Stochastic Aggregated Dynamic Model of Wind Generation with Correlated Wind Speeds

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***Abstract*—The paper describes a method to study the transient behavior of power systems with correlated wind speeds. With this aim, a generalised method to extract correlation from real-world measurement data is proposed. This method allows accommodating data with any sampling rate and any probability distribution. The case study discusses the impact of correlated wind speeds on power system dynamics by utilising the well-known two-area system modified to include a distribution network with wind power generation. The correlation of wind speeds is set up using real-world data. Simulation results show that a high level of correlation among the wind speeds worsens the dynamic performance of the system.**

***Index Terms*—Wind generation, stochastic differential algebraic equations (SDAEs), correlation, aggregated model, power system dynamics.**

I. Introduction

A. Motivation

Geographically close wind sites show similar variations in the wind speed [1]. Consequently, the power production of the wind power plants (WPPs) also shows a degree of correlation that depends on their location and proximity to each other. This correlation must be carefully considered when modelling aggregated WPPs [2], [3]. It is well known, for example, that inaccurate estimations of the power production of aggregated WPPs highly affect the results of the unit-commitment and, in turn, the market clearing price [4]. On the other hand, the modelling through stochastic differential equations (SDEs) of correlated wind speeds and the behavior of aggregated WPPs with correlated wind speeds have not been fully investigated so far. This paper fills this gap and provides a general approach to evaluate the correlation between wind speeds from wind speed measurement data. The effect of wind correlation on the aggregation of WPPs is also considered. This paper proposes a method to study power system short-term dynamics with correlated noise on wind speeds.

B. Literature Review

Due to the granularity of wind sites, wind power plants are typically connected to the grid in a tree-like topology as shown

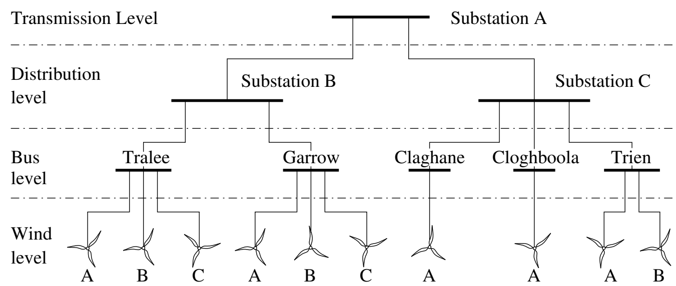

Fig. 1. Typical tree of a power grid with inclusion of wind power generation.

in Fig. 1. This hierarchical structure leads to several levels at which wind production can be aggregated. It is crucial, however, that independently from the level at which WPPs are aggregated, the statistical properties of the power injected into the grid by the aggregated WPP are similar to the ones obtained by simulating the detailed network.

In recent years, the modelling of correlated processes such as wind speeds and load power consumption has become an important field of research [1]–[3], [5], [6]. Research has also been carried out on the modelling of aggregated WPPs [7]–[14]. These works propose various techniques to model an aggregated WPP that reproduces the behavior of the detailed network, i.e., generates similar amount of active power at a given wind speed as in the case of the detailed network.

Some of the works cited above, e.g., [13] and [14], propose a way to calculate an equivalent wind speed that can be applied to the aggregated WPP to obtain the behavior of the active power similar to that generated by the WPPs of the original network. However, these works model power system dynamics through deterministic differential-algebraic equations (DAEs). The drawback of this approach is that the randomness in wind speeds is included into the set of DAEs only in the initialization step. Then the wind speed is assumed to remain constant during the simulation. This paper, on the other hand, focuses on modelling stochastic wind speed variations in the time scale of angle and voltage transient stability analysis.

These kinds of variations can be conveniently formulated in terms of SDEs. SDEs allow a precise modelling of the statistical behavior of the wind speed process in any time scale and with any PDF. Among the works that model the wind speed through SDEs, we cite [15]–[19]. These models can

This work is supported by the Science Foundation Ireland (SFI), by funding M. Adeen and F. Milano under Grant No. SFI/15/IA/3074, and by Sustainable Energy Authority of Ireland (SEAI), by funding F. Milano under the project FRESLIPS, Grant No. RDD/00681.

be then swiftly included in the set of DAEs that accurately model the power system dynamic response [20]–[22]. These works, however, do not model the correlation among stochastic processes and, hence, cannot study the impact of correlated wind speeds on the system dynamics.

