Frequency Regulation of Synchronous Machines

POWER SYSTEM MODELLING AND CONTROL (EEEN40550)

Prof. Federico Milano

Email: federico.milano@ucd.ie

Tel.: 01 716 1844

Room 157a – Engineering & Materials Science Centre

School of Electrical & Electronic Engineering

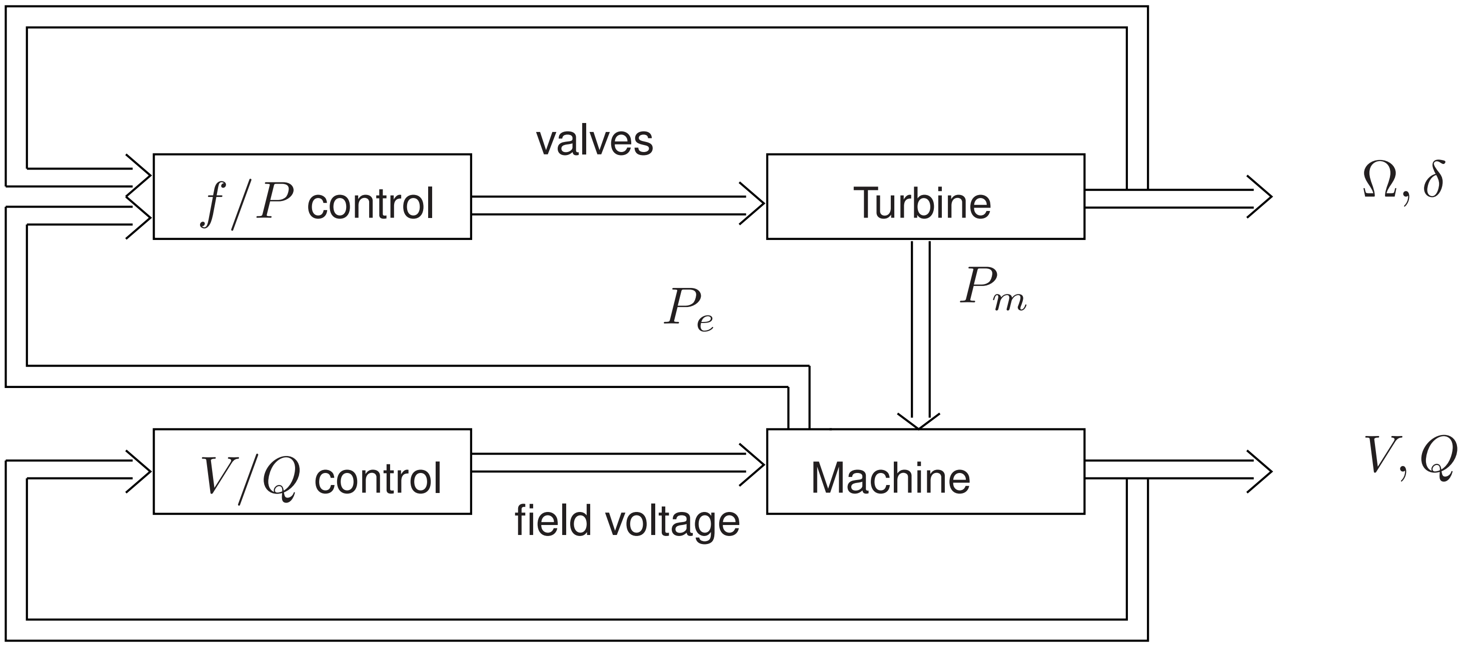
University College Dublin

Dublin, Ireland

Time Frames of Synchronous Machine Transients

Electrical transients	short circuits	$0.001 \text{ s} \div 0.1 \text{ s}$
Torsional oscillations	sub-synchronous resonance	$0.005 \text{ s} \div 0.5 \text{ s}$
Rotor fluxes	subtransient (damper winding)	$0.005 \text{ s} \div 0.3 \text{ s}$
	transient (field)	$0.2 \text{ s} \div 2 \text{ s}$
Electromechanical oscillations	OMIB	$0.1 \text{ s} \div 5 \text{ s}$
	Two machines	$2 \div 30 \text{ s}$
Voltage control	primary	$0.2 \div 5 \text{ s}$
	secondary	$0.2 \div 100 \text{ s}$
Frequency control	primary	$1 \div 20 \text{ s}$
	secondary	$10 \div 500 \text{ s}$

Interaction between Frequency and Voltage Controls - I



- The voltage (V/Q) control affects the frequency (f/P) control much more than what the f/P control affects the V/Q one.

$$f/P \quad \overset{\dashrightarrow}{\Longleftarrow} \quad V/Q$$

Interaction between Frequency and Voltage Controls - II

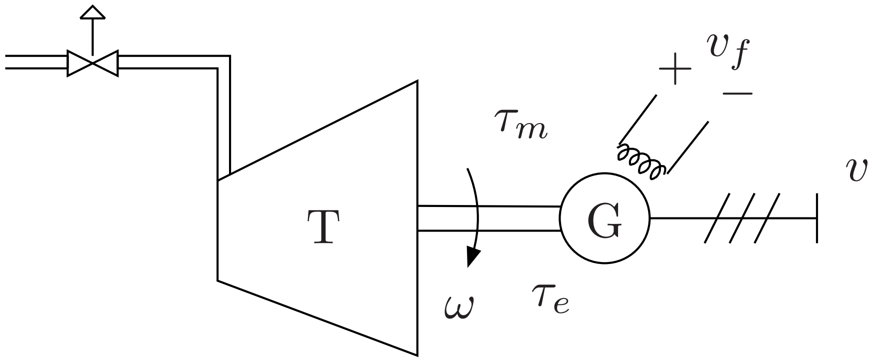
- In fact, we have:

$$
\begin{aligned}
P_e &= \frac{EV}{X} \sin \delta \\
E &\propto \Omega \Phi^{\max} \approx \text{constant} \qquad (\text{if } \Omega \uparrow, \ \Phi \downarrow)
\end{aligned}
$$

- Hence the f/P control does not affect *too much* the V/Q control.
- The f/P control is also much slower than the V/Q one.

Decoupled Frequency and Voltage Controls - I

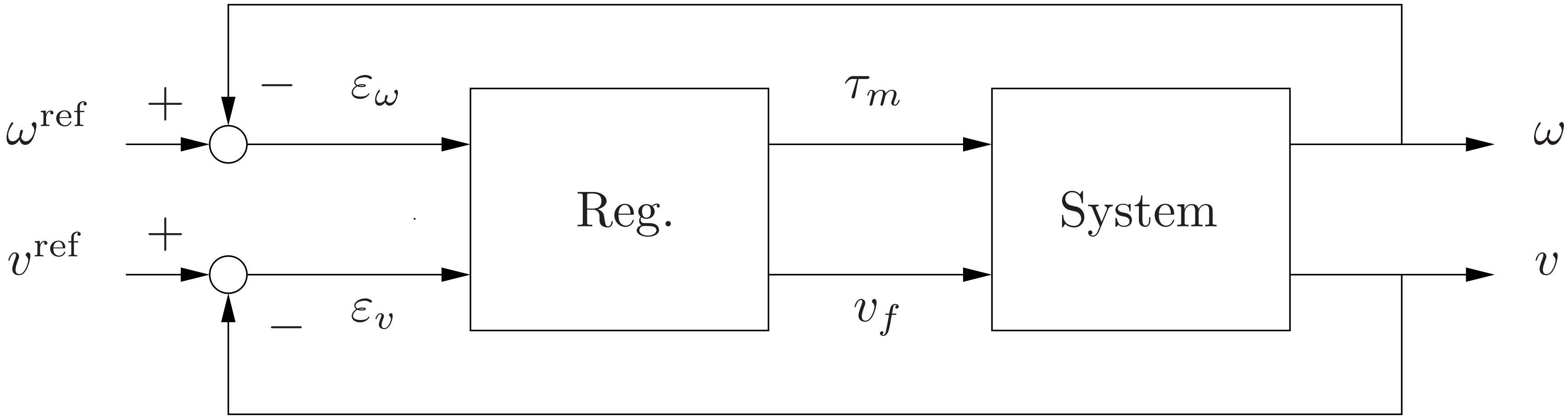
- The system is nonlinear, hence a fully decoupled control cannot be obtained.
- However, let's consider the linearisation at an operating point.
- We need the model of the system (all quantities are assumed in pu):



$$
\begin{cases}
\tau_m - \tau_e = M \frac{d\omega}{dt} \\
\tau_e = \tau_e(v_f, \omega) \\
v = v(v_f, \omega)
\end{cases}
$$

