

# Dynamic Index Assessment for Voltage Stability in Electric Power Systems

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**Abstract:** Voltage instability as a non linear dynamic phenomenon is here investigated, by introducing a dynamic load flow (LF) Jacobian matrix as a specific index to assess the non existence of a stable new equilibrium point in response to perturbations. Reference is made to the Lyapounov exponent and bifurcation theories to provide extension of steady-state based indices for voltage stability, this time taking care of system distance from collapse during transients caused by step perturbations. Theoretical results are validated on the IEEE 30-node test network, accounting for the effect of typically committed components and regulation loops, such as synchronous generators, and voltage dependent loads under load tap changing transformers. Comparison with traditional modal analysis is also provided for sake of completeness.

**Keywords:** power system dynamic stability, voltage control, bifurcation theory, Jacobian matrices, load flow analysis.

## I. INTRODUCTION

The dynamic phenomena causing voltage instability, occurring in electric power systems subjected to strong load demands, lead to a progressive decreasing of the voltage magnitude at one or more buses, resulting sometimes in network islanding, thus leading to local or global blackouts.

In Italy, even if dramatic events have not occurred yet, there is a growing concern about voltage instability risks especially during early summer periods, when large reactive power consumption due to air conditioning devices drives the EHV system towards low voltage profiles.

Since environmental and economic constraints limit the construction of new generation and transmission systems, and power demands are predicted to increase, the voltage instability problem appears to be more and more topical.

In addition to these facts, advanced power system remedial controls are of growing intent to assist on-line application of corrective remedial action to counteract severe system contingencies.

Until now corrective measures have involved almost exclusively interruptions by event driven protection devices whereas is well recognised the helpfulness of continuous monitoring and adoption of optimised control systems. This kind of control can be fruitful based on selective on-line indices, and this paper proposes an index evaluation approach to voltage instability capturing.

The typical *slowness* of the voltage instability processes suggests treating the problem as a steady-state one, and power flow formulation or Q-V curves were used to get node sensitivity information [1]. From this point of view, sensitivity and modal analysis of standard (or reduced) LF Jacobian matrices provides analytical tools in determining bus and branch participation factors even for large and complex networks [2-4].

However, dynamic aspects can not be neglected in the framework for voltage collapse analysis and an accurate mathematical model of the network components is required, if

large perturbations occur and short-medium term transients become topical [5-6]. As a consequence the overall system should be modelled by a set of differential and algebraic equations, in which the LF equations are included as constraints. Because of the non-linear nature of this set of equations, earliest approaches were based on linearisation techniques well documented and providing information about local steady-state equilibria [7-11].

In the last years, bifurcation theory has led to a more accurate analysis of the behaviour of systems subjected to small perturbations; the small signal approach has been therefore refined with sophisticated analytical tools, that can highlight voltage stability and enable to classify power system equations in simple well-assessed forms [12-19].

By the way, bifurcation theory is mainly a local analysis and fails to investigate dynamic behaviour due to large perturbations that seems to be an important aspect of voltage collapse phenomena.

Our intent is to merge the steady-state approaches based on LF Jacobians and dynamic ones, in view of exploiting the power flow equations without losing the information stated by differential equations. The aim is to implement a hybrid criterion that could combine numerical integration methods with the computation of stability indices, like companion procedures proposed for transient stability assessment.

In this paper, a dynamic LF Jacobian matrix is used for the derivation of time-varying stability indices. These ones can afford the diagnosis of incipient voltage instability in system characterised by possible saddle-node bifurcation points, as well as the localisation of nodes, to which components concerned to the critical dynamic processes are connected. Indeed, reference to Lyapounov exponent theory [20] allows extensions to large perturbations and enables continuous monitoring of voltage stability margins.

Finally, validations of theoretical assessment is first performed on a simple radial network with controlled load voltage, and afterwards on the 30-bus IEEE test network, properly stressed in order to drive the network towards diffuse voltage collapse, accounting for dynamic model for generators.

