On the Numerical Stability and Accuracy of One-Step Delay Approximations

Georgios Tzounas, *IEEE Member*, Federico Milano, *IEEE Fellow*

***Abstract*— The One-Step Delay Approximation (OSDA) technique consists in delaying certain variables of a power system model with time delays of magnitude equal to the step size of the Time Domain Integration (TDI). The objective of the OSDA is to reduce the mutual coupling of the system's equations and the density of the Jacobian matrix that needs to be factorized at each step of the TDI. In turn, the OSDA leads to a simulation speedup but may worsen the accuracy of the results. This paper provides a novel matrix-pencil based approach to assess the OSDA technique in terms of its impact on the accuracy and numerical stability of the TDI of power systems. The case study illustrates the proposed approach through the WSCC 9-bus benchmark system.**

***Index Terms*— Time Domain Integration (TDI), delay-based decoupling, One-Step Delay Approximation (OSDA), numerical stability, Small Signal Stability Analysis (SSSA).**

I. Introduction

A. Motivation

The transient stability analysis of power systems relies on the Time Domain Integration (TDI) of a set of nonlinear Differential Algebraic Equations (DAEs) [1]. Despite the ever increasing computational capability of workstations, assessing the performance of a power system by means of TDIs is one of the most time-consuming processes solved by system operators. Moreover, TDI requires to be evaluated continually, for every operating condition and for a large number of disturbance scenarios. Consequently, there is an incessant interest for techniques that can reduce the total computational cost of the TDI, as well as for novel tools to evaluate the impact of these techniques on the accuracy and numerical stability of the simulation. The latter is the focus of this paper.

B. Literature Review

Implicit numerical schemes are the most effective approach to integrate a power system model, mainly due to their numerical stability and their effectiveness in dealing with stiffness [2]. Employing an implicit scheme, however, implies the iterative solution at each step, of a set of non-linear algebraic equations. The DAEs that describe a power system are non-symmetrical and mutually coupled due to the meshed topology of the transmission grid and the actions of centralized secondary regulators. These features limit the performance of algorithms that exploit the bordered block diagonal structure of the Jacobian matrix of the DAEs, unless some form of relaxation that reduces the coupling of the DAEs is imposed, e.g. see [3]–[5]. These constraints make the solution of implicit schemes cumbersome unless some approximation is adopted. The simplest approximation is a "dishonest" approach, which consists in factorizing the Jacobian matrix of the DAEs only once per time step or only when a structural change (e.g., line outage) occurs in the system [6]. However, this approach only reduces the number of factorizations, not the complexity of the factorization *per se*. Another technique that has been proposed in the literature exploits the "localization" of dynamics [7].

In [8], a time delay-based technique is proposed to reduce the coupling and the density of the equations of power system DAE models while maintaining the accuracy of the simulation. This is achieved by introducing delays equal to the time step of the TDI to – typically slow – variables whose dynamics are not dominant in the time scale of interest. A good example of such one-step delay is the use of the Center-of-Inertia (COI) at the previous time step to decouple the equations of the rotor angles of synchronous machines [9]. We note that the decoupling properties of delayed variables is not a new concept, but a feature first exploited for the simulation of electromagnetic transients, e.g. see [10] and [11].

The accuracy of the OSDA in [8] is primarily evaluated by comparing the dynamic response of the original and time-delay system for certain variables and set of contingencies. However, an important aspect, i.e. the impact of the OSDA on the numerical stability and precision of the implicit TDI method applied, is not addressed in [8]. To provide a useful tool to address this point is the goal of the present paper. In this vein, a matrix pencil-based framework to evaluate the numerical distortion introduced by TDI methods to the dynamic modes of power systems is described in [12], while an extension of this technique for time-delay systems is presented in [13].

C. Contribution

This paper provides a tool to systematically assess the effect of the OSDA on the precision and numerical stability of integration methods employed for the time domain simulation of power systems. The proposed tool, which is based on Small-Signal Stability Analysis (SSSA), allows estimating the amount of numerical distortion introduced by the OSDA technique to the dynamic modes of the power system model under study, as well as to evaluate the significance of this amount when compared to the distortion induced by the integration method itself.

D. Organization

The remainder of the paper is organized as follows. Section II provides a background on power system TDI. Section III briefly summarizes the OSDA technique. The proposed approach to assess the impact of OSDA on the numerical stability and accuracy of the TDI is presented in Section III-A. The case study is discussed in Section IV. Finally, the conclusions are discussed in Section V.

G. Tzounas and F. Milano are with the School of Electrical & Electronic Engineering, University College Dublin (UCD), Belfield Campus, D04 V1W8, Ireland. E-mails: {georgios.tzounas, federico.milano}@ucd.ie

This work is supported by the European Commission, by funding G. Tzounas and F. Milano under the project EdgeFLEX, Grant No. 883710.

