynchronous operation ensures a near-continuous inertial response. When this combination is not adequately achieved, DDs react with one another, causing cycling behavior since (4) is not satisfied. This phenomenon is illustrated in Fig. 2, which shows the effect of total power injection on a bus for two combinations of DD sizes. One trivial solution to this issue is to use only small-sized DDs. However, this would require a much larger number of devices to meet the system's power demands. Instead, a proper balance among device sizes, granularity, and operational characteristics of the DDs ensures adequate functioning under SDD.

Figure 3 shows that system voltages remain within acceptable operational levels in all operative conditions. When the 3-phase fault occurs at bus 5, DDs effectively support the voltages at other buses by rapidly increasing power injection, as shown in Fig. 1. Faults lasting as long as 2 s with small resistance are uncommon in power systems, as protective mechanisms clear them within a few cycles. However, this test verifies that the complementary operation of DDs, fast-acting times, and granularity allows a robust response under such a severe disturbance.

B. CDD Scenario: All-island Irish transmission system

We use a dynamic model of the all-island Irish transmission system to demonstrate the performance of DDs in a real-world grid. The model includes 1479 buses, 1851 lines, 5 conventional power plants, and 302 wind power plants with a caseload of 1.8 GW. In all cases, DDs evaluate their participation every $\Delta t = 1$ s, $n = 100{,}000$ DDs, and the evaluation time of each DD is randomized in this interval.

The contingency is a disconnection of an equivalent load of 7% of the system. Figure 4 shows the dynamic performance

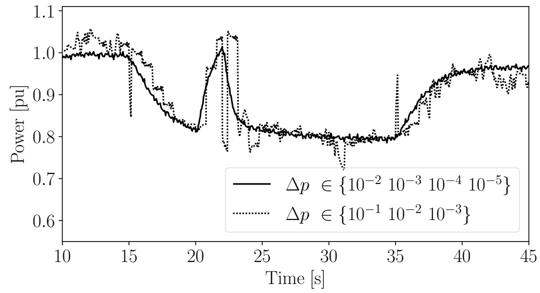

Fig. 2: Power injection for different DDs' size combination – Bus 3

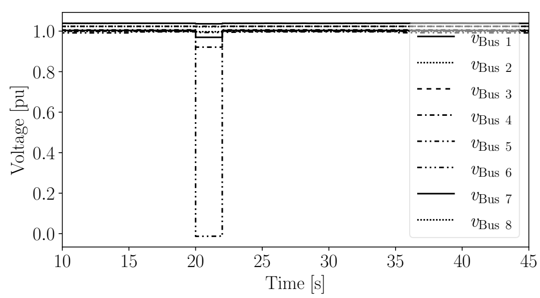

Fig. 3: Voltages on the busses – 9-bus WSCC System.

of the model of the Irish system as a base case and how its dynamic performancs is improved through CDD support.

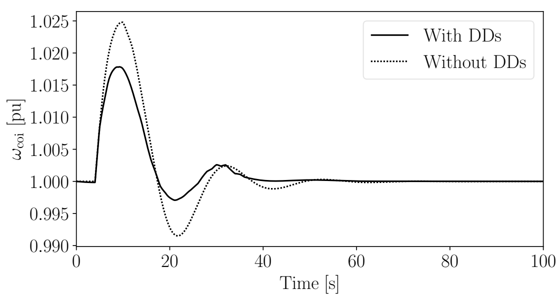

Fig. 4: Frequency support from DDs – Irish System.

Figure 5 shows that the switching of the DDs triggers slight voltage oscillations. The oscillations are relatively small due to the high granularity and balanced sizes of the DDs. They can be mitigated by connecting a dedicated device, as shown in Fig. 6, where an SVC is connected to bus Bedford.

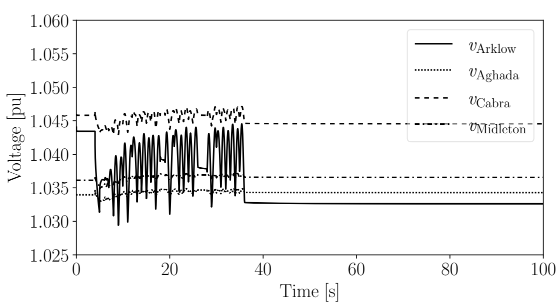

Fig. 5: Voltage on representative busses – Irish system.

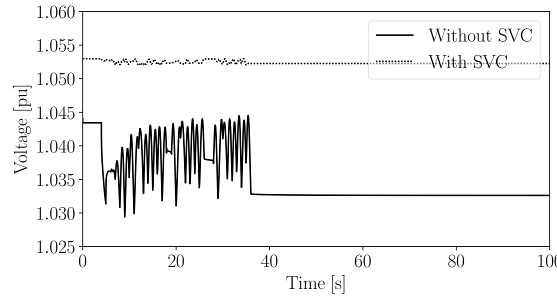

Fig. 6: Correction of voltage oscillation – Bus Arklow.

IV. Conclusions

This study demonstrates the potential of flexible discrete devices to provide synthetic inertia to power systems. DDs are shown to effectively manage frequency and voltage stability autonomously by injecting discrete power packets. The paper also shows that proper calibration of the DDs control can eliminate cycling effects, typical of the interaction of discrete controllers. The application of this approach to the WSCC 9-bus and the all-island Irish transmission systems confirms that DDs can offer scalable, adaptive support to meet dynamic stability requirements in power grids.

Future research will focus on integrating DDs with other fast-response technologies, potentially broadening DDs' application in various power system contexts.

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