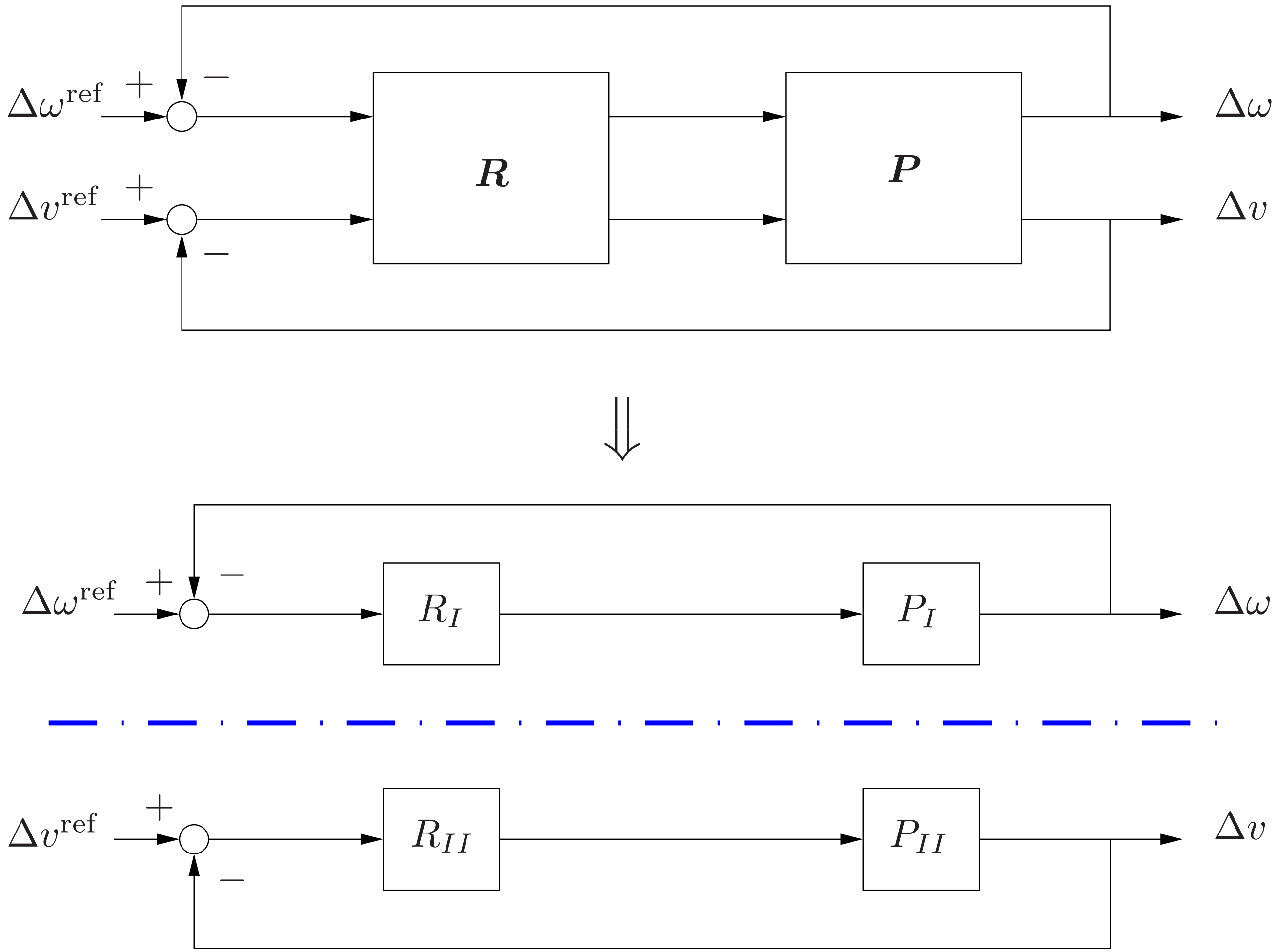
Decoupled Frequency and Voltage Controls - II

- General Scheme:



→ If we linearise the model of the system, we can obtain an independent control.

Principle of Decoupled Control



Linearization of the System at the O.P. - I

$$\Rightarrow \qquad \Delta\tau_m - \Delta\tau_e = M\frac{d\Delta\omega}{dt} \qquad (*)$$

$$\Rightarrow \Delta\tau_e = \left.\frac{\partial\tau_e}{\partial\omega}\right|_0 \Delta\omega + \left.\frac{\partial\tau_e}{\partial v_f}\right|_0 \Delta v_f \qquad (**)$$

$$\Rightarrow \quad \Delta v = \left.\frac{\partial v}{\partial\omega}\right|_0 \Delta\omega + \left.\frac{\partial v}{\partial v_f}\right|_0 \Delta v_f \qquad (***)$$

Substituting $(**)$ in $(*)$:

$$\Delta\tau_m = \left.\frac{\partial\tau_e}{\partial\omega}\right|_0 \Delta\omega + \left.\frac{\partial\tau_e}{\partial v_f}\right|_0 \Delta v_f + \underbrace{Ms\Delta\omega}_{s \text{ stands for Laplace operator}} \qquad (****)$$

Linearization of the System at the O.P. - II

- The outputs are $\Delta\omega$ and Δv, while the inputs are $\Delta\tau_m$ and Δv_f
- Hence we rewrite $(***)$ and $(****)$:

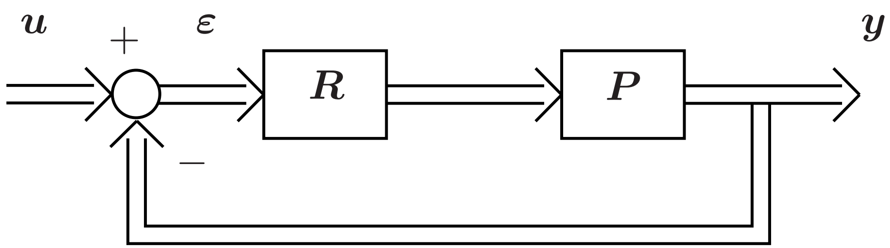
$$\Delta\omega = \frac{\Delta\tau_m}{Ms + \frac{\partial\tau_e}{\partial\omega}\big|_0} - \frac{\frac{\partial\tau_e}{\partial v_f}\big|_0}{Ms + \frac{\partial\tau_e}{\partial\omega}\big|_0}\Delta v_f$$

$$\Rightarrow \boxed{\Delta\omega = p_{11}\Delta\tau_m + p_{12}\Delta v_f}$$

$$\Delta v = \left\{\frac{\frac{\partial v}{\partial\omega}\big|_0}{Ms + \frac{\partial\tau_e}{\partial\omega}\big|_0}\right\}\Delta\tau_m + \left\{\frac{\partial v}{\partial v_f}\Big|_0 - \frac{\partial v}{\partial\omega}\Big|_0 \frac{\frac{\partial\tau_e}{\partial v_f}\big|_0}{Ms + \frac{\partial\tau_e}{\partial\omega}\big|_0}\right\}\Delta v_f$$

$$\Rightarrow \boxed{\Delta v = p_{21}\Delta\tau_m + p_{22}\Delta v_f}$$

Synthesis of the Regulator - I



- We know $\boldsymbol{P}$.
- Moreover: $\boldsymbol{y} = \boldsymbol{D}\boldsymbol{\varepsilon}$, where $\boldsymbol{D} = \boldsymbol{P}\boldsymbol{R}$

  we impose that:

$$\begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix} = \begin{bmatrix} d_{11} & 0 \\ 0 & d_{22} \end{bmatrix}$$

  Hence, $\boldsymbol{D}$ must be diagonal.

Synthesis of the Regulator - II

- We obtain:

$$\begin{cases} p_{11}r_{11} + p_{12}r_{21} = d_1 \\ p_{11}r_{12} + p_{12}r_{22} = 0 \\ p_{21}r_{11} + p_{22}r_{21} = 0 \\ p_{21}r_{21} + p_{22}r_{22} = d_2 \end{cases}$$

- Unknown quantities are $r_{11}, r_{12}, r_{21}, r_{22}$.
- d_1 and d_2 can be imposed by assuming the desired behaviour of the system.