## II. OUTLINES OF THE PROPOSED METHOD

The mathematical model of an electric power system can be described by a set of algebraic and differential equations, which are not linear:

$$\dot{x} = F(x, y) \quad (1)$$

$$0 = G(x, y) \quad (2)$$

where  $x \in R^n$ ,  $y \in R^m$  are vectors representing the state variables and the algebraic variables, and  $F: R^{n+m} \rightarrow R^n$  and  $G: R^{n+m} \rightarrow R^m$  are assumed to be smooth, i.e.  $F, G \in C^k$ ,  $k \geq 1$ .

The linearisation of (1) and (2) around a steady state operating point leads to the well-known form:

$$\begin{bmatrix} \dot{\Delta x} \\ 0 \end{bmatrix} = \begin{bmatrix} F_x & F_y \\ G_x & J_{LFV} \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} = A_C \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} \quad (3)$$

where  $J_{LFV}$  represents the LF Jacobian matrix where load models are taken into account and that is typically used in voltage collapse assessment.

The application of the Schur formula to the matrix  $A_C$ , with the hypothesis that  $F_x$  is not singular, leads to write the relationship:

$$\det(A_C) = \det(F_x) \det(J_{LFV} - G_x F_x^{-1} F_y) = \det(F_x) \det(J_{LFD}) \quad (4)$$

where a dynamic LF Jacobian matrix has been defined:

$$J_{LFD} = J_{LFV} - G_x F_x^{-1} F_y \quad (5)$$

In comparison with the traditional LF Jacobian matrices, the main advantages offered by the use of the dynamic LF Jacobian matrix, in a modal analysis approach, is that it ensures a link with the system differential equations, thus permitting the state variable dynamics to be taken into account. Moreover, since  $J_{LFD}$  preserves the structure of a LF Jacobian matrix, it can be reduced, in analogy with the procedure proposed in [4], into the matrix:

$$J_{LFD(R)} = J_{LFD(QV)} - J_{LFD(Q\theta)} J_{LFD(P\theta)}^{-1} J_{LFD(PV)} \quad (6)$$

Equation (6) defines a matrix whose dimension is equal to the number of the network buses, hence, estimating its eigenvalues and relative participation factors (as described in [4]) can be useful for localising physical nodes that are involved in instability phenomena.

The Jacobian matrix  $J_{LFD}$ , in addition with concepts of the bifurcation theory, is already been used in previous works [8, 11] for static or small signal studies.

With the intent of extending the conclusions that can be obtained by the bifurcation analysis beyond the neighbourhood of local equilibria, we adopt the approach that lead to Lyapounov exponent theory for studying time evolution of systems subjected to large perturbations [20].

In order to explain these concepts, let us represent with  $\Phi(x)$  the system equations (1) in which the algebraic equations (2) are assumed to be inverted for eliminating  $y$ , and take a reference trajectory  $x_r(t)$ , for which  $\dot{x}_r = \Phi(x_r)$  is satisfied. Let us now observe a neighbouring trajectory at an infinitesimal distance, which appertains to a small initial perturbation  $\tilde{x}(t_0)$ .

Neglecting higher-order terms, it can be asserted that:

$$\dot{\tilde{x}}(t) = (\partial\Phi / \partial x) \Big|_{x_r(t)} \tilde{x}(t) \quad (7)$$

where:

$$(\partial\Phi / \partial x) \Big|_{x_r(t)} = F_x \Big|_{x_r(t)} - F_y \Big|_{x_r(t)} [J_{LFV} \Big|_{x_r(t)}]^{-1} G_x \Big|_{x_r(t)} \quad (8)$$

Obviously, if the system is autonomous, as in the case of step perturbations, it is also  $\tilde{x}(t) \equiv \dot{x}_r(t)$ , hence the system behaviour can be stated by the stability of  $\tilde{x}(t)$ . However the general structure of equation (7) could be useful for stability studies for non-autonomous systems.

