A Framework to embed the Unit Commitment Problem into Time Domain Simulations

Abstract—**This paper proposes a software framework to embed the unit commitment problem into a power system dynamic simulator. A sub-hourly, mixed-integer linear programming Security Constrained Unit Commitment (SCUC) with a rolling horizon is utilized to account for the variations of the net load of the system. The SCUC is then included into time domain simulations to study the impact of the net-load variability and uncertainty on the dynamic behavior of the system using different scheduling time periods. A case study based on the 39-bus system illustrates the features of the proposed software framework.**

Index Terms—**Unit commitment, time domain simulation, power system dynamics, frequency stability.**

I. Introduction

A. Motivation

Conventional Unit Commitment (UC) problems are solved with an hourly time period [1]. This time scale is well decoupled from relevant power system dynamics. However, a sub-hourly UC is to be preferred in systems with high penetration of stochastic renewable energy sources (RES) [2] as these increase variability and uncertainty and at the same time decrease the overall inertia of the system [3]. A sub-hourly UC, e.g., 5 minute resolution, can overlap with long-term dynamics. While there exist several attempts to include simplified dynamic constraints into the UC problem, the other way round, i.e., embedding the UC problem into a fully-fledged transient stability analysis software tool has not been thoroughly discussed so far. This paper attempts to fill this gap.

B. Literature Review

The UC problem plays a crucial role in the secure operation of power systems. Most of the electric utilities in US clear the markets using a Security Constrained Unit Commitment (SCUC) [4]. The level of system security in these models is mainly defined by network constraints equations (e.g., line flow limits) and the amount of scheduled reserves. In order to guarantee the dynamic stability of low-inertia systems, additional dynamic constraints need to be added in the UC formulation [5]. With this aim, the authors in [6] proposed two frequency dynamic constraints, one for the maximum rate of change of frequency (RoCoF) and a second one for the minimum frequency. The purpose of the model proposed in [7] was to make sure that enough inertia is available online by introducing static and dynamic constraints on the total kinetic energy of the system. In [8], the inclusion of a linear frequency limit constraint in to SCUC was shown to be effective to keep frequency variations within acceptable limits. Other works considered frequency constraints in stochastic scheduling problems, e.g., [9], [10].

The aforementioned security constraints can help define unit scheduling that are acceptable from the stability point of view. Nevertheless, they do not provide a clear overview on the dynamic behavior of the system. Moreover, the SCUC discussed above are all modeled at an hourly scheduling.

In recent years, Transmission System Operators (TSOs) have acknowledged the need for sub-hourly scheduling (e.g., 15 minutes) to better accomodate the variability introduced by RES [11]. Such models are able to give more accurate results when it comes to unit cycling, which are expected to be higher than in traditional hourly models [12]. In [13], it was shown that using a rolling horizon approach, i.e., scheduling the system more frequently using a forecast moving window, the required reserves can be decreased. Finally, in [14] and [15], the solution of the UC is utilized to study the impact of RES with respect to active power imbalances and stability issues, respectively, but the dynamics of the system are oversimplified and linearized.

C. Contributions

All references mentioned above, study the impact of some sort of dynamic constraints on the electricity market. To the best of our knowledge, however, there is no systematic work that analyses the impact of the UC problem on the dynamic response of the power system. We thus discuss a software framework to solve the UC problem within the time domain simulations. This framework allows having a better overview of system reliability and testing the impact on stability and security of different UC models. In this paper, the focus is on the frequency response following a UC schedule.

D. Paper Organization

The remainder of the paper is organized as follows. Section II describes the mathematical formulation of SCUC and the power system model for transient stability analysis. The case studies on the modified IEEE 39-bus system and the respective results are discussed in Section III. Conclusions and outlines on future work are given in Section IV.

II. Modeling

A. Unit Commitment Formulation

The main objective of the UC problem is to minimize the total operating cost and determine for a given planning horizon the ON/OFF status of the generating units needed to match the demand. It is common practice to formulate the UC as a Mixed-Integer Linear Programming (MILP) problem thanks to the good performance of the available commercial solvers [16]. In this paper, a conventional MILP SCUC problem is used [17]. The mathematical formulation is recalled below.