Some simplifications:

- The elements of $\boldsymbol{P}$ can be simplified:

$$\frac{\partial v}{\partial \omega} \approx 0 \quad \Rightarrow \quad p_{21} \approx 0$$

$$p_{22} \approx \left. \frac{\partial v}{\partial v_f} \right|_0$$

- Moreover:

$$\frac{\partial \tau_e}{\partial \omega} \approx 0 \quad \Rightarrow \quad p_{11} \cong \frac{1}{Ms}$$

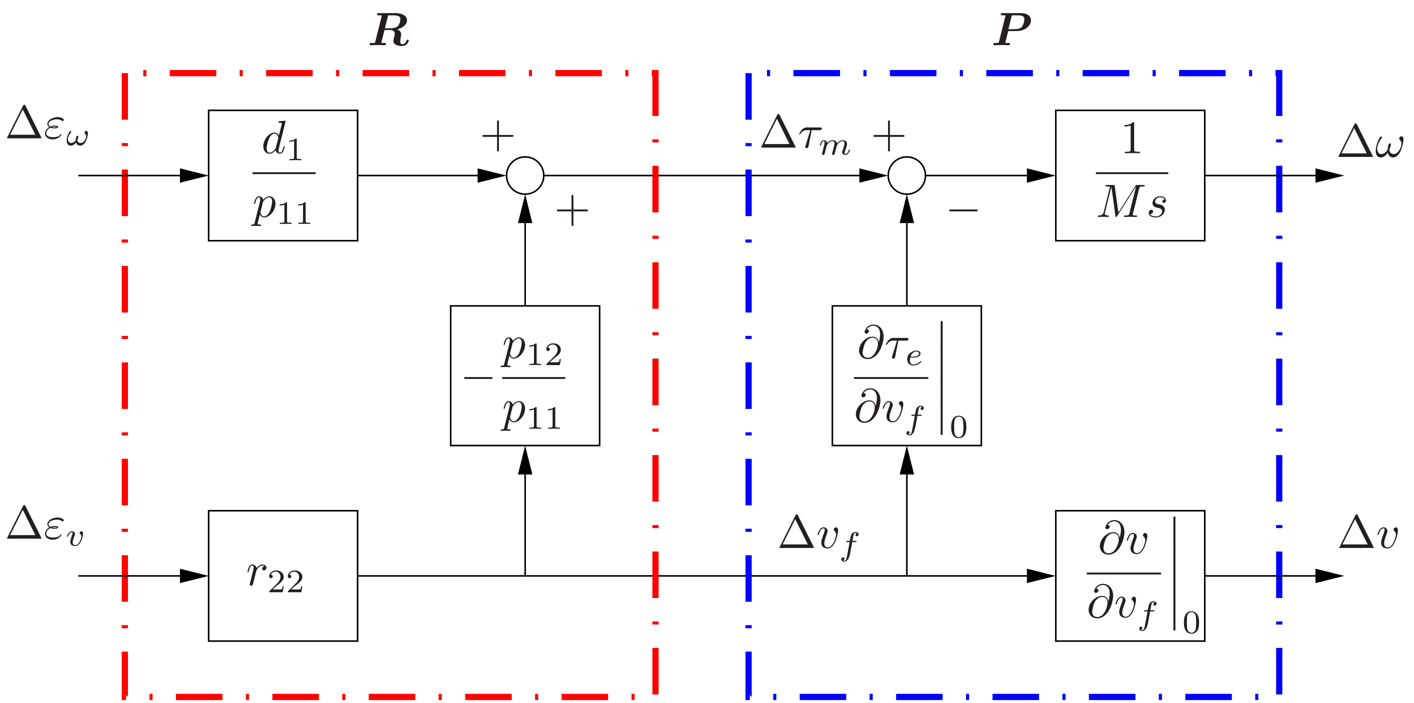
$$p_{12} \cong \left. \frac{\partial \tau_e}{\partial v_f} \right|_0 \frac{1}{Ms}$$

Synthesis of the Regulator - III

- Applying the simplifications above, we obtain:

$$\boldsymbol{R} \Rightarrow \begin{cases} r_{21} = 0 \\ r_{22} = \frac{d_2}{p_{22}} \\ r_{11} = \frac{d_1}{p_{11}} \\ r_{12} = -\frac{p_{12}}{p_{11}}\frac{d_2}{p_{22}} = -\frac{p_{12}}{p_{11}} r_{22} \end{cases}$$

Resulting Control Scheme



Observe that:
$$-\frac{p_{12}}{p_{11}} = \frac{-\left(-\left.\frac{\partial\tau_e}{\partial v_f}\right|_0 \cdot \frac{1}{Ms}\right)}{\frac{1}{Ms}} = \left.\frac{\partial\tau_e}{\partial v_f}\right|_0$$

Decoupled Frequency and Voltage Controls - III

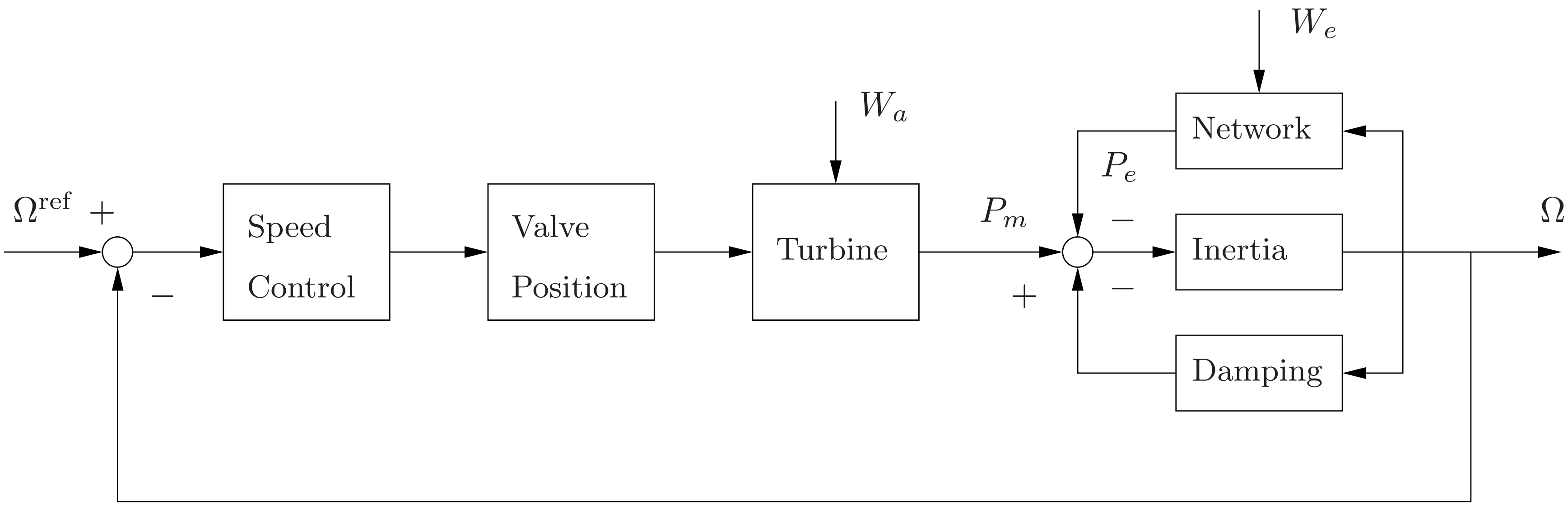
- Previous scheme is valid only for a particular O.P.
- To work for a general O.P., the control scheme must be adaptive
- We can obtain some results using a state-space approach:

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u}$$
$$\boldsymbol{y} = \boldsymbol{C}\boldsymbol{x} + \boldsymbol{D}\boldsymbol{u}$$

where $\boldsymbol{u} = [\tau_m, v_f]^T, \quad \boldsymbol{x} = [\delta, \omega]^T, \quad \boldsymbol{y} = [\omega, v]^T$

Primary Frequency Control of Synchronous Machines - I

- General scheme of the turbine governor and of the turbine/machine:

