Small-Signal Stability Analysis of Delayed Power System Stabilizers

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***Abstract*—This paper presents a stability analysis of power system stabilizers (PSS) for synchronous generators with inclusion of time delays. The paper shows that a time delay in the PSS feedback loop can improve the small-signal stability of a power system if the regulator gain is properly tuned. The paper provides a proof-of-principle analysis based on the classical model of the synchronous machine as well as a case study based on a detailed transient model of the IEEE 14-bus test system. The paper also provides a discussion on the practical implications that the properties of delayed PSS can have on the control of synchronous machines and of the whole power system.**

I. Introduction

Time-delay systems have been studied as early as 1920s. Increasing number of publications written on the subject, particularly in recent years, is evidence for the continuing interest of mathematicians and engineers in delayed systems [1]. One of the reasons for the importance of time delays is that they arise in a wide variety of physical systems and their effects on stability have been carefully investigated in various engineering applications, including signal processing and circuit design [2]–[6], as well as in biology, economics, and population dynamics [7].

The effects of time delays on power system stability and control have not been exhaustively studied to the best of our knowledge. Historically, the main studies pertaining to power systems and time delays focus mainly on long transmission lines (see for example, [8]). However, for transient analysis, controllers with delays are generally approximated with lag transfer functions, which do not capture the dead-time feature and the high-frequency response of delays. The effect of delays on wide area measurements and, in particular, on Power System Stabilizers (PSS) has actually been recognized as a relevant topic, see for example [9]–[11]. In the cited studies, the authors indicate the destabilizing effect of delays in the PSS control loop.

In spite of the "bad reputation" of delays as a source of destabilization, delays can also have surprisingly positive effects on improving system stability [6]. Some studies have shown that delays can benefit the closed-loop control, for instance, in damping and stabilization of ordinary differential equations [12] [13, Chapter 11], delayed resonators [14] and nonlinear limit cycle control [15]. Inspired by these studies, in particular by [12], this paper shows that time-delays in the feedback control loop of PSS devices do not necessarily imply a deterioration of the system transient response. Actually, we show that, if the time delay is known, a proper adjustment of the PSS transfer function gain can improve the overall system stability.

In this paper, we are interested in determining how time delays can affect the small-signal stability of power systems with inclusion of PSS controllers, and how such delays can be handled to avoid the occurrence of Hopf bifurcations followed by unstable or undamped oscillations (i.e., limit cycles). Both mathematical and computational aspects are taken into account so that the proposed procedures for small-signal analysis as well as for time domain integration can be applied to a power system of any size and complexity.

Specifically, the paper provides a parametric small-signal stability analysis of power systems with inclusion of time delays. This is done via well-known stability maps for the power system at hand [6]. With these maps, it becomes possible to display which parametric combinations render stability or instability in the power system. While stability in power systems was already studied without consideration of delays [16] and [17], and although stability maps have been used in many applications, to the best of out knowledge, these parametric maps are new for power systems subject to delays.

The main challenge in extracting the stability maps of systems with delays is due to the fact that the corresponding characteristic equation of the system at an equilibrium point is infinite dimensional; that is, the system has infinitely many eigenvalues. Yet, extraction of stability maps require studying some particular eigenvalues of the system, namely, those that are critical from stability point of view. Consequently, although not trivial, revealing stability maps is possible [7].

In PSS control, one however not only would like to achieve stability, but also satisfy some performance criterion. For instance, it is always desirable to have sufficient damping in the system. To be able to study such transient characteristics of the system with respect to delay and system parameters, one needs to find out the dominant modes, i.e., the rightmost eigenvalues of the state matrix of the system. This is a difficult task since the system is infinite dimensional, and hence the numerical computation of those eigenvalues is not always easy. In this paper, we resolve the problem of computing system's eigenvalues by means of the frequency-domain approach discussed in [11]. This approach is based on a discretization of a partial differential equation (PDE) representation of the delayed differential algebraic equation (DDAE), modeling the PSS control system [18]–[20]. The discretization allows computing an approximated but accurate set of the eigenvalues of the system that are relevant in terms of system stability and performance. That is, the computation reveals the rightmost eigenvalues of the system, thereby allowing one to infer system damping and settling time characteristics approximated with these eigenvalues.