II. Numerical Integration of Power Systems

Power system models are conventionally described by a set of DAEs, as follows:

$$\begin{aligned} \boldsymbol{x}' &= \boldsymbol{f}(\boldsymbol{x},\boldsymbol{y})\,, \\ \mathbf{0}_{m,1} &= \boldsymbol{g}(\boldsymbol{x},\boldsymbol{y})\,, \end{aligned} \tag{1}$$

where $\boldsymbol{x} = \boldsymbol{x}(t) \in \mathbb{R}^n$ are the states and $\boldsymbol{y} = \boldsymbol{y}(t) \in \mathbb{R}^m$ the algebraic variables; $\boldsymbol{f} : \mathbb{R}^{n+m} \to \mathbb{R}^n$, $\boldsymbol{g} : \mathbb{R}^{n+m} \to \mathbb{R}^m$, are sets of non-linear functions; and $\mathbf{0}_{m,1}$ is the $m \times 1$ zero matrix. In compact form, (1) can be rewritten as:

$$\mathbf{E}\,\mathbf{x}' = \boldsymbol{\phi}(\mathbf{x})\,, \tag{2}$$

where

$$\mathbf{x} = \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \end{bmatrix},\ \mathbf{E} = \begin{bmatrix} \boldsymbol{I}_n & \mathbf{0}_{n,m} \\ \mathbf{0}_{m,n} & \mathbf{0}_{m,m} \end{bmatrix},\ \boldsymbol{\phi}(\mathbf{x}) = \begin{bmatrix} \boldsymbol{f}(\boldsymbol{x},\boldsymbol{y}) \\ \boldsymbol{g}(\boldsymbol{x},\boldsymbol{y}) \end{bmatrix}. \tag{3}$$

A numerical method for the TDI of (2) is a discrete-time approximation employed to provide a solution for given initial conditions. Without loss of generality, in this paper we focus on one method, namely the Implicit Trapezoidal Method (ITM), which is known to be symmetrically A-stable. Applying the ITM to system (2), we have:

$$\boldsymbol{\eta}^{(i)} = \mathbf{E}\mathbf{x}_t^{(i)} - \mathbf{E}\mathbf{x}_{t-h} - 0.5h\boldsymbol{\phi}(\mathbf{x}_t^{(i)}) - 0.5h\boldsymbol{\phi}(\mathbf{x}_{t-h})\,, \tag{4}$$

where $h > 0$ is the integration time step. At each step, the following system needs to be solved:

$$\boldsymbol{\eta}^{(i)} = \mathbf{0}_{r,1}\,, \tag{5}$$

where the solution is found iteratively. For example, the i-th iteration of Newton's method applied to (5) is:

$$\mathbf{x}_t^{(i+1)} = \mathbf{x}_t^{(i)} - \left[\mathcal{A}^{(i)}\right]^{-1} \boldsymbol{\eta}^{(i)}\,, \tag{6}$$

where the Jacobian matrix $\mathcal{A}^{(i)}$ is defined as:

$$\mathcal{A}^{(i)} = \mathbf{E} - 0.5h\frac{\partial \boldsymbol{\phi}}{\partial \mathbf{x}_t^{(i)}}\,. \tag{7}$$

A. SSSA-Based Accuracy Assessment

In this section we briefly describe how the precision of the ITM when employed for the solution of (2) can be studied through SSSA [12]. Let $\mathbf{x}_o$ be an equilibrium point of (2). Linearization of (2) at $\mathbf{x}_o$ gives:

$$\mathbf{E}\,\Delta\mathbf{x}' = \mathbf{A}\,\Delta\mathbf{x}\,, \tag{8}$$

where $\mathbf{A} = \frac{\partial \boldsymbol{\phi}}{\partial \mathbf{x}_t}\big|_{\mathbf{x}_o}$. The eigenvalues of (8) are the solutions of the associated characteristic equation $\det(s\mathbf{E} - \mathbf{A}) = 0$, $s \in \mathbb{C}$, where the family of matrices

$$\boldsymbol{P}(s) = s\,\mathbf{E} - \mathbf{A} \tag{9}$$

is called the *matrix pencil* of (8), see [14].