1) Objective function: The objective function of the SCUC to be minimized consists of the fixed, variable, start-up and shut-down costs of the generating units, as follows:

$$\sum_{t\in\mathcal{T}}\sum_{g\in\mathcal{G}}(C_g^F z_{g,t}^F + C_g^V p_{g,t} + C_g^{SU} z_{g,t}^{SU} + C_g^{SD} z_{g,t}^{SD}) \qquad (1)$$

where t is the index for the time period; g is the index for the generating units; $\mathcal{T}$ is the set of time periods, e.g., $\{1,\dots,24\}$ hours; $\mathcal{G}$ is the set of generating units; $z_{g,t}^F$ is the binary variable that represents the status of the units in time period t, e.g., 1 if ON; $z_{g,t}^{SU}$ and $z_{g,t}^{SD}$ are the binary variables that represent the status of the units at the beginning of time period t, i.e., $z_{g,t}^{SU}=1$, $z_{g,t}^{SD}=0$ if the generator is up and $z_{g,t}^{SU}=0$, $z_{g,t}^{SD}=1$ is the generator is down; and $p_{g,t}$ is the continuous variable representing the active power production during time period t.

2) Binary variable constraints: These constraints are needed to ensure the consistency of the logic of binary variables. For example, if a unit is ON in a given period t, then it can only be switched OFF but not started-up in the following period. The constraints are:

$$z_{g,t}^{SU} - z_{g,t}^{SD} = z_{g,t}^F - z_{g,t-1}^F, \quad \forall g\in\mathcal{G}, \forall t\in\mathcal{T}, \qquad (2)$$

$$z_{g,t}^{SU} + z_{g,t}^{SD} \le 1, \quad \forall g\in\mathcal{G}, \forall t\in\mathcal{T}, \qquad (3)$$

$$z_{g,t}^F, z_{g,t}^{SU}, z_{g,t}^{SD} \in \{0,1\} \quad \forall g\in\mathcal{G}, \forall t\in\mathcal{T} \qquad (4)$$

Note that the sum of the start-up and shut-down binary variables cannot be greater than one. Note also that for the first time period, the initial status of the unit, namely $z_{g,0}^F$ has to be known in (2) and is thus an input datum. When the model steps forward (i.e., rolling horizon) the status of the units at the end of the horizon serve as an initial status for the next planning horizon, and so on.

3) Power bounds: Typical technical constraints of the generating units include their upper and lower limits:

$$P_g^{\min} z_{g,t}^F \le p_{g,t} \le P_g^{\max} z_{g,t}^F, \quad \forall g\in\mathcal{G}, \forall t\in\mathcal{T}, \qquad (5)$$

where, $P_g^{\min}$ and $P_g^{\max}$ are the minimum and maximum active power limits, respectively.

4) Ramping limits: Other important constraints of generation units are the so-called ramping limits. For instance, between two successive time periods, a unit output is bounded by a maximum value called the ramping-up limit. Also, a start-up limit is applied to the unit power output during its start-up time. Similar considerations apply to the ramping-down and shut-down ramping limits. The ramping limit constraints are:

$$p_{g,t} - p_{g,t-1} \le R_g^U z_{g,t-1}^F + R_g^{SU} z_{g,t}^{SU}, \ \forall g\in\mathcal{G}, \forall t\in\mathcal{T}, \qquad (6)$$

$$p_{g,t-1} - p_{g,t} \le R_g^D z_{g,t}^F + R_g^{SD} z_{g,t}^{SD}, \quad \forall g\in\mathcal{G}, \forall t\in\mathcal{T}, \qquad (7)$$

where, $R_g^U, R_g^{SU}, R_g^D, R_g^{SD}$ are the ramping-up, start-up ramping, ramping-down and shut-down ramping limits, respectively. Since ramping limits are generally given in per hour, we divide the hourly data by the relevant sub-hourly scheduling interval, e.g., by 4 in the 15 min case. Again, $z_{g,0}^F$ has to be assigned for the first time period.