The paper is organized as follows. Section II provides a proof of concept for the parametric analysis carried out in the paper and explores the stability regions in the delay versus control-gain plane for a simplified PSS-synchronous machine model. Section III extends the concepts presented in Section II to a real-world power system model. For this aim, the frequency-domain approach to compute the eigenvalues of a DDAE system is briefly outlined. Section IV provides a case study based on the IEEE 14-bus system. Finally, Section V provides a discussion on the practical implications of the small-stability analysis presented in the paper and duly draws relevant conclusion and future work directions.

Note that, with a slight abuse in notation, in the remainder of the paper, we use the term *stability* instead of *asymptotic stability* of the equilibrium point.

II. Theoretical Background – Proof of Concept

Time delay in a system usually has detrimental effects on the stability properties of that system. Sometimes even small values of delays that seem harmless to ignore in modeling a system, can lead to instability. For example, consider the following LTI system, which has a globally asymptotically stable equilibrium at the origin:

$$\dot{x}(t) + 2\dot{x}(t) = -x(t).$$

On the other hand, the trivial solution of the following neutral type functional differential equation

$$\dot{x}(t) + 2\dot{x}(t-\tau) = -x(t),$$

is unstable for any $\tau > 0$, see [**?**, p. 28] for a proof.

Consider the well-known simplified electromechanical model of a synchronous machine in steady state [21]:

$$2H\frac{\omega}{dt} = p_m - p_e(\delta), \tag{1}$$

where ω is the rotor speed, H is the machine inertia constant, p_m is the mechanical power, and p_e is the electromagnetical power defined as:

$$p_e(\delta) = \frac{e'_q v}{x'_d}\sin(\delta - \theta), \tag{2}$$

where δ is the rotor angle, v and θ are the machine bus terminal voltage magnitude and phase angle, respectively, e'_q is the internal fem, and x'_d is the d-axis transient reactance. Differentiating (1) leads to:

$$2H\frac{\Delta\omega}{dt} = -\frac{\partial p_e}{\partial \delta}\Delta\delta - \frac{\partial p_e}{\partial e'_q}\Delta e'_q - \frac{\partial p_e}{\partial v}\Delta v, \tag{3}$$

which can be further simplified as follows. Since, without the PSS and assuming an integral automatic voltage regulator, e'_q and v are constant, the above equation in Laplace s domain becomes:

$$2Hs\Delta\omega = -\frac{\partial p_e}{\partial \delta}\Delta\delta := -K\Delta\delta, \tag{4}$$

where

$$K = \frac{e'_q v}{x'_d}\cos(\delta_0 - \theta_0), \tag{5}$$

and we denote with δ_0 and θ_0 the rotor and bus voltage phase angles, respectively, at the equilibrium point. Since $\Delta\omega = s\Delta\delta$, we obtain the characteristic equation of the system as:

$$f(s) = s^2 + \frac{K}{2H} = 0, \tag{6}$$

which corresponds to an oscillator with roots on the imaginary axis of the complex plane.

As it is well-known, see, e.g., [21], the presence of a PSS control loop leads to a right-hand side term in (6) proportional to $e'_q \propto s\Delta\delta$. Assuming now that the feedback is affected by a delay term τ, then the system characteristic equation reads

$$f(s, e^{-\tau s}) = s^2 + Ase^{-\tau s} + \tilde{K} = 0. \tag{7}$$

where $\tilde{K} = K/(2H)$, A is proportional to the rotor-speed feedback-controller gain of the PSS, and $\tau \geq 0$ is the constant delay. Equation (7) can be interpreted as the characteristic equation of a feedback control system where an open loop oscillator dynamics with natural frequency $\sqrt{\tilde{K}}$ is controlled only by a derivative controller constructed based on delayed measurements of the output. This system also resembles to those studied in [6], [12].