How to model correlation on stochastic processes is introduced in [23] by modelling correlation on the active and reactive power consumption of loads by means of correlated SDEs. The model proposed in [23] can only model correlation between two stochastic processes. This limitation is removed in [24], which presents a systematic approach to set up correlated stochastic differential-algebraic equations (SDAEs) of arbitrary dimensions.

C. Contributions

Taking as starting point the approach described in [24] to set up SDEs with correlated processes, this work provides the following novel contributions.

- A general method to set up correlated wind speeds for arbitrary time scale and arbitrary PDF that can be incorporated in power system modelled as SDAEs and utilised to study their transient behavior.
- A SDE-based technique to properly set up an aggregated wind speed model such that the equivalent WPP, when driven by such an aggregated wind speed, accurately reproduces the statistical and dynamic behavior of the original network, i.e., detailed representation of the network.

The proposed method and aggregated wind model are duly tested using real-world measurement data provided by the Sustainable Energy Authority of Ireland (SEAI) [25] (see appendices A, and B for details on these data).

D. Organization

The remainder of the paper is organized as follows. Section II describes the procedure to generate correlated stochastic processes. In Section III, correlated stochastic processes are utilised to generate correlated wind speeds with arbitrary time-scale and arbitrary PDF. In particular, Sections III-A and III-B present the proposed method to set up correlated and aggregated wind speed models, respectively. The case study presented in Section IV discusses the impact of correlation on the trajectories of the wind speeds and of the power production of WPPs. The case study also performs time-domain simulations to study the transient behavior of the power system undergoing a contingency and with inclusion of correlated noise on the wind speeds. Section V draws relevant conclusions.

II. Modelling of Correlated Stochastic Processes

The dynamic behavior of power systems with inclusion of n correlated stochastic processes can be defined using the following set of n-dimensional correlated SDAEs:

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{\eta}) \,, \tag{1}$$

$$\boldsymbol{0} = \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{\eta}) \,, \tag{2}$$

$$\dot{\boldsymbol{\eta}} = \boldsymbol{a}(\boldsymbol{\eta}) + \boldsymbol{b}(\boldsymbol{\eta}) \odot \boldsymbol{\zeta} \,. \tag{3}$$

where $\boldsymbol{f} : \mathbb{R}^{l+m+n} \mapsto \mathbb{R}^m$, and $\boldsymbol{g} : \mathbb{R}^{l+m+n} \mapsto \mathbb{R}^l$ represent the differential, and algebraic equations, respectively. $\boldsymbol{x} \in \mathbb{R}^l$, and $\boldsymbol{y} \in \mathbb{R}^m$ are the vectors of state, and algebraic variables, respectively. Equation (3) describes a set of n-dimensional correlated SDEs constructed using correlated noise elements [24]. In (3), $\boldsymbol{\eta} \in \mathbb{R}^n$ is the vector of correlated stochastic processes; $\boldsymbol{a} : \mathbb{R}^n \mapsto \mathbb{R}^n$ is the vector of so called *drift* term; $\boldsymbol{b} : \mathbb{R}^n \mapsto \mathbb{R}^n$ is the vector of so-called *diffusion* term; $\odot$ represents the Hadamard product, i.e., the element-by-element product of two vectors; and $\boldsymbol{\zeta} \in \mathbb{R}^n$ is a vector of n-dimensional correlated *Gaussian white noise*, as follows:

$$\boldsymbol{\zeta} = \mathbf{C}\, \boldsymbol{\xi} \,, \tag{4}$$

where $\boldsymbol{\xi} \in \mathbb{R}^n$ is the vector of uncorrelated *Gaussian white noise*, and $\mathbf{C} \in \mathbb{R}^{n\times n}$ is lower triangular and can be computed through Cholesky decomposition of the correlation matrix $\mathbf{R} \in \mathbb{R}^{n\times n}$ such that:

$$\mathbf{R} = \mathbf{C}\, \mathbf{C}^T \,. \tag{5}$$

The correlation matrix $\mathbf{R}$ defines the correlation between the increments of noise elements of SDEs and is written as:

$$\mathbf{R} = \begin{bmatrix} 1 & r_{1,2} & r_{1,3} & \dots & r_{1,n} \\ r_{2,1} & 1 & r_{2,3} & \dots & r_{2,n} \\ r_{3,1} & r_{3,2} & 1 & \dots & r_{3,n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ r_{n,1} & r_{n,2} & r_{n,3} & \dots & 1 \end{bmatrix} ,$$

where $r_{i,j}$ represents the correlation between the increments of any two noise elements i.e., $r_{i,j} = \mathrm{corr}[dW_i(t), dW_j(t)]$, with $r_{i,j} = 1$ if $i = j$, since the correlation of any variable with itself is always 1. $\mathbf{R}$ is positive semi-definite for most of the power system applications. Note that if $\mathbf{R} = \mathbf{I}$, where $\mathbf{I}$ is the identity matrix, then the SDE in (3) becomes the conventional uncorrelated SDE. The details on the calculation of the elements of $\mathbf{R}$ from real-world data are given in Appendix B.