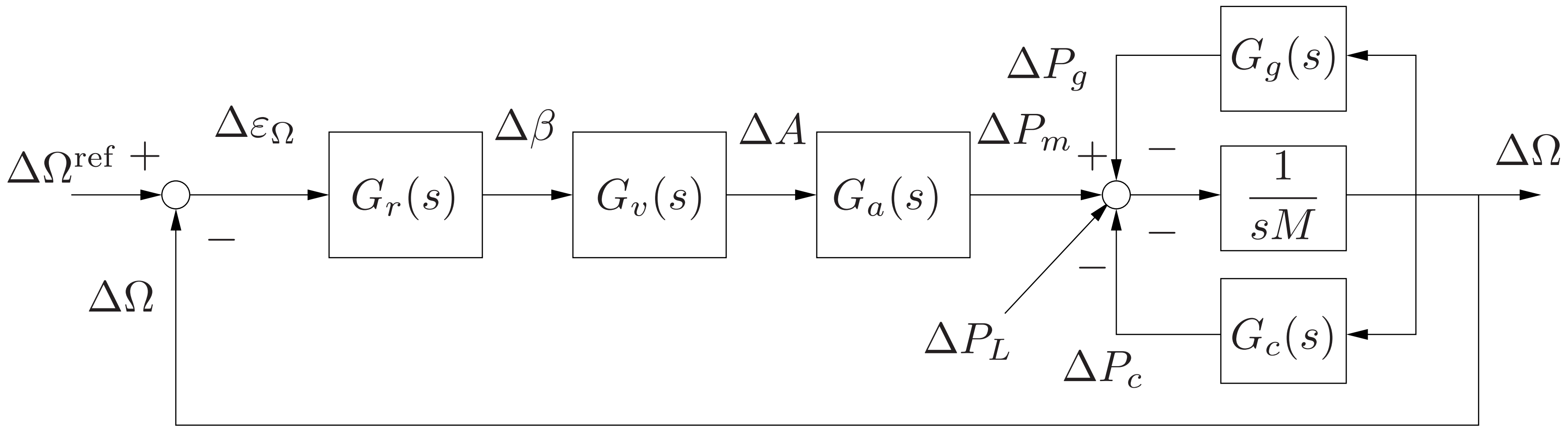


$$W_e = \text{perturbation within the network (e.g., load variation)}$$

$$W_a = \text{perturbation within the mechanical system}$$

Primary Frequency Control of Synchronous Machines - II

- Let's consider a linearisation around an operating point.



$$P_m - (P_e + P_g) = M\frac{\mathrm{d}\Omega}{\mathrm{d}t} \quad \rightarrow \quad \Delta P_m - (\Delta P_e + \Delta P_g) = M\frac{\mathrm{d}\Delta\Omega}{\mathrm{d}t}$$

where $P_e = P_c + P_L$

$$\Delta\Omega(s) = \frac{1}{sM}[\Delta P_m - (\Delta P_e + \Delta P_g)]$$

Primary Frequency Control of Synchronous Machines - III

- Where:

  $\Delta\beta$ = angular position for the valve regulation

  ΔA = valve position – cross section

  $G_c(s)$ = transfer function that takes into account the dependence of the load on frequency

  $G_g(s)$ = transfer function that takes into account mechanical losses, damping etc.

  $G_r(s) = \Delta\beta/\Delta\varepsilon_\Omega$

  $G_v(s) = \Delta A/\Delta\beta$

  $G_a(s) = \Delta P_m/\Delta A$

Primary Frequency Control of Synchronous Machines - IV

- We define $P_e = P_c + P_L \quad \Rightarrow \quad \Delta P_e = \Delta P_c + \Delta P_L$

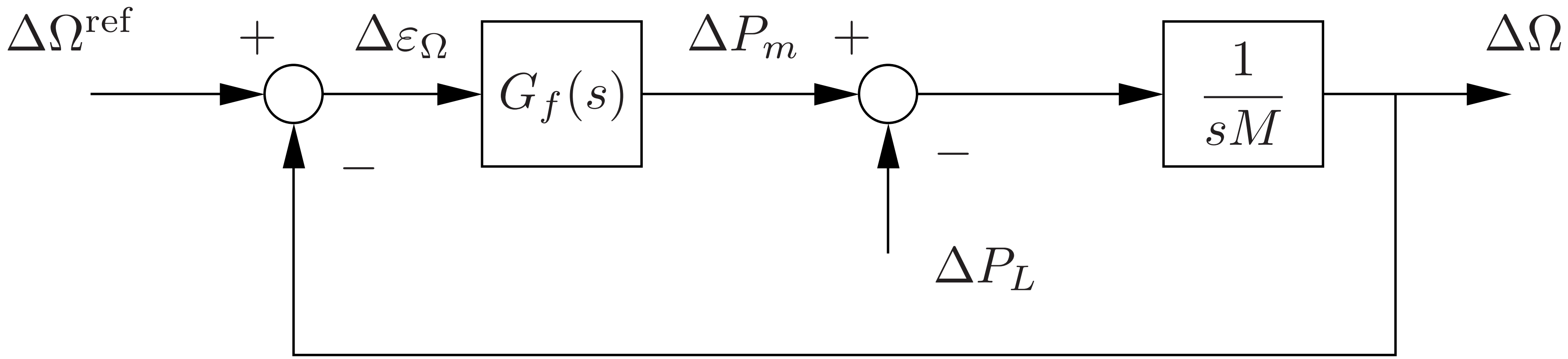
  P_L = load variation that does not depend on Ω

  P_c = load variation that depends on Ω

- For the sake of simplicity, let assume:

$$G_g(s) = 0$$
$$G_c(s) = 0$$

Simplified Primary Frequency Control Scheme



where:

$$G_f(s) \triangleq G_r(s)G_v(s)G_a(s)$$

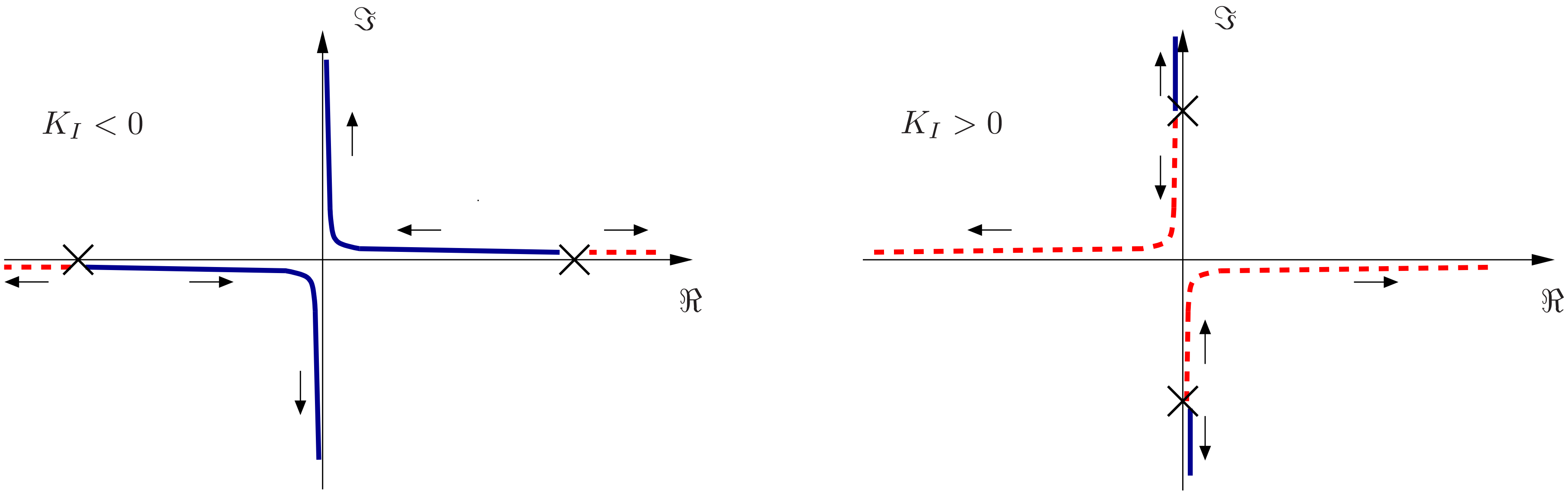
- We want a control scheme with the following features:
  - **fast**
  - **stable**
  - **zero static error**