5) Power balance: The following constraints ensures the active power balance at every node of the network:

$$\begin{aligned} &\sum_{g\in\mathcal{G}_n} p_{g,t} - \sum_{j\in\mathcal{D}_n} d_{j,t} = \\ &\sum_{m\in\mathcal{L}_n} B_{nm}(\delta_{n,t}-\delta_{m,t}), \ \ \forall n\in\mathcal{L}, \ \ \forall t\in\mathcal{T}, \end{aligned} \qquad (8)$$

where $d_{j,t}$ is the forecasted demand located at node n; $\mathcal{L}$ is the set of all branches; B_{nm} is the susceptance of transmission line $n-m$; and $\delta_{n,t}$ and $\delta_{m,t}$ are the voltage phase angles at nodes n and m, respectively.. The set $\mathcal{G}_n$ indicates the generators connected to bus n. Similarly, $\mathcal{D}_n$ and $\mathcal{L}_n$ are the demands and lines, respectively, connected to bus n.

6) Transmission lines limits: Generally, the power through a transmission line is limited by its thermal limit:

$$\begin{aligned} &-P_{nm}^{\max} \le B_{nm}(\delta_{nt}-\delta_{mt}) \le P_{nm}^{\max}, \\ &\forall n\in\mathcal{L}, \ \ \forall m\in\mathcal{L}_n, \ \ \forall t\in\mathcal{T}_n, \end{aligned} \qquad (9)$$

where, $P_{nm}^{\max}$ is the capacity limit of the line.

7) Security constraints: System operators usually schedule some spinning reserves in order to cope with unforeseen events, e.g., an unscheduled outage. So, for all time periods, the total generation available online has to be greater than the actual demand:

$$\sum_{g\in\mathcal{G}_r} P_g^{\max} z_{g,t}^F \ge \sum_{j\in\mathcal{D}} (d_{j,t}+R_{j,t}), \quad \forall r\in\mathcal{R}, \forall t\in\mathcal{T}, \qquad (10)$$

where $d_{j,t}$ is the system total forecasted demand; $R_{j,t}$ accounts for reserves; and $\mathcal{G}_r$ is the set of generators that provides reserve (in the following, we assume $\mathcal{G}_r \equiv \mathcal{G}$). For simplicity, the amount of reserve is assumed to be a percentage of the total demand. The reserve percentage value is lower for shorter scheduling timescales assuming that better forecast is available [13].

8) Reference angle: Finally, as it is well-known, the voltage phase angle at some node of the network has to be assigned:

$$\delta_{n,t} = 0, \quad \forall t\in\mathcal{T}, \qquad (11)$$

where n is the node chosen to be the reference angle.

9) Remarks on the UC model: Equations (1)-(11) form a conventional model of SCUC. The aim, in fact, is not to propose a novel formulation of the UC but rather to show how the UC can be embedded into the dynamic model of power systems. In the following, the SCUC is modeled using different sub-hourly time periods, ranging from 15 to 3.75 minutes. Moreover, a rolling (moving window) approach with a planning horizon of 24 hours is considered to account for better forecast. In other words, the SCUC problem (1)-(11) is solved at every time period t for the next 24 hours.

The average demand of each load is assumed to vary as a piece-wise linear function according to a predefined profile. To simulate uncertainty, the values $d_{j,t}$ utilized to solve the SCUC problem at each period differ from the actual demand of the loads by a given percentage. A normal distribution function with different standard deviations per period, say, σ_t, is used to generate the forecast error. The value of the standard deviation increases linearly as a function of t. Specifically, σ_t is null for current loading condition, i.e., $t = 0$, and is maximum for the last period of the planning horizon $\mathcal{T}$.

B. Power system model

Power systems can be modeled as a set of hybrid differential algebraic equations (HDAEs) [18], as follows:

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{u}, \boldsymbol{z}) \\ \boldsymbol{0} &= \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{u}, \boldsymbol{z}) \ , \end{aligned} \tag{12}$$

where $\boldsymbol{f}$ are the differential equations, $\boldsymbol{g}$ are the algebraic equations, $\boldsymbol{x}$, $\boldsymbol{x} \in \mathbb{R}^n$ are the state variables (e.g., generator rotor speeds), and $\boldsymbol{y}$, $\boldsymbol{y} \in \mathbb{R}^m$, are the algebraic variables (e.g., bus voltage angles); $\boldsymbol{u}$, $\boldsymbol{u} \in \mathbb{R}^q$, are the inputs (e.g., load forecast, generator bids); and $\boldsymbol{z}$, $\boldsymbol{z} \in \mathbb{R}^p$, are the discrete variables (e.g., status of the machines).