It has been stated that  $\tilde{x}(t)$  represents the difference between the reference trajectory and the new trajectory of the system, varied by an initial infinitesimal  $\tilde{x}(t_0)$ . This statement can seem in contrast with the intent of studying dynamics of power system subjected to large perturbations, but it is not the case. We just need the formal presence of an initial small variation  $\tilde{x}(t_0)$  to make possible the identification of (7) and the consequent inferences about system stability.

In order to explain this aspect, we limit our investigation to *saddle-node bifurcations*, and, in the following section, we will consider power systems where the presence of this bifurcation in dynamic equation has been proven to cause voltage instabilities [11, 17, 18].

It is possible to demonstrate [20] that this kind of bifurcation is *structurally* stable, where with structural we intend the ability of the system of proving robust with regard to small deviations. In particular saddle-node bifurcation is structurally stable with regard to small variations of a constant term, of lower-order terms and of higher-order terms in the Taylor series of the vector field in the neighbourhood of a non-hyperbolic fixed point.

This property allows assuming that the varied system does not behave differently from the reference trajectory, that is the real trajectory along which the system is evolving.

On the other hand, reference trajectories are not necessarily conditioned by small or infinitesimal parameter variations with respect to initial equilibrium point, but can be generated by any kind of perturbation.

The assumption about structural stability of system equations allows investigating the behaviour of non-linear systems during their time evolution. Furthermore, the presence of saddle-node bifurcation equilibrium points suggest that the state variable evolution is mainly monotonic and characterised by a convexity change if an instability occurs. Another consequence is that if one real eigenvalue of  $\partial\Phi/\partial x$  becomes and remains positive the state variables  $x_r(t)$  and  $\tilde{x}(t)$  diverge.

Furthermore, the change of sign of a real eigenvalue results in a change of sign the determinant of the Jacobian matrix  $\partial\Phi/\partial x$  and this fact leads to a criterion for detecting system instabilities.

Applying once again the Schur formula to  $A_C$ , and from (4), (5), (6) and (8), it can be derived that:

$$\det \left[ \frac{\partial\Phi}{\partial x} \right] = \det[F_x] \det[J_{LFD(P\theta)}] \frac{\det[J_{LFD(R)}]}{\det[J_{LFV}]} \quad (9)$$

where it has been assumed that the  $(P\theta)$  partition of  $J_{LFD}$  is not singular.

Equation (9) reveals the possibility of monitoring the time dependent function  $J_{LFD(R)}$  instead of  $\partial\Phi/\partial x$  with the aim of determining stability indices during dynamic evolution of the system.

Whereas the determinant produces a global information, the computation of the eigenvalues and eigenvectors of  $J_{LFD(R)}$  makes possible, as said above, the localisation of nodes more interested in instability processes. It has to be noted that in [4] this technique is applied to a reduced standard Jacobian matrix computed at an equilibrium point, whereas our computations are assumed to be made on a time-varying matrix whose variation is driven by system dynamic evolution.

In (9) appears also that  $J_{LFV}$  is assumed to be not singular, whereas it is well assessed the importance of this matrix in detecting singularity induced bifurcations, i.e. mismatch of the algebraic equations [15, 19]. By the way it will be shown in the following section that singularity induced bifurcation and relative voltage collapse happen only after that one eigenvalue of the dynamic LF Jacobian has changed sign and then after the detection of voltage instabilities.

Summarising, our method can be divided into three main points that are repeated at each time simulation step:

- determination of the system state variables;
- computation of eigenvalues of  $J_{LFD(R)}$  and their participation factors to network buses;
- check of eigenvalue changing sign.

When a change of sign is recognised, it is also detected an incipient instability, and system voltage collapse is going to happen if no corrective action is taken.