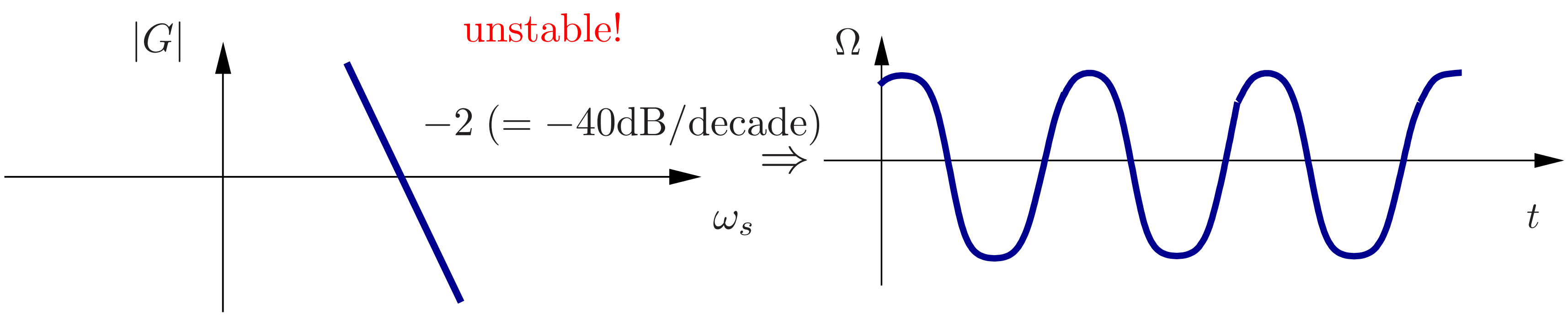
Synthesis of the Primary Frequency Control - I

- There are various possibilities for $G_f(s)$:
  1. $\dfrac{K_I}{s}$,   (integrator)
  2. $K_I \dfrac{1+sT_2}{s}$,   (PI)
  3. $K_I T_1 \dfrac{1+sT_2}{1+sT_1}$,   (lead-lag) $T_1 > T_2$ or $T_1 < T_2$
- In the following slides, we will consider each case.

Synthesis of the Primary Frequency Control - II

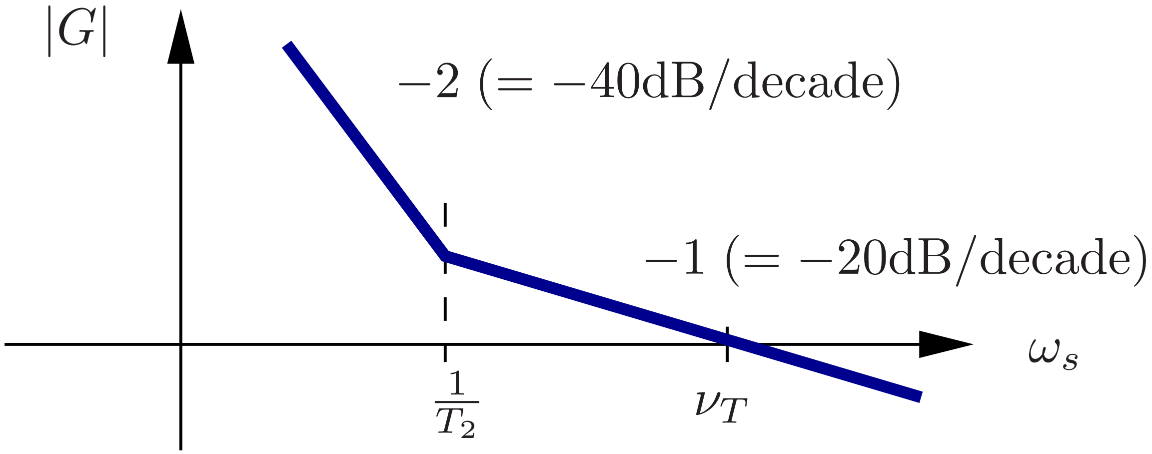
2) Integrator K_I/s then: $G(s) = G_f(s)\frac{1}{sM} = \frac{K_I}{s^2 M}$





Synthesis of the Primary Frequency Control - III

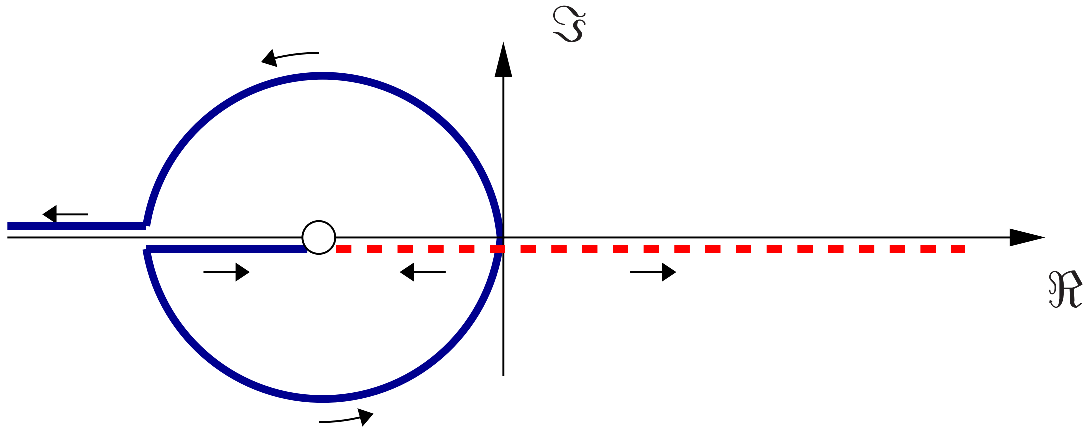
2) PI regulator: $G(s) = K_I \frac{1+sT_2}{s^2 M}$



stable if $\nu_T \gg \frac{1}{T_2}$

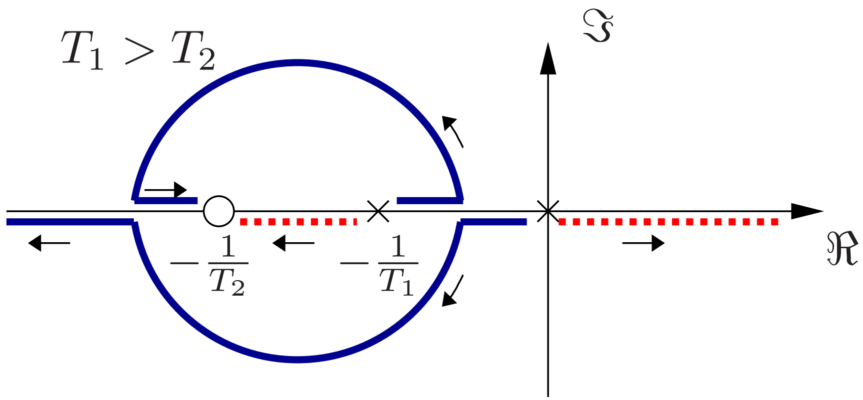
hence:

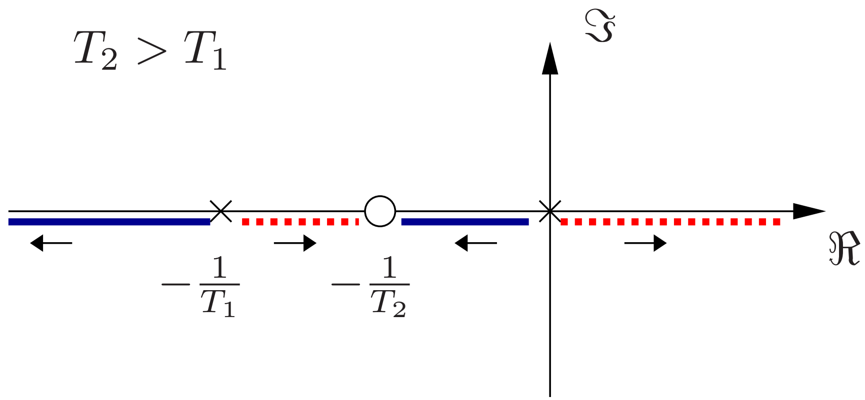
$$\frac{K_I}{s^2 M} sT_2 \Big|_{s=\nu_T} = 1$$