Consider the ITM applied for the TDI of (2), as described in (4). Linearization of (4) around the equilibrium gives:

$$(\mathbf{E} - 0.5h\mathbf{A})\,\Delta\mathbf{x}_t = (\mathbf{E} + 0.5h\mathbf{A})\,\Delta\mathbf{x}_{t-h}\,, \tag{10}$$

where we have taken into account that at $\mathbf{x}_o$ (5) is satisfied. Then, the matrix pencil of (10) is:

$$\boldsymbol{P}_{\rm ITM}(z) = z\,(\mathbf{E} - 0.5h\mathbf{A}) - (\mathbf{E} + 0.5h\mathbf{A})\,. \tag{11}$$

The eigenvalues of $\boldsymbol{P}_{\rm ITM}(z)$ represent the approximated by the ITM dynamic modes of (2), while the eigenvalues of $\boldsymbol{P}(s)$ represent the actual small-disturbance dynamics of the system. Thus, comparing the two pencils allows quantifying the numerical distortion introduced by the TDI method to the system dynamics. Note that $\boldsymbol{P}(s)$ is referred to the S-plane whereas $\boldsymbol{P}_{\rm ITM}(z)$ is referred to the Z-plane, hence the eigenvalues of the two pencils can be compared with the mapping $s_k^{\rm ITM} = \frac{1}{h}\log(z_k^{\rm ITM})$, where $z_k^{\rm ITM}$ is the eigenvalue of $\boldsymbol{P}_{\rm ITM}(z)$ that approximates the eigenvalue s_k of $\boldsymbol{P}(s)$.

III. One-Step Delay Approximation

The OSDA technique consists in replacing certain variables of system (2) with their values at the previous time step [8]. Such one-step delays do not model a physical phenomenon but are rather introduced with the goal of reducing the overall computational cost of the simulation. Including one-step delays in (2) yields the system:

$$\mathbf{E}\,\mathbf{x}' = \tilde{\boldsymbol{\phi}}(\mathbf{x},\mathbf{x}_d)\,, \tag{12}$$

where $\mathbf{x}_d = \mathbf{x}(t-h)$. In this work, (12) is basically an approximation of (2). Then, the OSDA relies on the fact that, using (12) leads to Newton iterations with a sparser Jacobian compared to (2). Applying the ITM to system (12), one has:

$$\begin{aligned} \mathbf{0}_{r,1} = \mathbf{E}\mathbf{x}_t^{(i)} - \mathbf{E}\mathbf{x}_{t-h} &- 0.5h\tilde{\boldsymbol{\phi}}(\mathbf{x}_t^{(i)},\mathbf{x}_{t-h}) \\ &- 0.5h\tilde{\boldsymbol{\phi}}(\mathbf{x}_{t-h},\mathbf{x}_{t-2h})\,. \end{aligned} \tag{13}$$

From (13), the Jacobian matrix of Newton's method is:

$$\tilde{\mathcal{A}}^{(i)} = \mathbf{E} - 0.5h\frac{\partial \tilde{\boldsymbol{\phi}}}{\partial \mathbf{x}_t^{(i)}}\,. \tag{14}$$

Then, the following relationship holds, [8]:

$$\frac{\partial \boldsymbol{\phi}}{\partial \mathbf{x}_t^{(i)}} = \frac{\partial \tilde{\boldsymbol{\phi}}}{\partial \mathbf{x}_t^{(i)}} + \frac{\partial \tilde{\boldsymbol{\phi}}}{\partial \mathbf{x}_{t-h}^{(i)}}\,. \tag{15}$$

Equation (15) implies that $\tilde{\mathcal{A}}^{(i)}$ is sparser than $\mathcal{A}^{(i)}$, see [8]. The goal of the OSDA is exactly to exploit this property by adding one-step delays to elements that satisfy the following conditions:

(i) When subject to small variations, they do not alter significantly the system's trajectories. In this regard, good candidates are variables whose dynamics are slower than the dynamics of interest. Systematic selection of the variables may be done, for example, through a modal analysis [8], [15], [16].

(ii) They produce dense rows/columns in the Jacobian matrix. This way, the sparsity increase is combined with a decoupling of the system's equations, which is an important advantage when exploiting parallelization techniques [17]. Relevant examples in power systems are the COI and the variables of secondary controllers, see [8], [9].

A qualitative comparison between (2) and (12) can be made by evaluating the small-signal dynamics of the two systems. Linearization of (12) around the equilibrium yields:

$$\mathbf{E}\,\Delta\mathbf{x}' = \mathbf{A}_0\,\Delta\mathbf{x} + \mathbf{A}_1\,\Delta\mathbf{x}_d\,, \tag{16}$$

TABLE I: Summary of matrix pencils discussed.