Equations (12) represent the conventional model of power systems for angle and voltage stability analysis. They includes dynamic models of synchronous machines, turbine governors, automatic voltage regulators, automatic generation control, and the discrete model of SCUC, just to mention some. These models are not discussed here for space limitation. The interested reader is referred to [18] for a detailed description of power system models.

C. Interaction between UC and DAEs

The solution of the SCUC, namely $p_{g,t}$, $\forall g \in \mathcal{G}$, is utilized to change the power set point of the turbine governors of the power plants. Figure 1 shows the connection and the interactions between SCUC, turbine governors, generators, demands and the rest of the grid. The SCUC is implemented in the Python language and solved using Gurobi [19], while all simulations are obtained using Dome, a Python-based software tool [20].

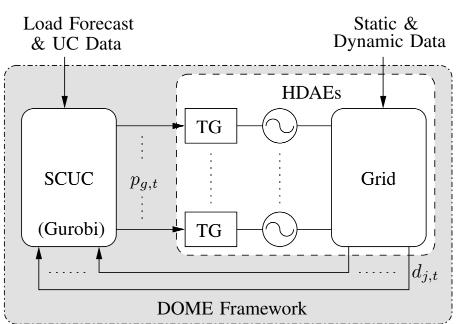

Fig. 1: Interaction between the SCUC problem and the dynamic model of the turbine governors, the synchronous machines and the grid.

III. Case Studies

A modified IEEE 39-bus system is used to demonstrate the effectiveness of the proposed framework. Several scenarios are considered to study the impact of the uncertainty of the net load, i.e., total load minus RES generation, on the dynamic behavior of the system. The first four scenarios compare the effects of the scheduling period of the UC problem on frequency variations. Periods of 15, 10, 7.5 and 3.75 minutes are considered. Then, the impact of net load volatility (short-term noise) on the frequency of the system is discussed. Finally, the combined impact of uncertainty and a large contingency on frequency deviations is shown.

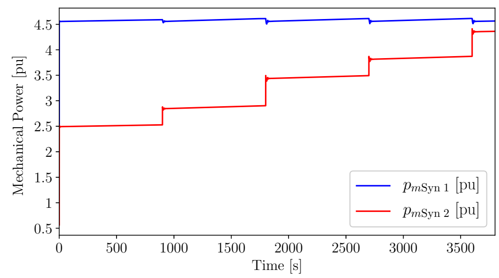

Fig. 2: Mechanical power of two relevant machines for the 15 minute scheduling period.

The data of the SCUC are based on [21]. Since these data do not correspond with that of the dynamic IEEE 39-bus system [22] (e.g., different loading levels), we adapted (scaled) the relevant dynamic data to that of 10-machine UC. While the rolling UC is solved for 24 hours at every period, in the discussions below, we show only the first hour of the planning horizon, which, according to these data has a demand forecast of 700 MW.

The total number of state and algebraic variables of the system for the first four case studies is 131 and 223, respectively, while in the fifth case study we have an increase in the number of state variables, from 131 to 169 (i.e., noise added to the 19 loads of the system). The total computing times to solve the TDS and the SCUC for the 15, 10, 7.5, 3.75 minute scheduling periods and the noise case are 1 min and 34 s, 1 min and 39 s, 1 min and 44 s, 2 min and 25 s and 4 min and 46 s, respectively.

A. 15 Minute Scheduling

In this scenario, SCUC is solved four times and a maximum of 30% of uncertainty is applied to the end of the first hour. Figure 2 shows the mechanical power of two relevant synchronous machines whereas Fig. 3 shows the transient behavior of the frequency of the center of inertia of the system (ω_{COI}).

Electro-mechanical oscillations occurs every 15 minutes due to a change in the operating points of the synchronous machines enforced by the SCUC. In the periods between two scheduling events of the SCUC, machine powers vary due to load ramps, but the frequency is almost steady state thanks to the action of primary and secondary frequency regulations.

The average value of the objective function for these four periods is found to be \$553,346.4.