It has to be noted that static studies based on scalar parameter sensitivity are generally the more traditional way to approach this kind of events, such as voltage collapse or voltage instability [3, 7]. Furthermore, since the instability process is characterised by a monotonic evolution of the state variables, simulation could be avoided and a direct detection of the final equilibrium point, if it exists, could be used.

By the way, classical static methods that manage to produce load margin curves or determine the point of collapse of the system must be repeated for each parameter taken into account, and generally are used in off-line studies and in predictive controls.

Our method, instead, appears to be independent from the choice of the parameter variation or from the applied perturbation, and could be likely utilised in on-line applications as a system monitoring for corrective evaluations. In this case, the acquisition of measures from the network and the estimation of the state variables could replace the first step of the method.

It is also to be noted that for very large networks, the computation of all eigenvalues of  $J_{LFD(R)}$  could be a considerably effort, but it is also been demonstrated [4] that only the lowest eigenvalues are really informative and for this aim algorithms that extract the lowest eigenvalues are been successfully developed.

Finally, we have to underline that the structure of  $J_{LFD(R)}$  has the same degree of sparsity of the more traditional LF Jacobians and only sparse matrix notation is used in the simulation examples to achieve dynamic indices. In such a way, the computation of  $J_{LFD(R)}$  appears to be reliable even for huge networks, whose sparsity degree is generally very high.

### III. VALIDATION ON TEST CASES

The possibility of monitoring voltage stability by the calculation of the determinant of the dynamic LF Jacobian matrix is investigated in this section through time domain simulations of power systems with dynamic components. The examples concern a simple radial system with an LTC transformer and the IEEE 30-bus test network with five synchronous machines. For sake of clarity, in the first system, it is defined the algebraic and differential model and it is pointed out the existence of an equilibrium point where a saddle-node bifurcation occurs. Finally, time domain simulation in which the system runs into voltage instability is presented, comparing time evolutions of the standard LF Jacobian with the  $\det(J_{LFD(R)})$  one. In the second example only results are reported and an utilisation of participation factors of  $J_{LFD(R)}$  eigenvalues is presented.

#### A. Static Load under Controlled LTC Transformer

Fig. 1 represents a simple radial network feeding a static load by an LTC transformer. The term  $x_L$  stands for the line reactance, whereas the effect of LTC internal voltage drop is neglected, for sake of simplicity.

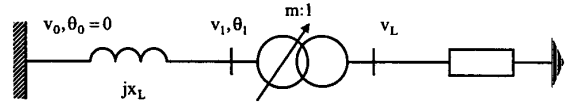


Fig. 1 - Equivalent circuit of the sample power system including an LTC transformer.

The tap ratio varies continuously without hard limits and the secondary voltage regulator is assumed to be proportional with one pole, as shown in Fig. 2. Indeed  $h \ll k$ , so that the behaviour of the regulator is quasi-integral.

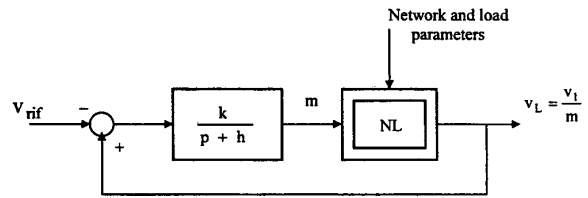


Fig. 2 - Typical control loop of an LTC transformer.

If the real and reactive load demands are modelled according to conventional voltage dependent laws, the following per unit equations can be identified:

$$\dot{m} = -hm + k \left( \frac{v_1}{m} - v_{rif} \right) \quad (10)$$

$$0 = P_L \left( \frac{v_1}{m} \right)^{\alpha_p} + P_1 \quad (11)$$

$$0 = Q_L \left( \frac{v_1}{m} \right)^{\alpha_q} + Q_1 \quad (12)$$

where  $P_1$  and  $Q_1$  are the power injections at node 1 and are linked to the network by the traditional LF equations, written

considering the voltage magnitude and phase angle at the ideal primary winding as the LF variables.