Synthesis of the Primary Frequency Control - IV

3) lead-lag controller. $G(s) = K_I \frac{1+sT_2}{sM(1+sT_1)}$



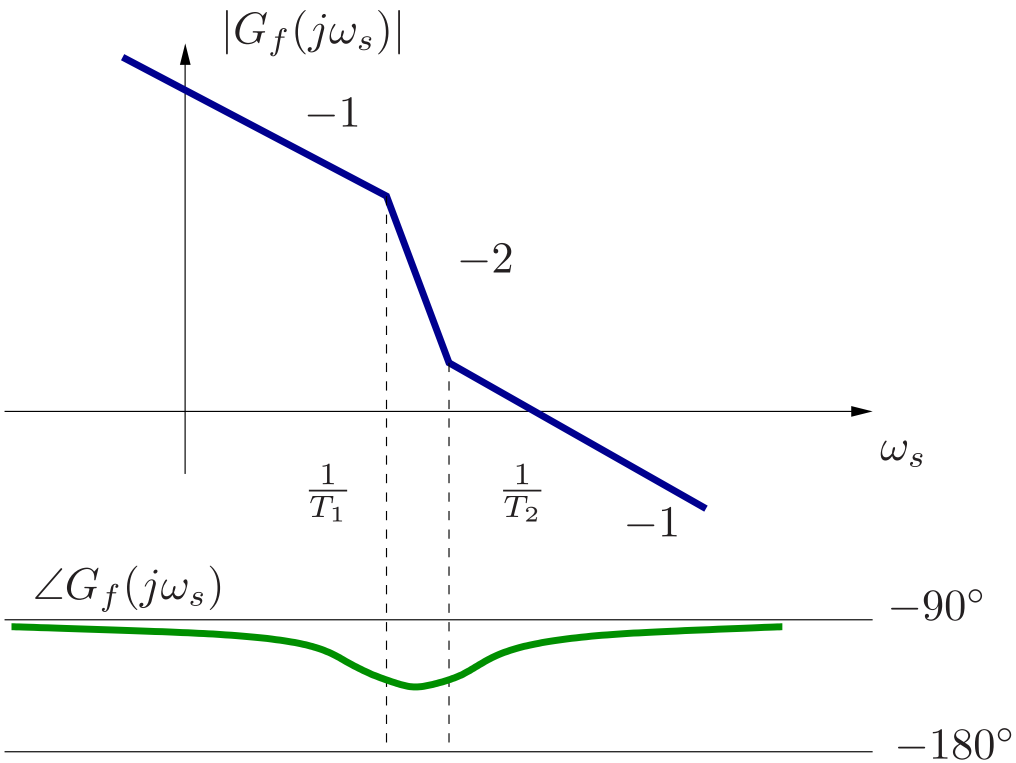


$$T_1 \simeq 10 \div 15 \text{ s}$$
$$T_2 \simeq 3 \div 5 \text{ s}$$

$$T_2 \simeq 10 \div 15 \text{ s}$$
$$T_1 \simeq 3 \div 5 \text{ s}$$

Synthesis of the Primary Frequency Control - V

- Bode's diagram:



$$\frac{b_p}{b_t} = \frac{T_2}{T_1}$$

$$G_f(s) = K_I T_1 \frac{1 + sT_2}{1 + sT_1}$$

$$-\left(\frac{\Delta\Omega}{\Delta P_m}\right)_{\Delta\Omega^{\mathrm{ref}}=0}(0) = \frac{1}{K_I T_1}$$

$$-\left(\frac{\Delta\Omega}{\Delta P_m}\right)_{\Delta\Omega^{\mathrm{ref}}=0}(\infty) = \frac{1}{K_I T_2}$$

Static and Dynamic Droops

- Let define the static droop b_p on machine bases:

$$b_p = \frac{P_n}{\Omega_n} \cdot \frac{1}{K_I T_1} \quad \Rightarrow \quad -\left(\frac{\Delta\Omega/\Omega_n}{\Delta P_m/P_n}\right)_{\Delta\Omega^{\text{ref}}=0}(0) = b_p$$

- Let define the dynamic droop b_t on machine bases:

$$b_t = \frac{P_n}{\Omega_n} \cdot \frac{1}{K_I T_2} \quad \Rightarrow \quad -\left(\frac{\Delta\Omega/\Omega_n}{\Delta P_m/P_n}\right)_{\Delta\Omega^{\text{ref}}=0}(\infty) = b_2$$

Droop Change of Bases

- The value of the droop depends on the power and frequency bases.
- If we use different bases, say, $\hat{P}_n$ and $\hat{\Omega}_n$, the droop $\hat{b}_p$ can be obtained as:

$$\hat{b}_p = \frac{\hat{P}_n}{\hat{\Omega}_n} \cdot \frac{\Omega_n}{P_n} \cdot b_p$$

- Then, assuming that the frequency is unique for all the AC network:

$$\hat{b}_p = \frac{\hat{P}_n}{P_n} \cdot b_p$$

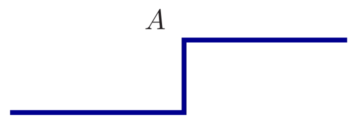
- Note that, machines tend to have similar b_p (on machine bases).

Static Error of the Lead-Lag control - I

- $$\frac{\Delta\Omega}{\Delta P_L} = \frac{-\frac{1}{sM}}{1 + \frac{1}{sM} K_I \frac{1+sT_2}{1+sT_1}}$$

  Note that: $\lim_{t\to\infty} \Delta\Omega(t) = \lim_{s\to 0} s\Delta\Omega(s)$

- If ΔP_L is a step of magnitude A, e.g.,



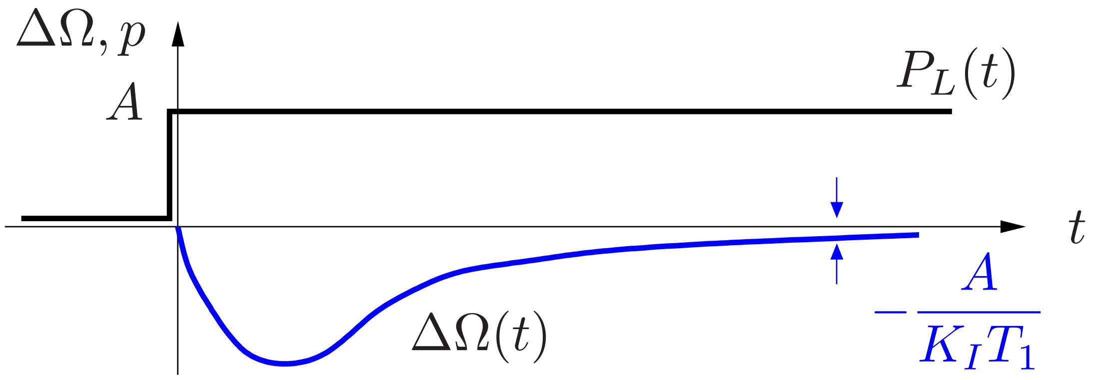
$$\Rightarrow \Delta P_L(s) = \frac{A}{s}$$

then:

$$\lim_{t\to\infty} \Delta\Omega(t) = \lim_{s\to 0} \frac{-\frac{1}{sM}}{1 + \frac{1}{sM} K_I \frac{1+sT_2}{1+sT_1}} s \frac{A}{s} = \frac{-A}{K_I T_1} \neq 0$$

$\Rightarrow$ The static error is not zero!

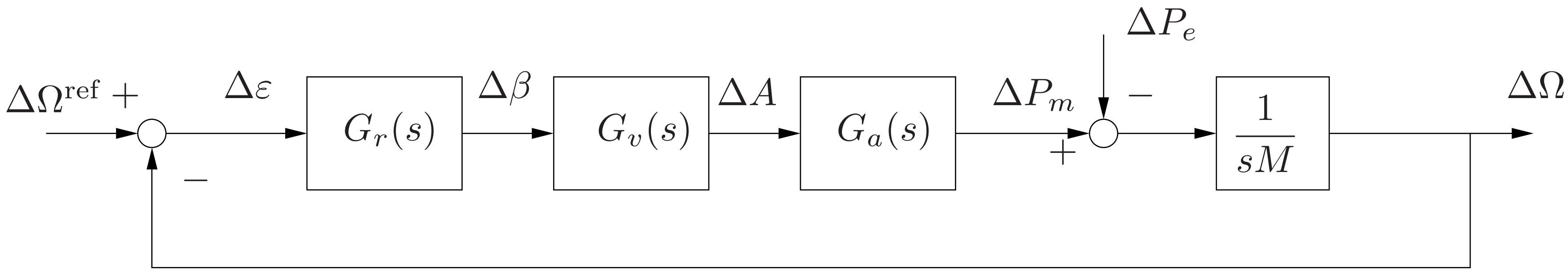
Static Error of the Lead-Lag Control - II



- Observe that in this simulation $\Delta\Omega^{\mathrm{ref}} = 0$, as it is in practise for the primary frequency control.
- If the system is islanded we need to use scheme (2).
- For interconnected networks, we use (3) for all machines and a secondary control to obtain $\Delta\Omega = 0$ in steady state.