We utilize the approaches in [6], [12], [22], [23] to reveal the stability map of (7) in the parameter space of A versus τ. To summarize, this mapping is obtained based on the following principles [24], [25]: (a) the system poles move on the complex plane continuously with respect to system parameters; (b) the system stability is preserved as delay τ transitions from zero to 0^+; and (c) the system may lose/recover stability only if at least one of its poles crosses over the imaginary axis of the complex plane. In light of these, one can determine the critical values of A and τ for which (7) produces imaginary eigenvalues $s = j\omega$ on the complex plane. That is, the characteristic equation in (7) needs to be solved for A and τ when $s = j\omega$, which reads

$$f(j\omega, e^{-j\tau\omega}) = (j\omega)^2 + jA\omega e^{-j\tau\omega} + \tilde{K} = 0. \tag{8}$$

Once the critical values of A and τ are solved from (8), corresponding to critical values of ω, with $\omega > 0$ without loss of generality, one can plot these critical points on τ versus A plane, on which countably many "regions" will form. That is, the critical values will decompose the parameter space into regions, where in each region any parametric combination will render the system to have a fixed number of unstable poles. The regions where this number is zero, i.e., the system has no unstable poles, are the regions where the system remains stable. This is the main spirit behind τ-decomposition theorem [6], which is instrumental in identifying stable and unstable regions.

Identification of stable and unstable regions requires a sensitivity analysis, namely, calculation of how the pole $s = j\omega$ moves across the imaginary axis as the corresponding critical delay value increases infinitesimally, while all the remaining parameters are kept fixed. Interested readers are referred to [6], [7], [22], [23], [26] for the details. Once the sensitivity analysis is completed, one has full information about how system stability transitions as one moves across the boundaries that decompose the parametric space. If/when sensitivity favors "stabilization" across a boundary, this would mean that crossing the boundary will reduce the number of unstable poles in the destination region, and if/when sensitivity shows "destabilization", the contrary happens. With the information available regarding the number of unstable poles of the system for the delay-free case ($\tau = 0$), one can then use the sensitivity information across the boundaries to calculate the number of unstable poles in all the regions on A-τ plane, and thus identify all the stability regions.

Figure 1 shows the stability map obtained by means of the above procedure. In shaded regions, the system is stable, and in the remaining regions it is unstable. The parameter values to generate the map are: $e'_q = 1.8$ pu, $x'_d = 0.8$ pu, $v_h = 1.0$ pu, $H = 2.0$ s, and $p_m = 1.0$ pu. These parameters lead to $K = 2.0156 \approx 2.0$. As expected, the delay-free system ($\tau = 0$) is stable for $A > 0$, as well as for small positive values of τ. Moreover, Figure 1 clearly shows that larger delays do not necessarily destabilize the system as long as the corresponding gain A is properly adjusted.

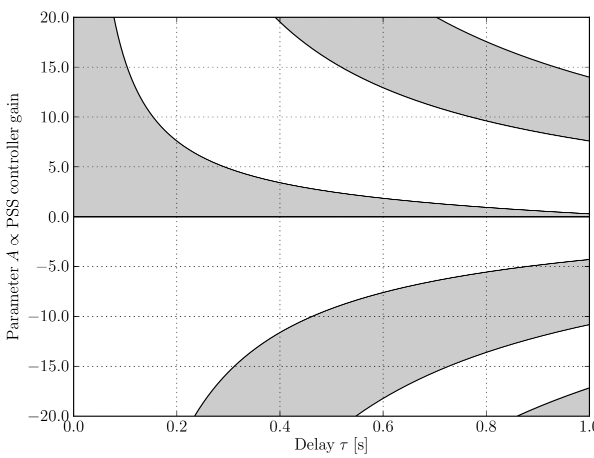

Fig. 1. Stability map on A-τ plane for $\tilde{K} \approx 2.0$. The power system is stable in the shaded regions.

III. Numerical Analysis for Large Power Systems

The example studied in Section II is a simplified one, as it captures the behavior of a joint system composed of a synchronous machine and the PSS control loop. This example is a good choice for two reasons: (i) its characteristic equation (7), which represents a delay differential equation (DDE) has such a relatively simple form that the critical roots of (8) could be determined analytically; and (ii) it motivates the rest of the paper regarding the intriguing mechanisms of delays on dynamic behavior.

In more general power control system models, however, the arising characteristic equation can be in a more complicated forms, e.g., the equations may have commensurate and multiple delays. To study the stability maps of such systems, various approaches can be adopted [7]. With this regard, we incorporate in our model the interactions among synchronous machines and the transmission system. This modeling part is borrowed from [11] as excerpts and provided below for completeness. The interested readers are referred to the cited study for a detailed discussion on the numerical small-signal stability analysis of delayed power system equations.