B. 10 Minute Scheduling

In this scenario, the hour is divided into 6 scheduling time intervals and uncertainty is added to the load but it is

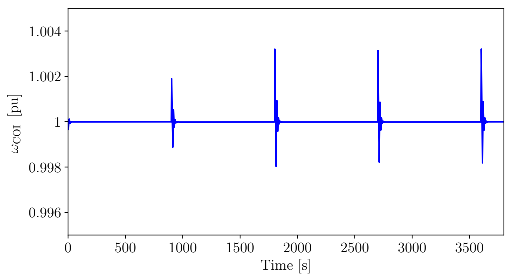

Fig. 3: Frequency of the center of inertia for 15 minute scheduling period.

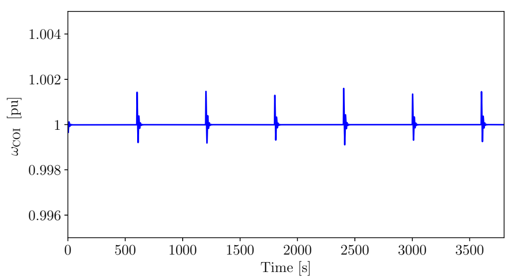

Fig. 4: Frequency of the center of inertia for the 10 minute UC scheduling period.

proportionally lower compared to the 15 minute scenario as a more reliable forecast is assumed to be available. Figure 4 shows that the amplitude and duration of the oscillations of the ω_{COI} decrases with respect to the 15 minute scenario. This is due to both lower uncertainty and lower load variations in the shorter period.

The average value of the objective function per period is $551,575.1, hence lower than for the 15 minute scenario. The decrease of the objective function is a consequence of the lower reserves and uncertainty and, in turn, of the shorter time period. It has to be noted that the net load consumption is assumed to vary linearly between two consecutive periods. If different paths are assumed, the solution of the sub-hourly UC problem can become more expensive [2]. A proper modelling of the transient behavior and the control of the system between consecutive solutions of the UC problem is thus crucial to avoid unnecessarily increasing the price of electricity.

C. 7.5 and 3.75 Minute Schedulings

The reuslts of these scenarios are shown in Figs. 5 and 6. The trend shown by the 10 minute scheduling is confirmed: shorter periods leads to lower frequency variations to the lower uncertainty. Also the cost of electricity decreases: the average values of the objective function for these two scenarios are $550,938.35 and $550,033.97, respectively. These results suggest that a smoother operation (e.g., more frequent solutions of the UC problem) is not just better from a dynamic point of view, but it is also more economical.

D. Effect of Noise

In this scenario, volatility is added to the load net consumption as proposed in [23]. This noise models both load and DER short-term fluctuations. Simulation results indicate that noise

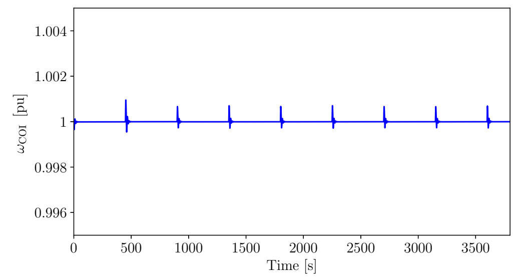

Fig. 5: Frequency of the center of inertia for 7.5 minute UC scheduling period.

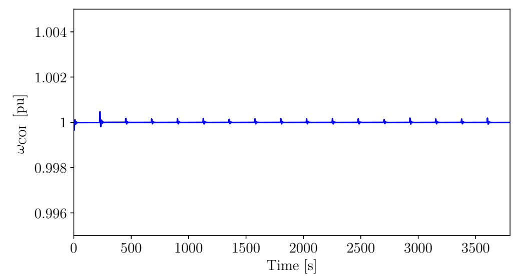

Fig. 6: Frequency of the center of inertia for 3.75 minute scheduling period.

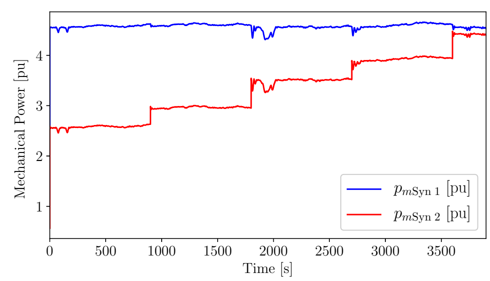

Fig. 7: Mechanical power of the relevant machines for 15 minute scheduling and noise.