Steady-State Characteristic of the Primary Frequency Control

- We have, in general:



where:

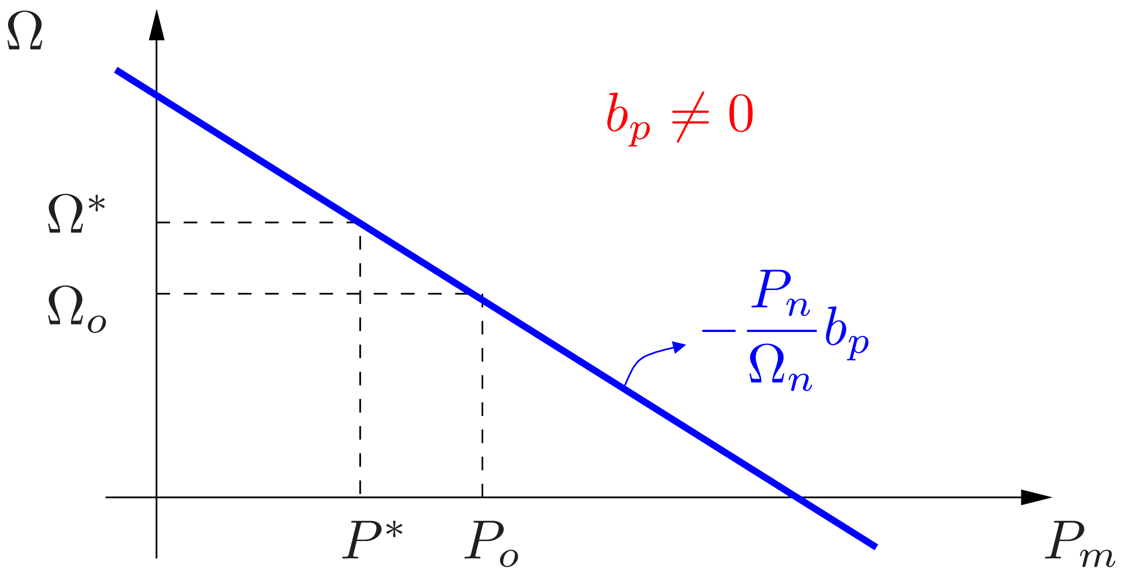
$$G_f(s) = G_r(s)G_v(s)G_a(s) = K_I T_1 \frac{1 + sT_2}{1 + sT_1}$$

- In steady-state:

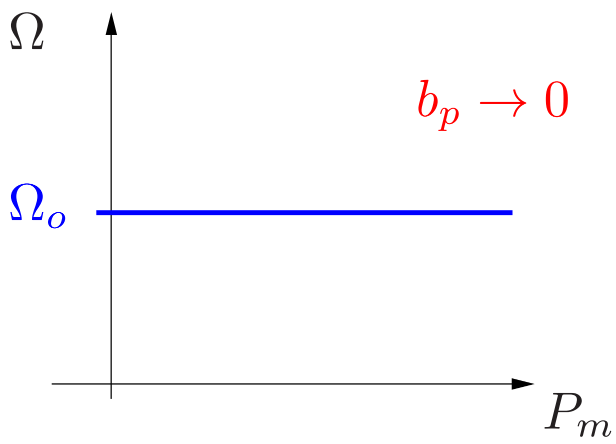
$$\Delta P_m = G_f(0)(\Delta\Omega^{\mathrm{ref}} - \Delta\Omega)$$

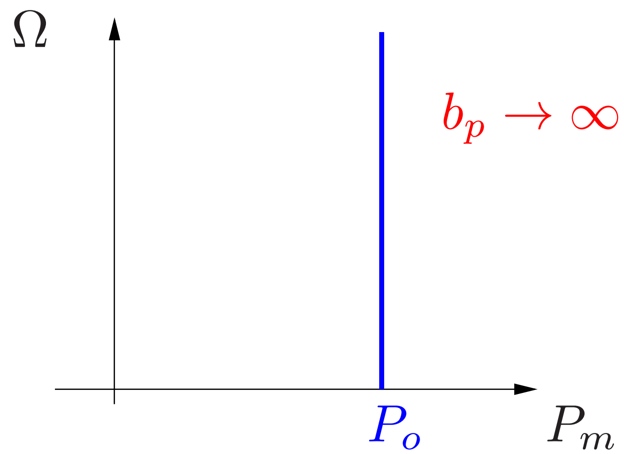
$$\Delta P_m = \frac{P_n}{\Omega_n}\frac{1}{b_p}(\Delta\Omega^{\mathrm{ref}} - \Delta\Omega)$$

Steady-State Characteristic of f/P Control



special cases:

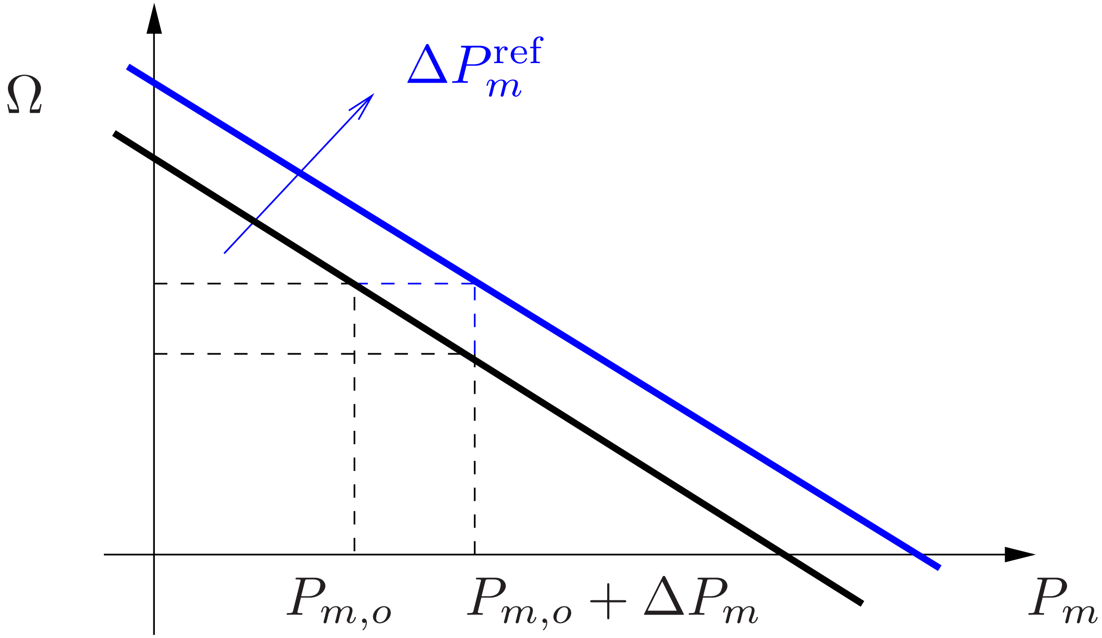




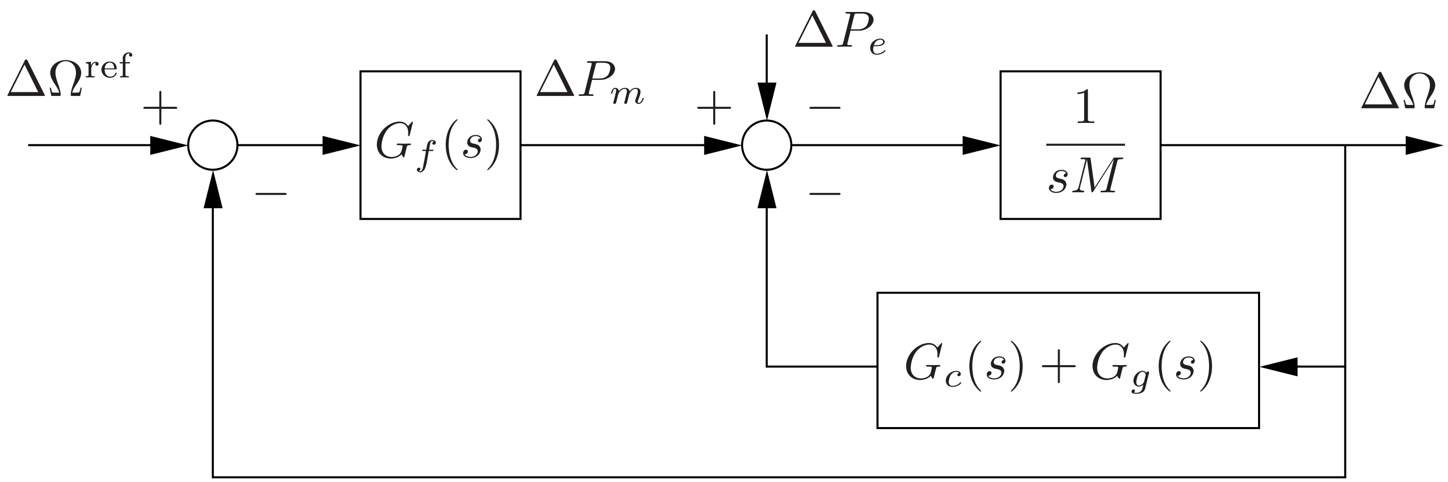
Change of the Operating Point

- Let's assume we set a different set point for the mechanical power:

$$\Delta P_m = \frac{P_n}{\Omega_n}\frac{1}{b_p}(\Delta\Omega^{\text{ref}} - \Delta\Omega) + \left[\Delta P_m^{\text{ref}}\right] \leftarrow \quad \text{change of O.P.}$$



Effect of $G_c(s)$ and $G_g(s)$



- Let's check if the effect of $G_c(s)$ and $G_g(s)$ can be neglected.