A. Standard Power System Model

The transient behavior of power systems is traditionally described through a set of differential algebraic equations (DAE) as follows:

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{u}), \\ \boldsymbol{0} &= \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{u}), \end{aligned} \tag{9}$$

where $\boldsymbol{f} : \mathbb{R}^{n+m+p} \mapsto \mathbb{R}^n$, $\boldsymbol{g} : \mathbb{R}^{n+m+p} \mapsto \mathbb{R}^m$, $\boldsymbol{x} \in \mathbb{R}^n$ are state variables, $\boldsymbol{y} \in \mathbb{R}^m$ are algebraic variables, and $\boldsymbol{u} \in \mathbb{R}^p$ are discrete variables modeling events, e.g., line outages and faults.

B. Delayed Power System Model

When delays affect the dynamics in (9), the delayed transient stability power system model becomes a set of delay differential-algebraic equations (DDAE) in index-1 Hessenberg form, as follows:

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{x}_d, \boldsymbol{y}_d, \boldsymbol{u}), \\ \boldsymbol{0} &= \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{x}_d, \boldsymbol{u}), \end{aligned} \tag{10}$$

where $\boldsymbol{x}_d$ and $\boldsymbol{y}_d$ are the *retarded* or *delayed* variables with respect to some state or algebraic variables, respectively. The model described in (10) is the index-1 Hessenberg form and is not the most general structure for DDAE. However, as shown in [11], the model (10) is appropriate to describe the transient stability of power systems and is thus used in the remainder of the paper.

C. Characteristic Equation of Delayed Power Systems

Assume now that, for a given event $\boldsymbol{u} = \boldsymbol{u}_0$, a stationary solution of (10) is known and has the form:

$$\begin{aligned} \boldsymbol{0} &= \boldsymbol{f}(\boldsymbol{x}_0, \boldsymbol{y}_0, \boldsymbol{x}_0, \boldsymbol{y}_0, \boldsymbol{u}_0), \\ \boldsymbol{0} &= \boldsymbol{g}(\boldsymbol{x}_0, \boldsymbol{y}_0, \boldsymbol{x}_0, \boldsymbol{u}_0), \end{aligned} \tag{11}$$

Then, linearizing (10) at the stationary solution yields:

$$\Delta\dot{\boldsymbol{x}} = \boldsymbol{f}_{\boldsymbol{x}}\Delta\boldsymbol{x} + \boldsymbol{f}_{\boldsymbol{x}_d}\Delta\boldsymbol{x}_d + \boldsymbol{f}_{\boldsymbol{y}}\Delta\boldsymbol{y} + \boldsymbol{f}_{\boldsymbol{y}_d}\Delta\boldsymbol{y}_d, \tag{12}$$

$$\boldsymbol{0} = \boldsymbol{g}_{\boldsymbol{x}}\Delta\boldsymbol{x} + \boldsymbol{g}_{\boldsymbol{x}_d}\Delta\boldsymbol{x}_d + \boldsymbol{g}_{\boldsymbol{y}}\Delta\boldsymbol{y}, \tag{13}$$

where, as usual, it can be assumed that $\boldsymbol{g}_{\boldsymbol{y}}$ is non-singular. Substituting (13) into (12) leads to:

$$\Delta\dot{\boldsymbol{x}} = \boldsymbol{A}_0\Delta\boldsymbol{x} + \boldsymbol{A}_1\Delta\boldsymbol{x}(t-\tau) + \boldsymbol{A}_2\Delta\boldsymbol{x}(t-2\tau), \tag{14}$$

where:

$$\boldsymbol{A}_0 = \boldsymbol{f_x} - \boldsymbol{f_y}\boldsymbol{g_y}^{-1}\boldsymbol{g_x}, \tag{15}$$
$$\boldsymbol{A}_1 = \boldsymbol{f}_{\boldsymbol{x}_d} - \boldsymbol{f_y}\boldsymbol{g_y}^{-1}\boldsymbol{g}_{\boldsymbol{x}_d} - \boldsymbol{f}_{\boldsymbol{y}_d}\boldsymbol{g_y}^{-1}\boldsymbol{g_x}, \tag{16}$$
$$\boldsymbol{A}_2 = -\boldsymbol{f}_{\boldsymbol{y}_d}\boldsymbol{g_y}^{-1}\boldsymbol{g}_{\boldsymbol{x}_d}. \tag{17}$$

The first matrix $\boldsymbol{A}_0$ is the well-known state matrix that is computed for standard DAE systems in the form of (9). The other two matrices are not null since the system at hand is affected by delays. The interested readers can find the details on how to determine (15)-(17) in [11].