Pencil	Notation	k-th eig. (S-plane)	Approximation caused by
$s\,\mathbf{E} - \mathbf{A}$	$\boldsymbol{P}(s)$	s_k	None (original system)
$z\,(\mathbf{E} - 0.5h\mathbf{A}) - (\mathbf{E} + 0.5h\mathbf{A})$	$\boldsymbol{P}_{\rm ITM}(z)$	$s_k^{\rm ITM} = \frac{1}{h}\log(z_k^{\rm ITM})$	ITM
$s\,\mathbf{E} - \mathbf{A}_0 - \mathbf{A}_1 e^{-sh}$	$\tilde{\boldsymbol{P}}(s)$	$\tilde{s}_k$	OSDA
$z\begin{bmatrix}\mathbf{I}_r & \mathbf{0}_{r,r}\\ \mathbf{0}_{r,r} & \mathbf{E} - \dfrac{h}{2}\mathbf{A}_0\end{bmatrix} - \begin{bmatrix}\mathbf{0}_{r,r} & \mathbf{I}_r\\ \dfrac{h}{2}\mathbf{A}_1 & \mathbf{E} + \dfrac{h}{2}(\mathbf{A}_0 + \mathbf{A}_1)\end{bmatrix}$	$\tilde{\boldsymbol{P}}_{\rm ITM}(z)$	$\tilde{s}_k^{\rm ITM} = \frac{1}{h}\log(\tilde{z}_k^{\rm ITM})$	ITM-OSDA

where $\mathbf{A}_0$, $\mathbf{A}_1$, are the Jacobians of the delay-free and delayed variables of the system, respectively, and $\mathbf{A} = \mathbf{A}_0 + \mathbf{A}_1$. The pencil of (16) is:

$$\tilde{\boldsymbol{P}}(s) = s\,\mathbf{E} - \mathbf{A}_0 - \mathbf{A}_1 e^{-sh}\,. \tag{17}$$

Thus, with the formulation used in this paper[1], this task corresponds to comparing the eigenvalues of $\boldsymbol{P}(s)$ and $\tilde{\boldsymbol{P}}(s)$.

A. Proposed Analysis

Comparing $\boldsymbol{P}(s)$, $\tilde{\boldsymbol{P}}(s)$, allows assessing the approximation introduced by the OSDA to the system's dynamic modes. Moreover, it allows extracting an estimate of the maximum admissible one-step delay so that the approximation is lower than a specified threshold. However, when employing the OSDA in a TDI, the ability to increase the time step while maintaining a good precision is also limited by the numerical approximation caused by the integration method *per se*. An estimation of the maximum admissible step size that also accounts for the latter cannot be provided through $\boldsymbol{P}(s)$, $\tilde{\boldsymbol{P}}(s)$.

In this section we propose an approach to assess the effect of OSDA on the accuracy and numerical stability of the TDI. For simplicity but without loss of generality we consider only the ITM. Note that studying the impact on numerical stability is relevant, since the asymptotic stability of the ITM, as well as of any A-stable natural Runge-Kutta method, is not guaranteed for delay differential equations [18].

Linearizing (13) around the equilibrium yields:

$$\begin{aligned}(\mathbf{E} - 0.5h\mathbf{A}_0)\,\Delta\mathbf{x}_t =& [\mathbf{E} + 0.5h(\mathbf{A}_0 + \mathbf{A}_1)]\,\Delta\mathbf{x}_{t-h} \\ &+ 0.5h\mathbf{A}_1\Delta\mathbf{x}_{t-2h}\,.\end{aligned} \tag{18}$$

We define $\mathbf{y}_t = [\Delta\mathbf{x}_{t-h}^{\rm T}\;\; \Delta\mathbf{x}_t^{\rm T}]^{\rm T}$, $\mathbf{y}_{t-h} = [\Delta\mathbf{x}_{t-2h}^{\rm T}\;\; \Delta\mathbf{x}_{t-h}^{\rm T}]^{\rm T}$. Then, (18) can be equivalently rewritten as:

$$\mathbf{F}\,\mathbf{y}_t = \mathbf{G}\,\mathbf{y}_{t-h}\,, \tag{19}$$

where

$$\mathbf{F} = \begin{bmatrix}\mathbf{I}_r & \mathbf{0}_{r,r}\\ \mathbf{0}_{r,r} & \mathbf{E} - \dfrac{h}{2}\mathbf{A}_0\end{bmatrix},\; \mathbf{G} = \begin{bmatrix}\mathbf{0}_{r,r} & \mathbf{I}_r\\ \dfrac{h}{2}\mathbf{A}_1 & \mathbf{E} + \dfrac{h}{2}(\mathbf{A}_0 + \mathbf{A}_1)\end{bmatrix}.$$

The matrix pencil of system (19) is:

$$\tilde{\boldsymbol{P}}_{\rm ITM}(z) = z\,\mathbf{F} - \mathbf{G}\,. \tag{20}$$

The eigenvalues of $\tilde{\boldsymbol{P}}_{\rm ITM}(z)$ account, in the Z-plane, for the approximation introduced to the dynamics of the original system by the ITM combined with the OSDA technique. Hereafter we will refer to this combination as ITM-OSDA.