The vector of uncorrelated white noise processes $\boldsymbol{\xi}(t)$ is defined as the time derivative of the Wiener process, as follows:

$$\boldsymbol{\xi}(t)\, dt = d\boldsymbol{W}(t) \,, \tag{6}$$

where $\boldsymbol{W} \in \mathbb{R}^{n_w}$ is a vector of uncorrelated standard Wiener processes i.e, there exists no correlation between the elements, say $W_i(t)$, $i = 1, \dots, n_w$, of $\boldsymbol{W}$. A standard Wiener process $W(t)$ has the following properties:

1) $W(0) = 0$.
2) $W(t)$ is a continuous function of t.
3) $W(t)$ has unbounded variation in every interval.
4) $W(t)$ has Gaussian increments, i.e., $\forall t \geq 0$, $dW = W(t+h) - W(t)$ is normally distributed with mean zero and variance h.

5) The increments of $W(t)$ are independent of past values of $W(t)$, i.e., $\forall t \geq 0$, $dW = W(t+h) - W(t)$ are independent of $W(s)$, $s \leq t$.

Equations (1)-(3) are nonlinear and are thus usually solved by numerical integration schemes. The integration of (3) involves the calculation of the increments of the Wiener process with sufficiently small time steps. Hence, the integral form of the SDE in (3) is obtained by substituting (6) into (3) as follows:

$$\boldsymbol{\eta}(t) = \int_t \boldsymbol{a}(\boldsymbol{\eta}(\tau))\, d\tau + \int_\eta \boldsymbol{b}(\boldsymbol{\eta}(\tau)) \odot [\mathbf{C} d\boldsymbol{\eta}(\tau)] \,, \qquad (7)$$

where the first integral involving the drift term is the conventional Riemann-Stieltjes' integral, and can be solved by employing any usual numerical integration schemes [26]. On the other hand, the integral of the diffusion term is carried out through Itô's calculus [27], [28]. The analytical solution of an Itô's integral is not known in most cases. Hence, numerical methods are employed. The most popular choice in power systems is the Euler-Maruyama scheme that involves time discretization [29], [30].

III. Modelling Correlated Wind Speeds

Modelling a stochastic process, such as the wind speed, requires the definition of appropriate functions for the drift and the diffusion terms of (7). The drift term of the wind speed process, observed from empirical data from different wind sites [31], can be defined through a mean-reverting function, as follows:

$$a_i(\eta_i) = -\alpha_i \left(\eta_i - \mu_i\right) \,, \quad i = 1, \ldots, n, \qquad (8)$$

where α_i is the autocorrelation coefficient, η_i is the i-th stochastic process, and μ_i is the mean of the i-th stochastic process. The principal effect of the mean reversion is to bound the standard deviation of the diffusion term of the SDE. This property is thus appropriate for processes that show constant standard deviations in stationary conditions, such as the wind speed.

Once the drift term is defined, the diffusion term is determined based on the required PDF of the underlying stochastic process using a technique such as that based on the stationary Fokker-Plank equation in [32] or a memoryless transformation as in [16].

The resulting SDE that models the wind speed as a stochastic stationary process with desired drift term in (8), and the diffusion term for arbitrary PDF is written as:

$$\dot{w}_i = -\alpha_{w_i}(w_i - \mu_{w_i}) + b_{w_i}(w_i)\xi_{w_i} \,, \quad i = 1, \ldots, n, \qquad (9)$$

where w_i is the wind speed, and $b_{w_i}(w_i)$ is the diffusion term, which in general depends on w_i. Finally, the set of SDEs that describe the correlated wind speeds with arbitrary PDFs can be written as:

$$\dot{\boldsymbol{w}} = -\boldsymbol{\alpha}_w \odot (\boldsymbol{w} - \boldsymbol{\mu}_w) + \boldsymbol{b}_w(\boldsymbol{w}) \odot \boldsymbol{\zeta}_w \,, \qquad (10)$$

where $\boldsymbol{w}$ is the vector of wind speeds, $\boldsymbol{\alpha}_w$ is the vector of autocorrelation coefficient; $\boldsymbol{\mu}_w$ is the vector of mean; $\boldsymbol{b}_w(\boldsymbol{w})$ is the vector of diffusion term; and $\boldsymbol{\zeta}_w$ is the vector of correlated white noise defined in (4). Note that (10) is valid for wind speeds with different PDFs, i.e., for $b_{w_i}(w_i) \neq b_{w_j}(w_j)$. The interested reader can find the values of μ_w and $b_w(w)$ for several distributions relevant to model wind speeds in [31].