The linearised equations lead to the following Jacobian matrices:

$$J_{LFD} = J_{LFV} - \begin{pmatrix} 0 & \alpha_p P_L \left( \frac{v_1^{\alpha_p}}{m^{\alpha_p+1}} \frac{k/m}{h + (kv_1/m^2)} \right) \\ 0 & \alpha_q Q_L \left( \frac{v_1^{\alpha_q}}{m^{\alpha_q+1}} \frac{k/m}{h + (kv_1/m^2)} \right) \end{pmatrix} \quad (13)$$

If we assume constant impedance load, i.e.  $\alpha_p \alpha_q = 2$ , the differential equation becomes:

$$\dot{m} = -hm + k \left[ v_0 \frac{m}{\sqrt{x_L^2 (P_L^2 + Q_L^2) + 2Q_L x_L m^2 + m^4}} - v_{rif} \right] \quad (14)$$

Assuming  $Q_L$  as the parameter and keeping other quantities constant ( $v_{rif} = 1$ ,  $v_0 = 1$ ,  $P_L = 0.6$ ,  $x_L = 0.3$ ,  $k = 0.1$ ,  $h = 0.001$ ), the system presents one possible saddle-node bifurcation point occurring at  $m = 0.531$  and  $Q_L = 0.725$ .

Fig. 3 shows the dynamic evolution of the system when it is forced by a step variation of the reactive power  $Q_L$  from 0.3 to 0.8 (p.u.): since the reactive power demand increases over the value for which bifurcation occurs, the introduced stability index, namely  $\det(J_{LFD})$ , change sign (Fig. 4) and the system subsequently gets into voltage instability, well before the voltage collapse detected by the zero crossing of  $\det(J_{LFV})$ .

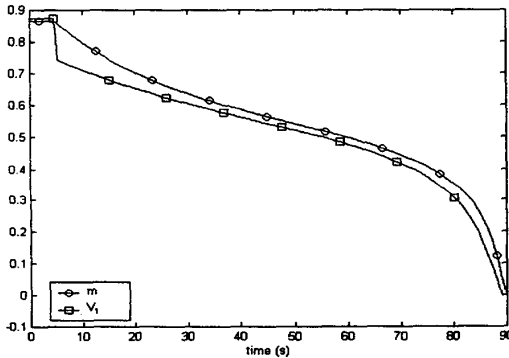


Fig. 3 - Dynamic evolution of tap ratio ( $m$ ), voltage at node 1 ( $v_1$ ).

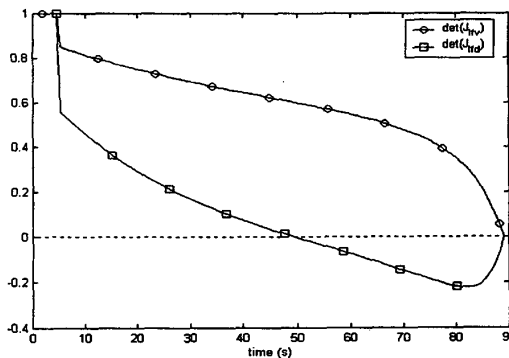


Fig. 4 - Dynamic evolution of the determinant of both  $J_{LFV}$  and  $J_{LFD}$ .

## B. Overloaded IEEE 30-node test network

The presented voltage stability assessment method is now extended to a multi-dimensional problem, that is the standard topology of IEEE 30-node test network, reported in Fig. 5, with a rough overloading factor of 140 % with regard to the usually proposed steady-state system profile.