Equation (14) is a particular case of the standard form of the linear delay differential equations:

$$\dot{\boldsymbol{x}} = \boldsymbol{A}_0\boldsymbol{x}(t) + \sum_{i=1}^{\nu} \boldsymbol{A}_i\boldsymbol{x}(t-\tau_i), \tag{18}$$

where, in this case, $\nu = 2$, $\tau_1 = \tau$ and $\tau_2 = 2\tau$. The substitution of a sample solution of the form $\boldsymbol{x} = e^{st}\boldsymbol{v}$, with $\boldsymbol{v}$ being a non-trivial possibly complex vector of dimension $n \times 1$, leads to the *characteristic equation* of (18):

$$\det \Delta(s) = 0, \tag{19}$$

where

$$\Delta(s) = s\boldsymbol{I}_n - \boldsymbol{A}_0 - \sum_{i=1}^{\nu} \boldsymbol{A}_i e^{-s\tau_i}, \tag{20}$$

is called the *characteristic matrix* [27], and $\boldsymbol{I}_n$ is the identity matrix of dimension $n \times n$. The solutions s of (19) are called the *characteristic roots* or *spectrum*, similar to the finite-dimensional case (i.e., the case for which $\boldsymbol{A}_i = \boldsymbol{0}\ \forall i = 1, \ldots, \nu$). However, since (19) is transcendental, i.e., it has infinitely many roots, one can only approximate the dynamic behavior of the system based on its rightmost eigenvalues.

Similar to the finite-dimensional case, the stability of (18) is guaranteed if and only if all the roots s of (19) have negative real parts. Although the stability condition seems to be simple, its assessment is quite difficult since it is impossible to find all the roots of (19). Yet, some elegant approaches have already been developed in the literature, to address this problem. These approaches utilize three useful properties of the characteristic equation [6], [27], as follows:

1) The power control system represented by (19) is of retarded type. That is, the highest derivative of the state in (18) is not affected by delay terms. This guarantees that the stability properties of the system at hand will be preserved as delay transitions from zero to a very small positive number.
2) Equation (19) only has a finite number of characteristic roots in any vertical strip of the complex plane, given by $\{\lambda \in \mathbb{C} : \alpha < \Re(\lambda) < \beta\}$.
3) There exists a number $\gamma \in \mathbb{R}$ such that all characteristic roots of (19) are confined to the half-plane $\{\lambda \in \mathbb{C} : \Re(\lambda) < \gamma\}$.

The above properties imply that the number of roots of the characteristic equation in the right-half of the complex plane is finite and if $\gamma < 0$, all the system eigenvalues have negative real parts, indicating that the system is stable.

D. Approximated Solution of the Characteristic Equation

Unfortunately, an analytic solution of γ from the characteristic equation is not possible. Hence, in this paper we use the technique proposed in [18], [19], [28] based on recasting (18) as an abstract Cauchy problem. This approach consists in transforming the original problem of computing the roots of a retarded functional differential equation as a matrix eigenvalue problem of a PDE system of infinite dimension and then discretizing such system by means of a finite element method.

The technical details of the discretization idea are suppressed here, referring the readers to the cited studies. The outcome of this approach is as follows:

- For stability analysis purposes, the approach should be carried out to detect the parametric settings for which the system has its rightmost eigenvalues ideally with zero real parts, corresponding to the boundaries separating stable from unstable regions. Note that stability analysis can also be performed following the ideas presented in [7], [22], [23] and the references therein.
- For performance analysis purposes, e.g., for studying the damping characteristics of the system, one could use the same approach to compute the real part of the rightmost eigenvalues of the system, in the parameter space, and using these eigenvalues, it would be possible to approximate damping and settling time properties of the system in the parameter space. Please also refer to [7] for other techniques that can be used to compute the system's rightmost eigenvalues.