Let $\tilde{z}_k^{\rm ITM}$ be the eigenvalue of $\tilde{\boldsymbol{P}}_{\rm ITM}(z)$ that approximates the k-th mode of (2). The latter is represented by the eigenvalue s_k. Then, $\tilde{z}_k^{\rm ITM}$ becomes comparable to s_k by using a simple plane transformation, see also the relevant discussion in Section II-A. Finally, a summary of the matrix pencils $\boldsymbol{P}(s)$, $\boldsymbol{P}_{\rm ITM}(z)$, $\tilde{\boldsymbol{P}}(s)$ and $\tilde{\boldsymbol{P}}_{\rm ITM}(z)$ is provided in Table I. In the remainder of the paper, we study how $\tilde{s}_k^{\rm ITM}$ and $s_k^{\rm ITM}$ compare to each other and to the original system mode s_k.

IV. Case Study

This section presents an application of the proposed approach on the WSCC 9-bus system. The system consists of 6 transmission lines, 3 transformers, and 3 Synchronous Machines (SMs) equipped with Turbine Governors (TGs) and Automatic Voltage Regulations (AVRs). During transients, loads are modeled as constant admittances. SMs in this study are referred to the system's COI, i.e. the differential equation of the rotor angle of the i-th SM is:

$$f_{(\delta_i')} := \delta_i' = \Omega_b\big(\omega_i - \omega_{\rm COI}\big)\,, \tag{21}$$

where Ω_b is the nominal synchronous angular frequency in rad/s. The COI speed is defined by the algebraic equation:

$$g_{(\omega_{\rm COI})} := 0 = \omega_{\rm COI} - \sum_{i=1}^{3}\frac{M_i}{M_{\rm T}}\omega_i\,, \tag{22}$$

where ω_i, M_i are the rotor speed and the starting time of the i-th SM, respectively; and $M_{\rm T} = M_1 + M_2 + M_3$. Finally, SMs are assumed to provide secondary frequency regulation through an Automatic Generation Control (AGC). The AGC produces a dynamic active power command $p_{\rm s}$ as follows:

$$f_{(p_{\rm s}')} := p_{\rm s}' = K_i\left(\omega^{\rm ref} - \omega_{\rm COI}\right)\,. \tag{23}$$

The power command $p_{\rm s}$ is distributed to the TGs proportionally to their droops [19]. The power order $p_{{\rm r},i}$ received by the i-th TG is defined by the algebraic equation:

$$g_{(p_{{\rm r},i})} := 0 = p_{{\rm r},i} - \frac{R_i}{R_{\rm T}}p_{\rm s}\,, \tag{24}$$

where R_i is the droop of the i-th TG; and $R_{\rm T} = R_1 + R_2 + R_3$.

A. Example 1

OSDA in this section is performed by delaying (i) ω_i in (22), which removes $\partial g_{(\omega_{\rm COI})}/\partial\omega_i$ from the Jacobian, (ii) $\omega_{\rm COI}$ in (21), (23), which removes $\partial f_{(\dot{\delta}_i)}/\partial\omega_{\rm COI}$, $\partial f_{(p_{\rm s}')}/\partial\omega_{\rm COI}$, and (iii) $p_{\rm s}$ in (24), which removes $\partial g_{(p_{{\rm r},i})}/\partial p_{\rm s}$. This setup is not

[1] In this work, algebraic variables are treated as states with zero time constants and thus they are not eliminated from the linearized model. This preserves sparsity, while also preventing the birth of infinite spurious delays, a modeling issue that is present in the formulation of [8].

TABLE II: 15 rightmost eigenvalues and relative errors.