A. Correlated Wind Speeds with given PDF

The correlation matrix $\mathbf{R}$ is the core mathematical object that allows defining the correlation between stochastic processes in (7). The elements of the correlation matrix are defined based on measurement data. With this aim, we utilise the analytical solution of (9) to extract the noise elements of processes with arbitrary PDF.

The solution of (9) can be established by multiplying (9) by $e^{\alpha_w t}$, and re-arranging as:

$$\begin{aligned} &\alpha_w w(t) e^{\alpha_w t} dt + e^{\alpha_w t} dw(t) \\ &= e^{\alpha_w t} \left[\mu_w \alpha_w + b_w\left(w(t)\right)\, dW_w(t)\right] . \end{aligned} \qquad (11)$$

Note that

$$d\left(e^{\alpha t}\, w(t)\right) = \alpha w(t) e^{\alpha t} dt + e^{\alpha t} dw(t) \,. \qquad (12)$$

Hence, substituting (12) into (11) and integrating, one obtains:

$$\begin{aligned} w(t) = w(0) e^{-\alpha_w t} &+ \int_0^t \mu_w \alpha_w e^{\alpha_w (s-t)} ds \\ &+ \int_0^t b_w\left(w(s)\right) e^{\alpha_w (s-t)} dW(s) \,, \end{aligned} \qquad (13)$$

where $w(0)$ is the initial value of the process at $t = 0$. The first integral is the conventional Riemann-Stieltjes' integral, and integrates to $\mu_w\left(1 - e^{-\alpha_w t}\right)$. The second integral is expressed as an Itô's integral. Using Itô's isometry [27], [28] the second integral integrates to a normal random variable with mean zero and variance given as:

$$\begin{aligned} &E\left[\int_0^t b_w\left(w(s)\right) e^{\alpha_w (s-t)} dW_w(s)\right]^2 \\ &\qquad = \frac{b_w^2\left(w(t)\right)}{2\alpha_w}\left(1 - e^{-2\alpha_w t}\right) . \end{aligned} \qquad (14)$$

Thus, the analytical solution of (9) is written as:

$$\begin{aligned} w(t) = w(0)\, e^{-\alpha_w t} &+ \mu_w\left(1 - e^{-\alpha_w t}\right) \\ &+ b_w\big(w_w(t)\big)\, \psi_w(t) \sqrt{\frac{1 - e^{-2\alpha_w t}}{2\alpha_w}} \,, \end{aligned} \qquad (15)$$

where $\psi_w(t)$ is the random variable, which is distributed normally with zero mean and unit variance. $\psi_w(t)$ can be extracted from (15) and written as:

$$\psi_w(t) = \frac{w(t) - w(0)\, e^{-\alpha_w t} - \mu_w\left(1 - e^{-\alpha_w t}\right)}{b_w\big(w(t)\big) \sqrt{\frac{1 - e^{-2\alpha_w t}}{2\alpha_w}}} \,. \qquad (16)$$

Equation (16) is employed to estimate the noise element $\psi_w(t)$ from the empirical data, provided the underlying process can be defined using (9).

The solution provided in (15) is valid for an arbitrary time interval $[0, t]$ and any initial condition. It can also be applied to an arbitrarily chosen time step Δt beginning at t_{i-1} and ending at t_i. We assume equidistantly spaced time steps such that $\forall i \in \mathbb{Z}_+, t_i - t_{i-1} = \Delta t > 0$. To calculate the increment in the stochastic process at an arbitrarily chosen time step Δt, we assume that the value of the process at the previous time step t_{i-1} serves as the initial condition for time step t_i. Therefore, the increment in the stochastic process for the time step Δt is calculated using (15) as:

$$\begin{aligned} dw(t_i) =& w(t_{i-1})\, e^{-\alpha_w \Delta t} + \mu_w \left(1 - e^{-\alpha_w \Delta t}\right) \\ &+ b_w\big(w(t_{i-1})\big)\, \psi_w(t_i) \sqrt{\frac{1 - e^{-2\alpha_w \Delta t}}{2\alpha_w}}\, . \end{aligned} \tag{17}$$