To better illustrate the approach, we first assume that (10) has only one delay τ common to all retarded variables. Moreover, it is assumed that $\boldsymbol{A}_2 = \boldsymbol{0}$. This hypothesis is actually a consequence of considering that in (10) only algebraic variables depend on the delay value. Hence, (10) simplifies to:

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{y}_d, \boldsymbol{u}), \\ \boldsymbol{0} &= \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{u}), \end{aligned} \tag{21}$$

and from (16) and (17) one obtains:

$$\boldsymbol{A}_1 = -\boldsymbol{f}_{\boldsymbol{y}_d}\boldsymbol{g_y}^{-1}\boldsymbol{g_x}, \qquad \boldsymbol{A}_2 = \boldsymbol{0}, \tag{22}$$

from which, (19) becomes:

$$\det\Delta(s) = \det\left(s\boldsymbol{I}_n - \boldsymbol{A}_0 - \boldsymbol{A}_1 e^{-s\tau}\right). \tag{23}$$

Then, one has to choose the numbers of nodes N, which associated with a Chebyshev's discretization scheme. This number affects the precision and the computational time needed in the method, as explained below. Let $\boldsymbol{D}_N$ be Chebyshev's differentiation matrix of order N and define

$$\boldsymbol{M} = \left[\begin{array}{ccccc} \multicolumn{5}{c}{\hat{\boldsymbol{C}} \otimes \boldsymbol{I}_n} \\ \hline \boldsymbol{A}_1 & \boldsymbol{0} & \ldots & \boldsymbol{0} & \boldsymbol{A}_0 \end{array}\right], \tag{24}$$

where $\otimes$ denotes Kronecker's product; $\boldsymbol{I}_n$ is the identity matrix of order n; and $\hat{\boldsymbol{C}}$ is a matrix composed of the first $N-1$ rows of $\boldsymbol{C}$ defined as follows:

$$\boldsymbol{C} = -2\boldsymbol{D}_N/\tau, \tag{25}$$

Then, the eigenvalues of $\boldsymbol{M}$ approximate the spectrum of (23).

Without entering into mathematical details, one can see $\boldsymbol{M}$ as the discretization of a PDE system where the continuous variable, say ξ, corresponds to the time delay. Then ξ is discretized along a grid of N points. The position of such points are defined by Chebyshev's polynomial interpolation. The last n rows of $\boldsymbol{M}$ correspond to the PDE boundary conditions $\xi = \tau$ (e.g., $\boldsymbol{A}_1$) and $\xi = 0$ (e.g., $\boldsymbol{A}_0$), respectively. This suggests also how to generalize $\boldsymbol{M}$ for a case of characteristic equations with $\nu > 1$. For example, the matrix $\boldsymbol{M}$ for $\nu = 2$ can be formulated as follows:

$$\boldsymbol{M} = \left[\frac{\hat{\boldsymbol{C}} \otimes \boldsymbol{I}_n}{\boldsymbol{A}_2 \quad \boldsymbol{0} \dots \boldsymbol{0} \quad \boldsymbol{A}_1 \quad \boldsymbol{0} \dots \boldsymbol{0} \quad \boldsymbol{A}_0} \right], \tag{26}$$

where $N + 1$ must be odd to allow $\boldsymbol{A}_1$ being in the central node of Chebyshev's grid.

As expected, the general case with multiple delays can be implemented by increasing N and, hence, the size of the matrix $\boldsymbol{M}$, and modifying accordingly its last n rows. The interested reader can find further insights on the multiple delay case in [19] and [20]. For the sake of simplicity, in this paper, only the case of a single delay is considered. This assumption is justified by the fact that the delay originates from a single device type, i.e., the PSS.

IV. Case Study

The system considered in this paper is the IEEE 14-bus system, which consists of two generators, three synchronous compensators, two two-winding and one three-winding transformers, fifteen transmission lines, eleven loads and one shunt capacitor (see Fig. 2). The system also includes generator controllers, such as the primary voltage regulators. All dynamic data of this system as well as a detailed discussion of its transient behavior can be found in [21].