	$h = 0.01$ s				$h = 0.2$ s			
	ITM		ITM-OSDA		ITM		ITM-OSDA	
s_k	$s_k^{\rm ITM}$	$\epsilon_{\rm ITM}[\%]$	$\tilde{s}_k^{\rm ITM}$	$\tilde{\epsilon}_{\rm ITM}[\%]$	$s_k^{\rm ITM}$	$\epsilon_{\rm ITM}[\%]$	$\tilde{s}_k^{\rm ITM}$	$\tilde{\epsilon}_{\rm ITM}[\%]$
$-0.0200\ (\times 2)$	$-0.0200\ (\times 2)$	0.000	$-0.0200\ (\times 2)$	0.000	$-0.0200\ (\times 2)$	0.001	$-0.0200\ (\times 2)$	0.001
-0.0555	-0.0555	0.000	-0.0556	0.000	-0.0556	0.001	-0.0564	1.487
$-0.0997 \pm \jmath 0.1332$	$-0.0997 \pm \jmath 0.1332$	0.000	$-0.0994 \pm \jmath 0.1334$	0.221	$-0.0997 \pm \jmath 0.1332$	0.009	$-0.0926 \pm \jmath 0.1356$	4.468
$-0.1699 \pm \jmath 7.6696$	$-0.1697 \pm \jmath 7.6658$	0.049	$-0.1698 \pm \jmath 7.6659$	0.049	$-0.1070 \pm \jmath 6.5435$	14.701	$-0.1070 \pm \jmath 6.5436$	14.701
$-0.4263 \pm \jmath 0.4945$	$-0.4263 \pm \jmath 0.4945$	0.000	$-0.4263 \pm \jmath 0.4945$	0.000	$-0.4255 \pm \jmath 0.4950$	0.142	$-0.4255 \pm \jmath 0.4950$	0.143
$-0.4394 \pm \jmath 0.7390$	$-0.4394 \pm \jmath 0.7390$	0.001	$-0.4394 \pm \jmath 0.7390$	0.001	$-0.4373 \pm \jmath 0.7391$	0.246	$-0.4373 \pm \jmath 0.7391$	0.247
$-0.4441 \pm \jmath 1.2110$	$-0.4441 \pm \jmath 1.2110$	0.001	$-0.4441 \pm \jmath 1.2110$	0.001	$-0.4380 \pm \jmath 1.2075$	0.550	$-0.4380 \pm \jmath 1.2075$	0.551
$-0.6720 \pm \jmath 11.6396$	$-0.6697 \pm \jmath 11.6266$	0.113	$-0.6697 \pm \jmath 11.6266$	0.113	$-0.2851 \pm \jmath 8.6197$	26.114	$-0.2851 \pm \jmath 8.6197$	26.114

expected to notably impact on TDI accuracy, since COI and AGC dynamics are slower than the dynamics of interest [8].

We carry out a TDI[2] to show the impact of OSDA on the transient response of the system. To this aim, we consider a three-phase fault at bus 4, applied at $t = 1$ s and cleared after 80 ms by tripping the line between buses 4 and 5. Two step sizes are considered for the TDI, 0.01 and 0.2 s. The rotor speed of the SM at bus 2 is shown in Fig. 1. Increasing the step size from 0.01 to 0.2 s causes a significant distortion of the trajectory by the ITM. Compared to this distortion, the additional approximation introduced by the OSDA is negligible. In fact, the trajectories corresponding to the original and the OSDA system are basically indistinguishable for $h = 0.01$ s, while they show only a slight deviation for $h = 0.2$ s.

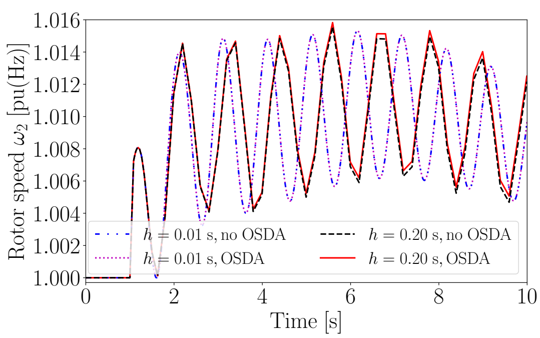

Fig. 1: Transient of ω_2 after the fault, ITM (with and without OSDA).

We discuss the effect of the ITM and OSDA on the accuracy of the TDI based on the proposed approach. Table II shows the relative percentage error introduced by the ITM to the 15 rightmost eigenvalues of the system when employed without and with the OSDA. The relative errors are defined as follows:

$$\tilde{\epsilon}_{\rm ITM} = \frac{|\tilde{s}_1^{\rm ITM} - s_1|}{|s_1|} \,, \quad \epsilon_{\rm ITM} = \frac{|s_1^{\rm ITM} - s_1|}{|s_1|} \,. \tag{25}$$

Results indicate that for $h = 0.01$ s, the ITM introduces a very small numerical distortion to the dynamic modes of the system. The conclusion is the same when the ITM is combined with the OSDA (ITM-OSDA). On the other hand, for a large time step ($h = 0.2$ s), the error caused by the ITM is significant and, most importantly, impacts the system's critical modes. This is also clearly shown in Fig. 2. The two most poorly

[2]Simulations are executed with the power system analysis software tool Dome [20].

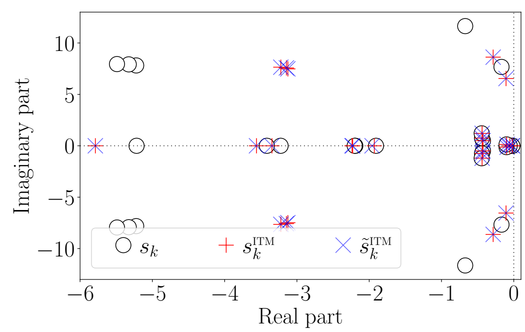

Fig. 2: Eigenvalue analysis, $h = 0.2$ s.

damped modes of the system are electromechanical and are represented by the complex pairs $-0.1699 \pm \jmath 7.6696$ and $-0.6720 \pm \jmath 11.6396$ with damping ratios 2.215%, 5.764%, respectively. The error $\epsilon_{\rm ITM}$ for these pairs is 14.7% and 26.1%, respectively. Compared to $\epsilon_{\rm ITM}$, the additional error due to the use of the OSDA, i.e. $\tilde{\epsilon}_{\rm ITM} - \epsilon_{\rm ITM}$, is negligible.