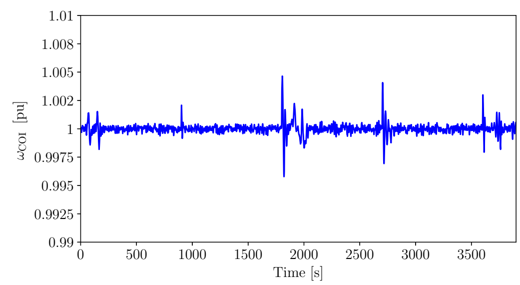

Fig. 8: Frequency of the center of inertia for 15 minutes scheduling and noise.

can significantly increase frequency variations. Figures 7 and 8 show the worst case scenario, namely the 15 minute scheduling with noise. In addition, the average value of the objective function is $556,689.32 and so slightly higher compared to the 15 minutes case without noise.

E. Effect of Contingencies

A contingency, i.e., the outage of line 1, occurring at 1,802 s and cleared after 200 ms for the scenarios of 15 and 3.75 minute scheduling scenarios illustrates the impact of the UC on the stability of the system. The time at which the contingency occurs is chosen on purpose to be in the seconds after the

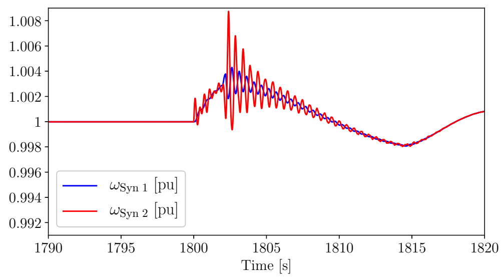

Fig. 9: Frequency of two synchronous machines following a contingency for the 15 minute scheduling.

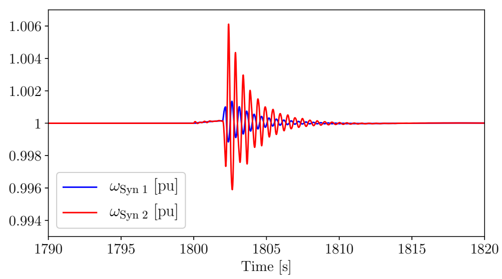

Fig. 10: Frequency of two synchronous machines following a contingency for the 3.75 minute scheduling.

solution of the SCUC problem. Figures 9 and 10 show the rotor speeds of the machines following the SCUC scheduling and the contingency. As expected, the highest impact of the contingency on the frequency occurs if a 15 minute scheduling period is used.

IV. Conclusions

This work presents a framework to include the UC problem into a time domain simulator. The framework allows studying the impact of the net load variability and uncertainty on the dynamic behavior of the system. In the proposed approach, the UC problem is modelled as a slow "discrete controller" that responds to time varying loading condition of the grid by changing the power set points of the turbine governors. In particular, a sub-hourly MILP SCUC is used to accomodate the variable net load. Results show that reducing the time intervals at which the UC problem is solved help reduce frequency variations, reduce electricity price and mitigate the impact of volatility and contingencies.

We see this work as a promising starting point towards bulding a flexible platform – within the Dome software tool – on which we can implement and test different control strategies and UC models. Future work will focus on improving technical constraints of generating units, e.g., minimum up and down times; using the output of DAEs to adjust the UC; and explicitly incorporate uncertainty, e.g., stochastic UC formulation.

References

[1] Y. Fu, M. Shahidehpour, and Z. Li, "Security-constrained unit commitment with ac constraints," *IEEE Trans. on Power Systems*, vol. 20, no. 3, pp. 1538–1550, Aug 2005.

[2] N. Troy, D. Flynn, and M. O'Malley, "The importance of sub-hourly modeling with a high penetration of wind generation," in *IEEE PES General Meeting*, July 2012, pp. 1–6.

[3] F. Milano, F. Dörfler, G. Hug, D. J. Hill, and G. Verbič, "Foundations and challenges of low-inertia systems (invited paper)," in *Power Systems Computation Conference (PSCC)*, June 2018, pp. 1–25.