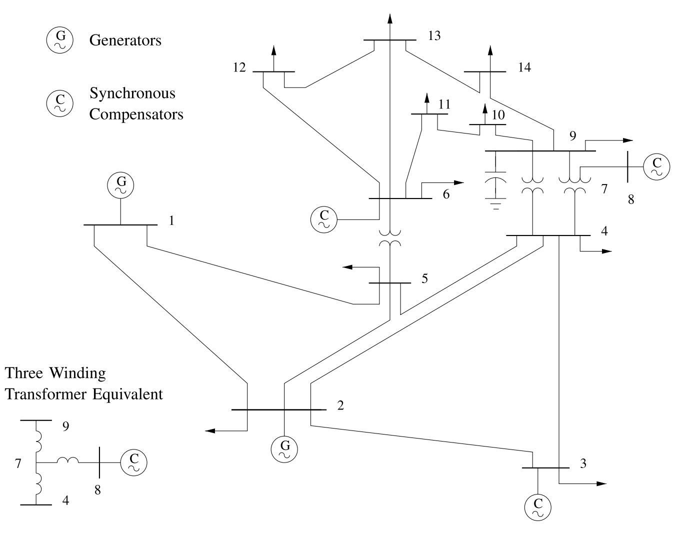

Fig. 2. IEEE 14-bus test system. The system includes a PSS device connected to generator 1.

In typical PSSs, the input signal is the synchronous machine rotor speed ω, which, in our formulation is a state variable. In most cases, the rotor speed is measured locally, i.e., it is the rotor speed of the machine where the PSS is installed. However, there exist wide area measurement systems (WAMS) where remote signals are used, e.g., the frequency of a pilot bus [9]. Local measurements have at most a few ms delay while remote measurements can be affected by a delay of up to 100 ms or more [9]. In this paper we consider that the input signal of the PSS of generator 1 is obtained through a WAMS. A typical PSS control scheme includes a washout filter and two lead-lag blocks. The resulting control scheme diagram of the PSS is shown in Fig. 3. Observe that the DDAE that describes the PSS satisfies the index-1 Hessenberg form (10) where $x_d = \omega(t - \tau_\omega)$. The interested reader can find a detailed description of the system equations in [11].

Thus the retarded measure of ω propagates into the PSS equations, as follows:

$$\begin{aligned}
\dot{v}_1 &= -(K_w \omega(t - \tau_\omega) + v_1)/T_w \qquad (27)\\
\dot{v}_2 &= ((1 - \frac{T_1}{T_2})(K_w \omega(t - \tau_\omega) + v_1) - v_2)/T_2 \\
\dot{v}_3 &= ((1 - \frac{T_3}{T_4})(v_2 + (\frac{T_1}{T_2}(K_w \omega(t - \tau_\omega) + v_1))) - v_3)/T_4 \\
0 &= v_3 + \frac{T_3}{T_4}(v_2 + \frac{T_1}{T_2}(K_w \omega(t - \tau_\omega) + v_1)) - v_s
\end{aligned}$$

where v_1, v_2 and v_3 are state variables introduced by the PSS washout filter and by lead-lag blocks, and other parameters are illustrated in Fig. 3. Observe that equations in (27) are in the form of (10) with $\boldsymbol{x} = (v_1, v_2, v_3)$, $x_d = \omega(t - \tau_\omega)$, and $y = v_s$.

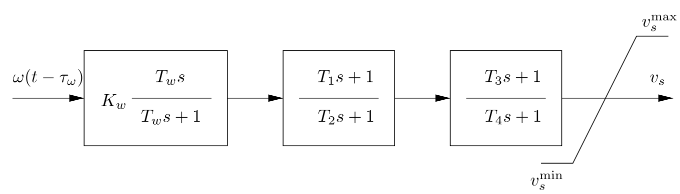

Fig. 3. Power system stabilizer control diagram [21].

Applying the approaches summarized above, the small-signal stability region for the delayed IEEE 14-bus system is found as in Fig. 4. The shaded regions indicate stable equilibria. The shape of the stable region is similar to the one of the simplified system depicted in Fig. 1 only for small positive values of τ_ω, but for larger values of τ_ω, there is noticeable difference. Moreover, inspecting Fig. 4, one finds out as a general rule that in order to keep the IEEE 14-bus system in the stability region for larger τ_ω values, one should decrease K_w. Furthermore, there exists a region on the stability map, corresponding to relatively large values of τ_ω, for which the system stability can still be maintained with the selection of negative values of the PSS gain K_w. Finally, it is remarked that a properly damped response, assuming that 5% is an adequate damping threshold, is attainable in this case, as indicated by the dark gray region in Fig. 4.[1]

The stable region for $\tau_\omega \in (0.2, 0.5)$ ms shows a cusp for $(\tau_\omega, K_w) \approx (0.3325, -5.067)$. This is a bifurcation point: the descending branch remains stable for $\tau_\omega > 0.3325$, but the

[1]Damping ratio ζ is approximated here as the negative cosine of the angle formed by the complex vector defined by the imaginary and real part of the stable rightmost complex eigenvalues z; $\zeta = -\cos(\arctan(\Im(z)/\Re(z)))$.