Similarly, the increment of $\psi(t_i)$ for the time step Δt is written as:

$$d\psi_w(t_i) = \frac{w(t_i) - w(t_{i-1})\, e^{-\alpha_w \Delta t} - \mu_w \left(1 - e^{-\alpha_w \Delta t}\right)}{b_w\big(w(t_{i-1})\big) \sqrt{\frac{1 - e^{-2\alpha_w \Delta t}}{2\alpha_w}}}\, . \tag{18}$$

The applicability of (18) to calculate noise elements from wind speed measurement data for arbitrary PDFs is discussed in Appendix A.

B. Aggregated Wind Speed Model

The aggregated wind speed process is obtained as the average of the underlying wind speed processes. This method of averaging the underlying wind speeds has also been utilised in [13] and [14]. However, these references consider neither wind speed dynamics nor correlated wind speeds. In this paper, on the other hand, we are interested in correlated wind speed processes modelled through correlated stochastic processes in the time scale of transients. The aggregated wind speed process is thus modelled as a stochastic process that is an average of the underlying individual wind speed processes modelled through the set of correlated SDEs as proposed in (10). The proposed aggregated wind speed model is built using (10) and (15), as follows:

$$\begin{aligned} w_{\text{agg}}(t) = \frac{1}{n} \sum_{i=1}^{n} \Bigg[& \mu_{w_i} + e^{-\alpha_{w_i} t}\big(w_i(0) - \mu_{w_i}\big) \\ & + \psi_{w_i} \sum_{j=1}^{n} \left(\sigma_{w_j} c_{j,i}\right) \Bigg]\, . \end{aligned} \tag{19}$$

where $c_{i,j}$ is the i,j element of matrix $\mathbf{C}$, and σ_{w_i} is the standard deviation of the i-th wind speed process from (14). The procedure discussed above is applied in Appendix C to generate an aggregated wind speed that generates similar statistical properties of aggregated power production as in the case of detailed network.

IV. Case Study

This case study aims at evaluating the impact of correlated wind speeds, modelled through correlated SDAEs, on the dynamic of power system. The effect of correlation on the dynamic of the wind speeds themselves and the power production of WPPs is discussed in Section IV-A. Then, the impact of a distribution network and correlated wind generation on the dynamic response of the a modified version of the two-area system is discussed in Section IV-B.

A. Effect of Correlation on Wind Speeds and WPP Power Production

The effect of correlation on the dynamic of wind speeds, modelled through correlated SDEs, is illustrated in Fig. 2. The wind speeds illustrated in Fig. 2 are modelled through beta distribution. The parameters of beta distribution for modelling noise on wind speeds in the time scale of power system transient are reported in [16]. Figure 2 shows that the profiles of wind speeds become closer to each other as the level of correlation is increased, and thus changing the dynamics of the wind speeds.

As discussed in Section II, the level of correlation modelled on stochastic processes does not modify the statistical properties of the stochastic processes. As proof of concept, Fig. 3 illustrates the PDF of the two wind speed processes realised, as shown in Fig. 2, through correlated SDEs. Figure 3 shows that the statistical properties of the wind speed processes remain independent of the level of correlation being modelled among them.

The effect of correlation among wind speeds is transferred to the wind active power injections p_g of the WPPs. However, this does not modify the statistical properties of p_g, as the statistical properties of the wind speeds remain unaltered. Figure 4 illustrates the time profile of p_g for different levels of correlation modelled on the underlying wind speeds. In Fig. 4, it can be observed that, as the level of correlation among wind

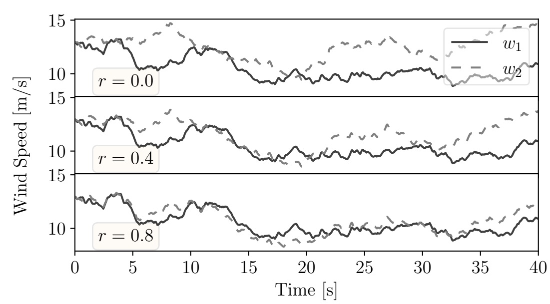

Fig. 2. Wind speeds of two wind sites modelled through correlated SDEs for different levels of correlation r.