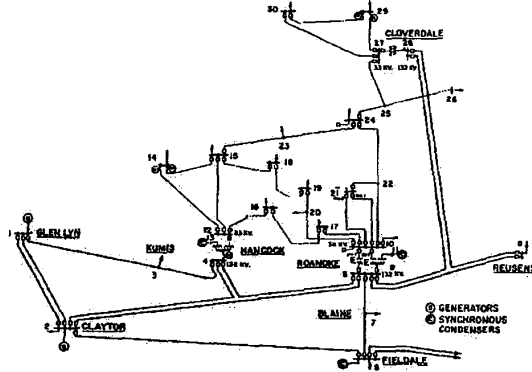


Fig. 5 - IEEE 30-node test network.

The overall test case provides a set of differential equations, initially associated to components like synchronous machines and condensers, each represented by a third-order dynamic model; on the whole, we must account for 15 state variables, among which 5, i.e.  $e'_p$ , are closely related to the voltage stability problem.

In order to cause voltage instability phenomena, it is assumed a nearly sudden increase, placed at each node presenting loads, of required reactive power from 140 % to 170 % of the reference load profile. No modifications are assumed for real power requirements, considering the well-known decoupling effect, which reduces the influence of active power perturbations on node voltage dynamics. If otherwise considered, we will exclusively notice a sort of noise effect, of limited amount, on voltage stability index behaviour, due to electromechanical oscillations.

Fig. 6 reports the evolution of node voltages in response to the previously introduced contingency.

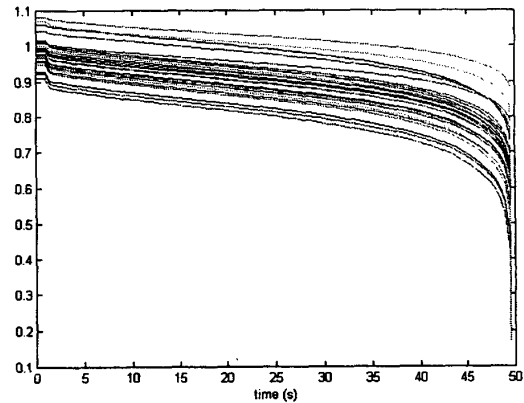


Fig. 6 - Voltage profiles at network nodes.

Insight of the presented dynamic behaviours reveals that each voltage magnitude shows an aperiodic profile and that loss of system stability is visually associated to a couple of detectable phenomena. The first one is an almost contemporaneous, according to the presented time scale, change in curve convexities, placed nearly at 20 s after the simulated contingency. This lead towards a general diverging effect on network voltage profile. The second main effect, obviously correlated to the first cited one, and located around 50 s, is a common collapse of node voltages, analytically linked to the loss of causality of the algebraic model.

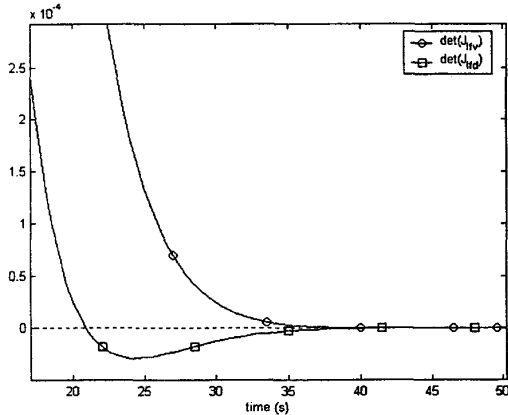


Fig. 7 – Dynamic evolution of the determinant of both  $J_{LFV}$  and  $J_{LFD}$  zoomed around instability detection.

Fig. 7 shows, as introduced in chapter II, a zoom of time evolution of the determinant of both  $J_{LFD}$  and  $J_{LFV}$ , confirming how loss of causality occurs after the detection of instability.

According to the previously proposed theoretical developments, it results quite significant to detail on  $J_{LFD(R)}$  eigenvalues, considered in their temporal evolution. With regard to this point, Fig. 8 proposes, obviously on the same time frame, the smallest eigenvalues, closely related to voltage stability assessment, since presumably better candidates for sign changing. In fact, a sorting algorithm is included in eigenvalues determination, so that at every time step a eigenvalue classification is provided, with the associated chance to link them to specific nodes, by computing participation factors.