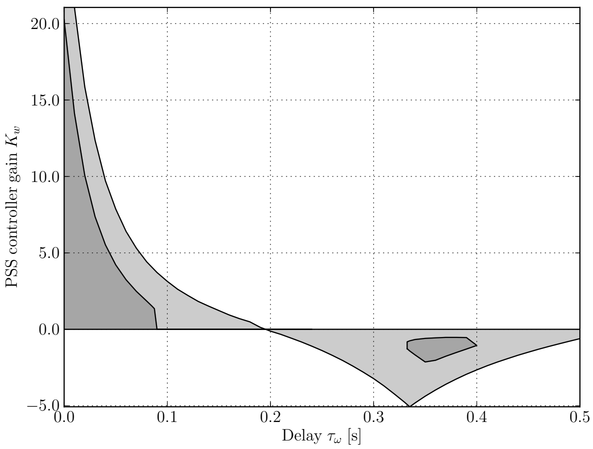

Fig. 4. Stability map of the K_w-τ_ω plane for the IEEE 14-bus system. Shaded regions are stable. Dark shaded regions indicate a damping greater than 5%.

other branch cuts the stability region. No stable points can be found for $K_w < -5.067$.

To solve the stability map shown in Fig. 4, the software Dome [29] has been used, which implements the frequency domain approach discussed in Section III. The number of points of the Chebyshev differentiation matrix is $N = 10$, which leads to a matrix $\boldsymbol{M}$ of order 520×520. On a Dell Precision T1650 equipped with 4-core Intel Xeon CPU 3.50GHz and 8 GB of RAM, the solution of the rightmost eigenvalue spectrum for each given point in the parameters space takes around 0.45 s. The small-signal stability boundaries has been calculated using a simple bisection method with a tolerance of 10^{-3} on the real part of the critical eigenvalues.

To properly determine the whole stable region, an eigenvalue analysis for a grid of points in the rectangle defined by $\tau_\omega \in [0, 0.5]$ ms and $K_w \in [-5, 20]$ has been carried out. Note that we have not detected other stable regions beside the one shown in Fig. 4. However, even if such regions would exist under different parametric settings, they would be "islanded" regions, and thus would be likely unreachable as system trajectories could not get to such disjointed stable regions without passing through an unstable path. Hence the stability region depicted in Fig. 4 is the only one that has practical interest.

V. Practical Implications of Delayed PSS and Final Remarks

The stability analysis presented in Sections II and IV allow drawing general conclusions and lead to practical implications, as follows:

- The system studied in Fig. 1 remains stable for $0 \leq \tau \leq 1$ s, provided that the gain A is properly adjusted. However, although stable, the response of the system in terms of damping can be unacceptable for high values of τ. In fact, as τ increases, A has to decrease to keep the system stable. The limit case is $A \to 0$ as $\tau \to \infty$ which means that the PSS control loop is open and, hence, the system transient behavior is driven by the sole synchronous machine, which is generally poorly damped.
- In case of remote PSS input signals (see, for example, [9]), estimating the value of time delay would allow properly tuning the gain A so that the effect of the delay on the system dynamic response could be minimized.
- The effect of delays depends on $\tilde{K}$ and thus on relevant parameters of the synchronous machines such as p_m, v_h and e'_q. This fact can be taken into account to define a proper tuning of A in case of changes in the operating point of the synchronous machine.
- To intentionally add delay to a control loop is generally not acceptable. However, the stability map shown in Fig. 4 suggests that, in case the measured PSS input signal is affected inevitably by a relatively large delay (e.g., $\tau \in (0.1, 0.3)$ s), then it could be convenient to introduce an additional delay, and to accordingly change the control gain K_w, in order to improve the overall system small-signal stability. This adaptive control requires an estimation of the delay and, apart from that, it can be easily implemented by means of a look-up table based on the results obtained from the small-signal stability analysis.

The analysis presented in this paper can be extended towards several directions. For example, the effect of multiple time-delays and their joint interaction on system stability are still open questions. With this aim, a multi-dimensional analysis has to be carried out. Future work will focus on the definition of robust controllers that take advantage of the effects of delays to properly damp synchronous machine oscillations.

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