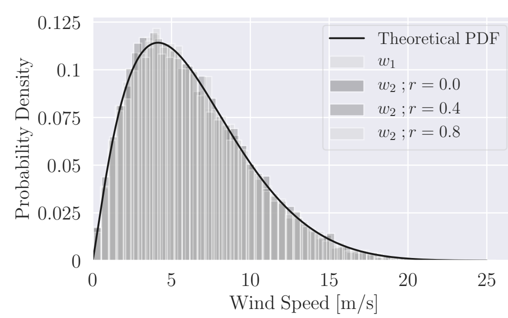

Fig. 3. Probability density function of wind speeds shown in Fig. 2.

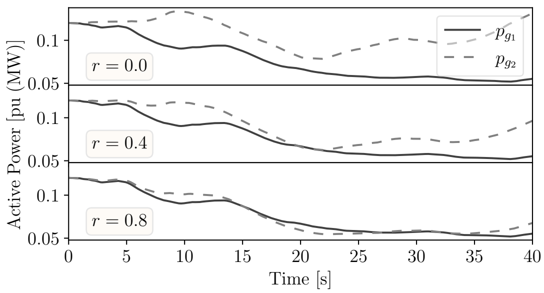

Fig. 4. Active power injection of WPPs for different levels of correlation r among wind speeds.

speeds is increased, the time profiles of p_g come closer to each other, which modifies the dynamic behavior of p_g.

B. Impact of Correlated Wind Speeds on the Power System Dynamic Response

The power system considered in this section is the well-known two-area system, originally introduced in [33]. This system is composed of 11 buses and 4 synchronous generators. The generators are split into two areas connected through a weak tie-line. All the synchronous machines are modelled via VI-order model, and are equipped with turbine governors, and automatic voltage regulator of type IEEE DC-I. Loads are modelled as constant impedances.

In this study, the original system is modified to include wind generation. With this aim, the wind generation network is modelled as in Fig. 1. Then Substation A is connected to bus 9 of the two-area system. The WPPs are modelled through variable-speed doubly-fed induction generators. Finally, the correlation matrix $\mathbf{R}$ of wind speeds is needed to complete the setup of the model. The detailed procedure to construct $\mathbf{R}$ based on real-world data is provided in Appendix B.

To study the impact of correlated wind speeds on the power system dynamic, the following two cases are proposed:

- Case 1 considers no correlation among wind speeds.
- Case 2 considers correlation among all wind speeds.

The power system dynamic simulations for the two cases are performed using the Monte Carlo method. 1000 stochastic time domain simulations per case are solved. For the integration of the deterministic part of the SDAE, in (1), implicit trapezoidal integration scheme with a time step of $\Delta t = 0.01$ s is utilised. The Maruyama-Euler integration scheme is employed to integrate the stochastic part of SDAEs, where a step size of $h = 0.01$ s is used for the realization of the Wiener processes. Each simulations is solved for 100 s of simulated times.

First we consider a scenario where the system is perturbed only with noise due to wind variations. The standard deviation of the frequency of the center of inertia ω_{Coi} for both cases is illustrated in Fig. 5. The standard deviation of ω_{Coi} shows an increase with the increase in the level of correlation among the wind speeds. This indicates that correlated wind speeds are capable of modifying the distribution of power system quantities without modifying the distribution of p_g of WPPs.

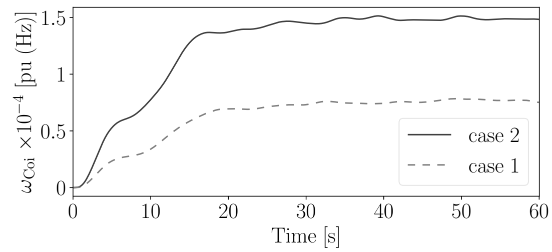

Fig. 5. The standard deviation of the frequency of the center of inertia ω_{Coi} for the two cases.

In this second scenario, we consider both noise and a contingency. The contingency consists in the trip of the line connecting buses 8 and 9 at time $t = 30$ s. The voltage profile at Bus 8 for the two cases of correlation is illustrated in Figs. 6 and 7. Figures 6 and 7 show the trajectories of the bus voltage magnitude v at bus 8 along with the mean of the trajectories for the system modelled through correlated SDAEs, for the two cases. The trajectory of the bus voltage magnitude at bus 8 for the system modelled through set of deterministic DAEs using constant wind speeds is also shown in Figs. 6 and 7.