[4] Y. Fu, Z. Li, and L. Wu, "Modeling and solution of the large-scale security-constrained unit commitment," *IEEE Trans. on Power Systems*, vol. 28, no. 4, pp. 3524–3533, Nov 2013.

[5] C. Cardozo, L. Capely, and P. Dessante, "Frequency constrained unit commitment," *Energy Systems*, vol. 8, no. 1, pp. 31–56, Feb 2017.

[6] R. Doherty, G. Lalor, and M. O'Malley, "Frequency control in competitive electricity market dispatch," *IEEE Trans. on Power Systems*, vol. 20, no. 3, pp. 1588–1596, Aug 2005.

[7] P. Daly, D. Flynn, and N. Cunniffe, "Inertia considerations within unit commitment and economic dispatch for systems with high non-synchronous penetrations," in *IEEE PowerTech*, June 2015, pp. 1–6.

[8] H. Ahmadi and H. Ghasemi, "Security-constrained unit commitment with linearized system frequency limit constraints," *IEEE Trans. on Power Systems*, vol. 29, no. 4, pp. 1536–1545, July 2014.

[9] Y. Lee and R. Baldick, "A frequency-constrained stochastic economic dispatch model," *IEEE Trans. on Power Systems*, vol. 28, no. 3, pp. 2301–2312, Aug 2013.

[10] F. Teng, V. Trovato, and G. Strbac, "Stochastic scheduling with inertia-dependent fast frequency response requirements," *IEEE Trans. on Power Systems*, vol. 31, no. 2, pp. 1557–1566, March 2016.

[11] A. C. Melhorn, M. Li, P. Carroll, and D. Flynn, "Validating unit commitment models: A case for benchmark test systems," in *IEEE PES General Meeting*, July 2016, pp. 1–5.

[12] C. O'Dwyer and D. Flynn, "Using energy storage to manage high net load variability at sub-hourly time-scales," *IEEE Trans. on Power Systems*, vol. 30, no. 4, pp. 2139–2148, July 2015.

[13] J. Kiviluoma, M. O'Malley, A. Tuohy, P. Meibom, M. Milligan, B. Lange, H. Holttinen, and M. Gibescu, "Impact of wind power on the unit commitment, operating reserves, and market design," in *IEEE PES General Meeting*, July 2011, pp. 1–8.

[14] E. Ela and M. O'Malley, "Studying the variability and uncertainty impacts of variable generation at multiple timescales," *IEEE Trans. on Power Systems*, vol. 27, no. 3, pp. 1324–1333, Aug 2012.

[15] S. Riaz, G. Verbič, and A. C. Chapman, "Computationally efficient market simulation tool for future grid scenario analysis," *IEEE Trans. on Smart Grid*, pp. 1–1, 2018.

[16] M. Carrión and J. M. Arroyo, "A computationally efficient mixed-integer linear formulation for the thermal unit commitment problem," *IEEE Trans. on Power Systems*, vol. 21, no. 3, pp. 1371–1378, Aug 2006.

[17] A. Conejo and L. Baringo, *Power System Operations*. Springer International Publishing, 2017.

[18] F. Milano, *Power System Modelling and Scripting*. London: Springer, 2010.

[19] L. Gurobi Optimization, "Gurobi optimizer reference manual," 2018. [Online]. Available: http://www.gurobi.com

[20] F. Milano, "A Python-based Software Tool for Power System Analysis," in *IEEE PES General Meeting*, Vancouver, BC, July 2013.

[21] Y. Chen, F. Liu, W. Wei, S. Mei, and N. Chang, "Robust unit commitment for large-scale wind generation and run-off-river hydropower," *CSEE Journal of Power and Energy Systems*, vol. 2, no. 4, pp. 66–75, Dec. 2016.

[22] Illinois Center for a Smarter Electric Grid (ICSEG), "IEEE 39-Bus System," URL: http://publish.illinois.edu/smartergrid/ieee-39-bus-system/.

[23] F. Milano and R. Zárate Miñano, "A Systematic Method to Model Power Systems as Stochastic Differential Algebraic Equations," *IEEE Trans. on Power Systems*, vol. 28, no. 4, pp. 4537–4544, June 2013.