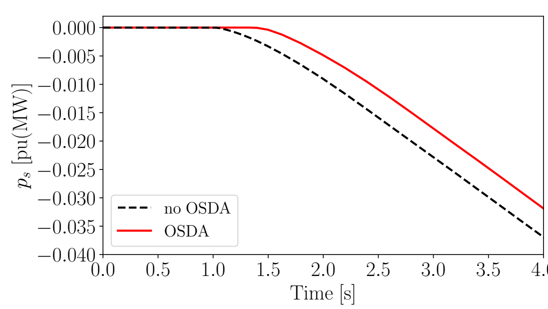

Fig. 3: Transient of p_s following the fault, ITM, $h = 0.2$ s.

For $h = 0.2$ s, the mode mostly impacted by the OSDA is the pair $-0.0997 \pm \jmath 0.1332$, for which $\tilde{\epsilon}_{\rm ITM} - \epsilon_{\rm ITM} = 4.459\%$. Participation analysis [16] shows that the variable mostly contributing to this mode is the AGC power command p_s. Figure 3 shows the trajectory of p_s for $h = 0.2$ s with and without the OSDA (we assume the same disturbance with Fig. 1) and indicates that, as expected, the impact of the one-step delay increases for larger values of h.

It is relevant to note that the proposed approach is based on SSSA and is thus technically valid only around steady-state solutions. That said, the structure of the modes of a power system as well as the qualitative properties of TDI methods are features that tend to be "robust" for varying operating conditions and hence, we can reasonably accept that results provide rough yet accurate estimates of the modes' distortion.

B. Example 2

The results above indicate that the OSDA does not substantially impact on the TDI accuracy, which was expected for the selected setup [8]. To further illustrate the features of the proposed approach, we consider an example of poor OSDA setup. In particular, apart from the elements already delayed, we also delay the input voltage signals of the AVRs, whose accurate representation is very important in transient stability analysis. The rightmost eigenvalues and the corresponding distorted eigenvalues for $h = 0.2$ s are presented in Fig. 4. Relative error analysis shows that the complex pair mostly impacted by the OSDA is $-0.4441 \pm \jmath 1.211$, for which $\tilde{\epsilon}_{\rm ITM} - \epsilon_{\rm ITM} = 14.2\%$. This mode is largely linked to the dynamics of the stator's transient voltage of the SM at bus 1.

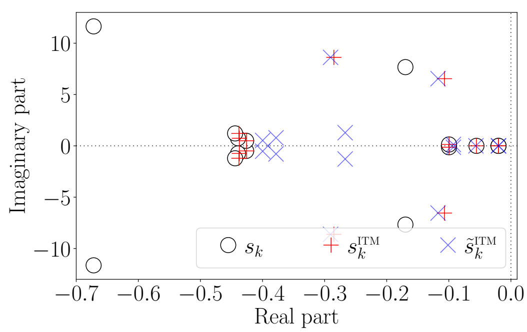

Fig. 4: Eigenvalue analysis, $h = 0.2$ s.

We focus on two modes, (i) the most poorly damped mode $-0.1699 \pm \jmath 7.6696$ and (ii) the mode mostly impacted by the OSDA, i.e. $-0.4441 \pm \jmath 1.211$. We will refer to these modes as M1 and M2, respectively. For M1, M2, Fig. 5 presents the relative errors introduced by ITM ($\epsilon_{\rm ITM}$) and ITM-OSDA ($\tilde{\epsilon}_{\rm ITM}$) as functions of the time step size. We see that the OSDA causes a significant distortion to M2, which in practice seriously impacts our ability to increase the step size while maintaining precision. Finally, a rough yet accurate estimation

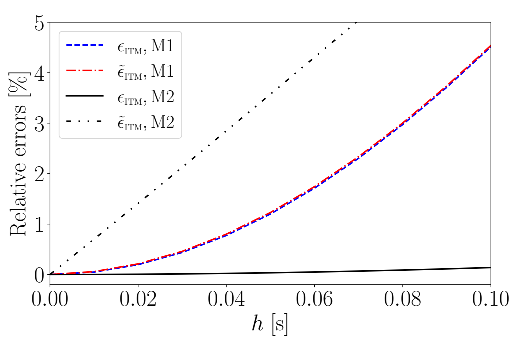

Fig. 5: Relative errors, $h = 0.2$ s.

of the maximum admissible time step size $h_{\rm max}$ may be obtained by setting an upper threshold to the relative error. For example, considering the 15 rightmost eigenvalues, we obtain that an error threshold of 0.7% yields $h_{\rm max} = 0.01$ s.