The mean trajectory of v coincides with the deterministic trajectory in both the cases. This was to be expected as the level of correlation among wind speeds does not impact on the wind speed average values. On the other hand, the standard deviation of v increases as the wind correlation increases.

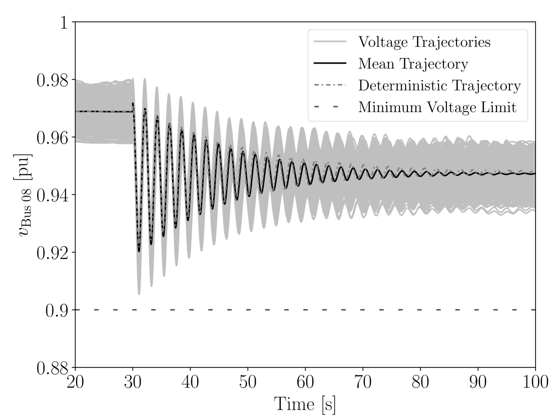

Fig. 6. Voltage profile at bus 8 for case 1.

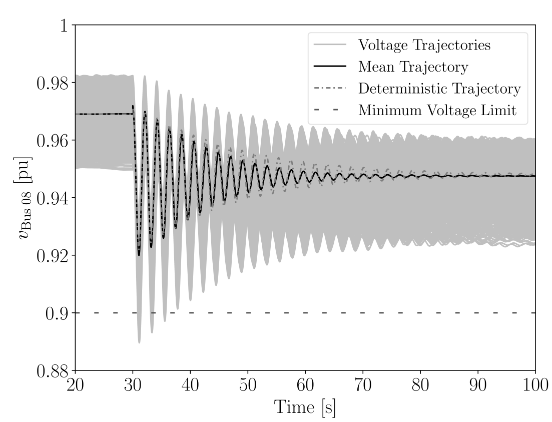

Fig. 7. Voltage profile at bus 8 for case 2.

This increase of the standard deviation causes 59 (5.9 %) trajectories of v to violate the minimum voltage limit for at least 5 s for case2 (see Fig. 7). Note that the results presented in this section are obtained by simulating the wind generation network in detail as in Fig. 1. Moreover, the results obtained through the aggregated wind generation are validated against those obtained through detiled network in Appendix C.

V. Conclusions

This paper introduces a general technique to evaluate the correlation among wind speeds by extracting the noise elements from wind speed measurement data. The proposed technique is general in the sense that it can be used for arbitrary sampling rate, and arbitrary PDF. Then, the paper describes how to utilise correlated SDEs to set up accurate aggregated wind speed processes and WPP models.

The case study is based on real-world data and demonstrates the usefulness of the proposed method to study the impact of correlated wind speeds on power system dynamics. Through time domain simulations, the case study shows that the correlated wind speeds are capable of modifying the distribution of relevant system quantities, e.g., the frequency of center of inertia. Finally, simulations carried out on a modified version of the two-area system indicate that the correlation on wind speeds can also make the effect of contingencies more severe.

Appendix

A. Data Analysis

This appendix illustrates the procedure presented in Section III-A to extract the noise elements $\psi_w(t)$ from the wind speed data. For this purpose, a variety of wind speed measurement data exhibiting different fitting PDF types and different time scales ranging from 1 second to 1 hour are utilised, and shown in Table I. The wind speed data presented in Table I is obtained through different open source platforms [16].

The fitting PDF types for the data in Table I were obtained by applying the Kolmogorov-Smirnov test. The parameters of the fitting PDF type were obtained through the maximum likelihood estimation method [16]. A curve fitting technique was utilised to calculate the autocorrelation coefficient of the wind speed by calculating the autocorrelation function of the wind speed measurement data [16], [31].

The noise elements $\psi_w(t)$ were extracted from the wind speed data, shown in Table I, by utilising the procedure presented in Section III-A. The PDF of $\psi_w(t)$ obtained from the wind measurement data, presented in Table I, is shown in Fig. 8. Figure 8 illustrates that the PDF of $\psi_w(t)$ follows the normal random variable with zero mean and unit variance, and is in accordance with the discussion in Section III-A.