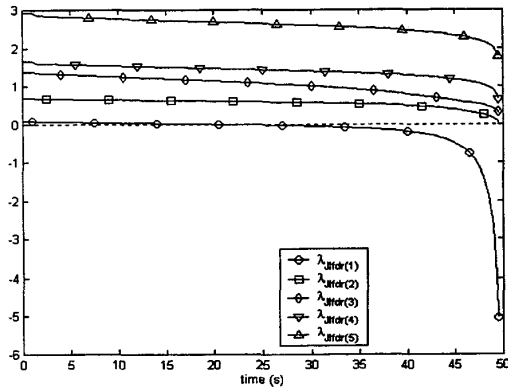


Fig. 8 – Time evolution of the lower eigenvalues of the reduced dynamic LF Jacobian matrix.

In other words, eigenvalue dynamic behaviour could present order modifications, but this occurrence is surely detected, and consequently correlated to a specific network zone.

The previously introduced participation factors are the instruments to connect system dynamic instability to relevant topology: each eigenvalue presents prevailing participation factors towards specific network areas, that is a voltage instability phenomenon is immediately associated to a part of the system, and subsequently, as Fig. 9 confirms, more and more clearly connected to a single node.

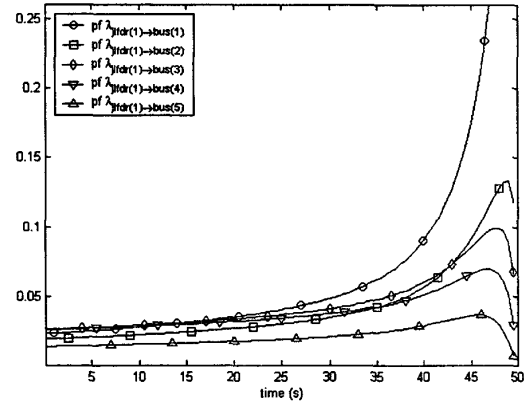


Fig. 9 – Participation factors of the lowest eigenvalue of the reduced dynamic LF Jacobian matrix.

Electromechanical oscillations affect in a really limited way the proposed dynamic behaviours, resulting in small ripple effects. As a proof, simulations have been performed considering a first order dynamic model for all rotating machines, with no significant differences in the interesting quantities.

On the other hand, since  $J_{LFD(R)}$  eigenvalues could be used for on-line applications, the remarks on electromechanical oscillations would justify the use of a signal low-pass filter to avoid inaccuracies, or continuous on/off effect, in sign changing detection.

Account for AVR or over-excitation limiters (OXL) on synchronous generators, even if advisable for more complete machine modelling, does not affect the overall system behaviour and the proposed conclusions. This occurs since AVR and/or OXL do not introduce additional saddle-node bifurcations, but only could result in Hopf ones [18].

Moreover, AVR saturation leads the dynamic form of the system to the third order computed one, as well as OXL intervention only results in a smoother occurrence of voltage collapse.

#### IV. CONCLUSION

This paper has presented the description of a criterion that allows estimating the occurrence of voltage instability processes in electrical power systems.

Because a particular dynamic LF Jacobian matrix is used, typical concepts of static approaches are combined to standard time domain simulations.

The proposed method appears to be independent from the choice of parameter variations or from applied perturbations,

and could be likely utilised in on-line applications as a system monitoring for corrective evaluations.

The criterion has been successfully applied to the IEEE 30-node test network with five synchronous machines. The computation of the smallest eigenvalues and relative participation factors of the reduced dynamic LF Jacobian matrix has proved to be informative about the localisation of nodes, to which components concerned to the critical dynamic processes are connected.

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## VI BIOGRAPHIES

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