V. Conclusion

This paper presents a novel tool to assess the effect of the OSDA on the precision of the TDI of DAE power system models. The proposed approach is based on SSSA matrix pencils and thus avoids a full evaluation of the effect of one-step delays through time-domain simulations, which would require running a large set of disturbance scenarios and assessing the trajectories of many variables. The features of the proposed tool is tested on the WSCC 9-bus system integrated through the ITM, but the approach is general enough to be applied, in principle, to any numerical method and DAE model. In future work we will further exploit matrix pencil-based techniques to assess the stability and accuracy of numerical methods.

References

[1] P. Kundur, *Power system stability and control*. New York: Mc-Grall Hill, 1994.

[2] B. Stott, "Power system dynamic response calculations," *Proceedings of the IEEE*, vol. 67, no. 2, pp. 219–241, Feb. 1979.

[3] M. Ilic'-Spong, M. L. Crow, and M. A. Pai, "Transient stability simulation by waveform relaxation methods," *IEEE Transactions on Power Systems*, vol. 2, no. 4, pp. 943–949, 1987.

[4] M. La Scala, A. Bose, D. Tylavsky, and J. Chai, "A highly parallel method for transient stability analysis," *IEEE Transactions on Power Systems*, vol. 5, no. 4, pp. 1439–1446, 1990.

[5] P. Aristidou, D. Fabozzi, and T. Van Cutsem, "Dynamic simulation of large-scale power systems using a parallel schur-complement-based decomposition method," *IEEE Transactions on Parallel and Distributed Systems*, vol. 25, no. 10, pp. 2561–2570, 2014.

[6] F. Milano, *Power system modelling and scripting*, ser. Power Systems. Springer Berlin Heidelberg, 2010. [Online]. Available: https://books.google.ie/books?id=MQu7IqoLrfYC

[7] D. Fabozzi, A. S. Chieh, B. Haut, and T. Van Cutsem, "Accelerated and localized newton schemes for faster dynamic simulation of large power systems," *IEEE Transactions on Power Systems*, vol. 28, no. 4, pp. 4936–4947, 2013.

[8] G. Tzounas and F. Milano, "Delay-based decoupling of power system models for transient stability analysis," *IEEE Transactions on Power Systems*, vol. 36, no. 1, pp. 464–473, 2021.

[9] D. Fabozzi and T. Van Cutsem, "On angle references in long-term time-domain simulations," *IEEE Transactions on Power Systems*, vol. 26, no. 1, pp. 483–484, Feb. 2011.

[10] N. Watson and J. Arrillaga, *Power systems electromagnetic transients simulation*. London, UK: The IET, 2003.

[11] J. Mahseredjian, L. Dube, Ming Zou, S. Dennetiere, and G. Joos, "Simultaneous solution of control system equations in EMTP," *IEEE Transactions on Power Systems*, vol. 21, no. 1, pp. 117–124, Feb 2006.

[12] G. Tzounas, I. Dassios, and F. Milano, "Small-signal stability analysis of numerical integration methods," *IEEE Transactions on Power Systems*, 2021, under review. Available online at: faraday1.ucd.ie/tdistab.pdf.

[13] ——, "Stability analysis of implicit integration methods for power systems with time delays," *Electric Power Systems Research*, 2021, under review. Available online at: faraday1.ucd.ie/ddestabmap.pdf.

[14] F. Milano, I. Dassios, M. Liu, and G. Tzounas, *Eigenvalue problems in power systems*. CRC Press, Taylor & Francis Group, 2020.

[15] H. Hamdan and A. Hamdan, "On the coupling measures between modes and state variables and subsynchronous resonance," *Electric Power System Research*, vol. 13, no. 3, pp. 165 – 171, 1987.

[16] G. Tzounas, I. Dassios, and F. Milano, "Modal participation factors of algebraic variables," *IEEE Transactions on Power Systems*, vol. 35, no. 1, pp. 742–750, 2020.

[17] J. Fong and C. Pottle, "Parallel processing of power system analysis problems via simple parallel microcomputer structures," *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-97, no. 5, pp. 1834–1841, Sep. 1978.

[18] A. Bellen and M. Zennaro, *Numerical Methods for Delay Differential Equations*. Oxford, UK: Oxford Science Publications, 2003.

[19] C. W. Taylor and R. L. Cresap, "Real-time power system simulation for automatic generation control," *IEEE Transactions on Power Apparatus and Systems*, vol. 95, no. 1, pp. 375–384, Jan. 1976.

[20] F. Milano, "A Python-based software tool for power system analysis," in *Proceedings of the IEEE PES General Meeting*, Jul. 2013.