B. Construction of the Correlation Matrix

As explained in Section II, the elements of the correlation matrix $\mathbf{R}$ represent the correlation between the increments of the noise elements, i.e., $d\psi_{w_i}(t)$ and $d\psi_{w_j}(t)$. The noise elements are calculated from measurement data using (18). Once the time series of $d\psi_{w_i}(t)$ is obtained, each element of

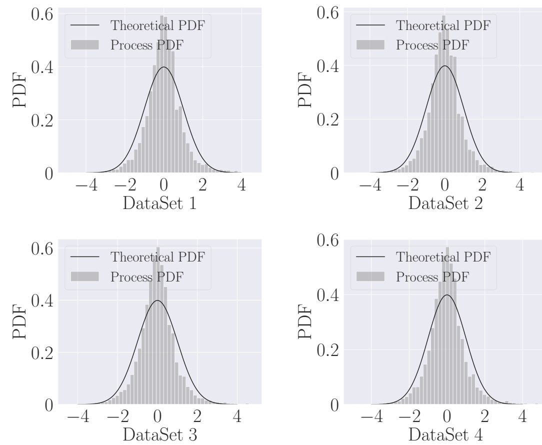

Fig. 8. PDF of $\psi_{w_i}(t)$ obtained from measurement data, in Table I, using (18).

$\mathbf{R}$, i.e., $r_{i,j} = \mathrm{corr}[d\psi_{w_i}(t), d\psi_{w_j}(t)]$ is calculated by employing Pearson correlation coefficient. To construct $\mathbf{R}$, wind speed measurement data from ten wind sites in Ireland are obtained. These data are available on the website of SEAI [25]. The relationship between distance, and correlation of the wind speed data is illustrated in Fig. 9. The correlation between the noise elements of the wind speeds shows an exponential decay w.r.t. the distance between them. This exponential decrease in the correlation w.r.t. the distance is consistent with the results reported in other studies [1], [3], [34].

C. Wind Power Aggregation and Validation

This appendix illustrates the effectiveness of the wind speed aggregation model presented in Section C. The goal of the wind speed aggregation is to accurately model wind speed process aggregated at different levels of the grid such as bus,

TABLE I
Sampling rates and PDF types of measurement data

Data Set	Sampling Rate	PDF Type
1	1 hour	1-parameter Rayleig
2	10 minutes	3-parameter Gamma
3	1 minute	3-parameter Gamma
4	1 second	3-parameter Beta

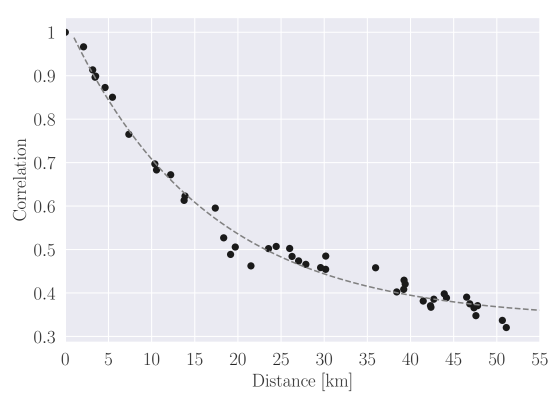

Fig. 9. Distance vs correlation between wind sites. Note: The dashed line is an exponential fit.

distribution, and transmission, with correlated wind speeds. With this aim, we compare the standard deviation of the trajectories of active power σ_p generated at different levels of the grid by simulating the entire network to σ_p generated by the aggregated WPP driven by the aggregated wind speed process.

The network of WPPs utilised in this paper is shown in Fig. 1. The network is formed in a hierarchical manner. The wind production is aggregated at different levels of the network, i.e., bus, distribution, and transmission. The aggregated WPP is then driven by the wind aggregated process. The wind aggregation model is considered to work with high accuracy if σ_p of wind generation obtained through aggregating WPPs in different regions of the grid is close to σ_p obtained by individually modelling WPPs in the network. The values of σ_p calculated for detailed and aggregated WPPs along with the errors are presented in Table II. The results shown in Table II are obtained considering the data of the first row of Table Table I. The low values of the errors shown in Table II is an evidence of the accuracy of the proposed aggregated wind speed model.

TABLE II
STANDARD DEVIATION OF AGGREGATED WPP POWER GENERATION.

Aggregation Level	Location	SPM		Error
		Detailed	Aggregated	
Bus	Tralee	57.73	60.71	2.78
	Garrow	61.8	63.72	3.1
	Trien	62.31	64.4	3.35
Distribution	Substation B	44.21	45.53	2.88
	Substation C	51.66	52.59	1.8
Transmission	Substation A	46.64	47.36	1.53

SPM: Standard deviation of p_g expressed in percent of the mean value.
Error: Absolute Normalised Error in % between detailed and aggregated.

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