- Without $G_c(s)$ and $G_g(s)$, we have:

$$\frac{\Delta\Omega}{\Delta P_e} = \frac{-\frac{1}{sM}}{1 + \frac{1}{sM} G_f(s)} = \frac{-1}{G_f(s) + sM}$$

Effect of $G_c(s)$ and $G_g(s)$

- With $G_c(s)$ and $G_g(s)$, we have:

$$\frac{\Delta\Omega}{\Delta P_e} = \frac{-1}{G_f(s) + G_c(s) + G_g(s) + sM}$$

- In steady-state ($s \to 0$), we have:

$$\frac{-1}{G_f(0) + G_c(0) + G_g(0)}$$

- Sometimes, $G_f(0)$ is defined as regulating energy, then $G_f(0) + G_c(0) + G_g(0)$ is the total regulating energy.

- In general $G_c(0)$ and $G_g(0) \ll G_f(0)$

Effect of $G_c(s)$ and $G_g(s)$

- Moreover, observe that:

$$\frac{1}{G_f(0)} > \frac{1}{G_f(0) + G_c(0) + G_g(0)}$$

  so, the static error is a bit smaller if we consider G_c and G_g.

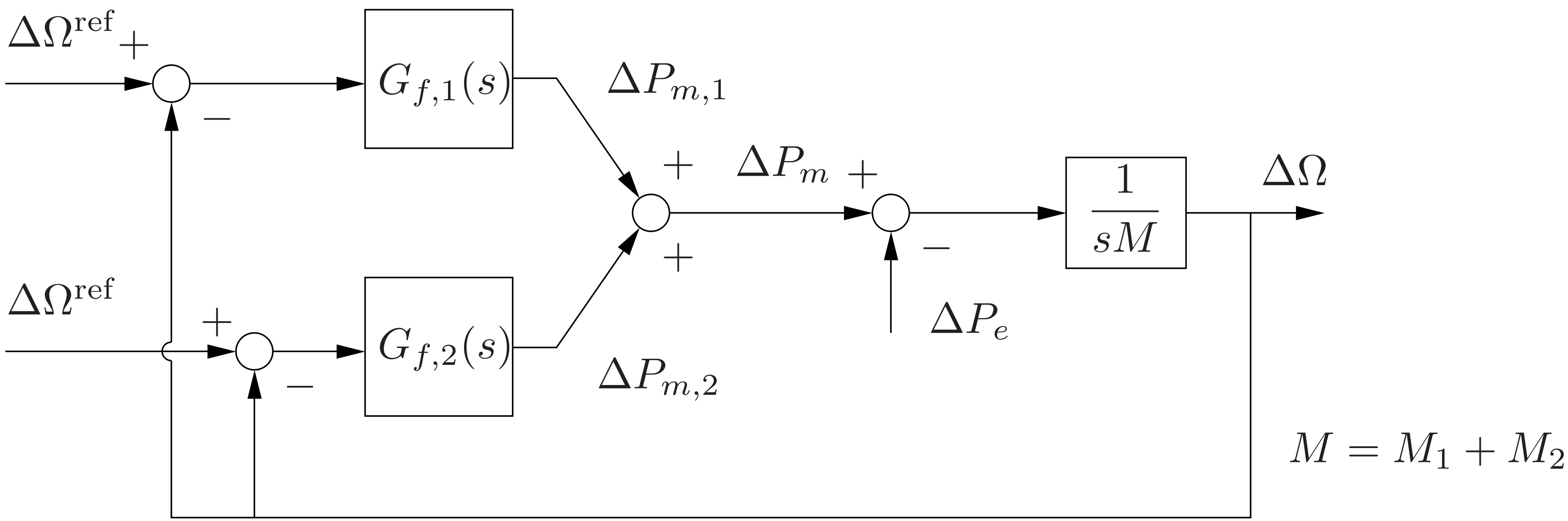
- From the dynamic point of view, $G_g(s)$ is actually very negligible.
- Less intuitive is to neglect $G_c(0)$ $\rightarrow$ it takes into account load inertia.
- We can take it into account by defining:

$$M^* = (1.15 \div 1.20)\ M$$

- *Load Frequency Control* (LFC) is expected to play a major role in the near future.

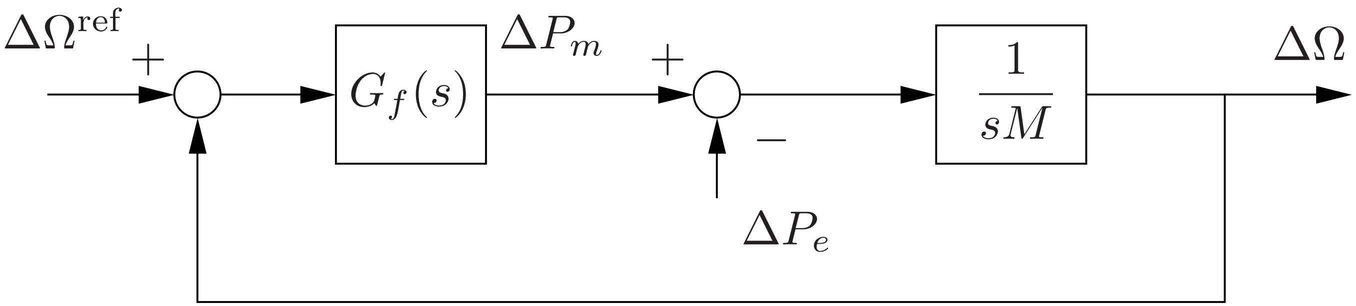
Parallel of Synchronous Machines

- Let's assume that the system frequency is unique (no electromechanical transients).



→ Let's define a unique $G_f(s)$

Parallel of Synchronous Machines



- Where we define:

$$G_f(s) = G_{f,1}(s) + G_{f,2}(s)$$

- If we have n machines:

$$M = \sum_{i=1}^{n} M_i$$

$$\Delta P_m = \sum_{i=1}^{n} \Delta P_{m,i}$$

$$G_f(s) = \sum_{i=1}^{n} G_{f,i}(s)$$

Parallel of Synchronous Machines

$\Rightarrow$ Let's compute b_p, T_1, T_2 for the resulting $G_f(s)$

- We know $b_{p,i}, T_{1,i}, T_{2,i}$ of the n machines:

$$G_f(s) = \sum_{i=1}^{n} \frac{P_{n,i}}{\Omega_{n,i}} \cdot \frac{1}{b_{p,i}} \cdot \frac{1 + sT_{2,i}}{1 + sT_{1,i}}$$

- Since $T_2 = 3 \div 5$ s and $T_1 = 10 \div 15$ s, let assume that $T_{2,i}$ and $T_{1,i}$ are equal to mean values, say T_{2m} and T_{1m}.
- Then Ω_n is the same for all machines.
- Hence:

$$\frac{P_n}{b_p} = \sum_{i=1}^{n} \frac{P_{n,i}}{b_{p,i}} \quad \Rightarrow \quad b_p = \frac{P_n}{\sum_{i=1}^{n} \frac{P_{n,i}}{b_{p,i}}} \quad \text{where} \quad P_n = \sum_{i=1}^{n} P_{n,i}$$

Parallel of Synchronous Machines

- So the resulting total transfer function $G_f(s)$ is:

$$G_f(s) = \frac{P_n}{\Omega_n} \cdot \frac{1}{b_p} \cdot \frac{1 + sT_{2m}}{1 + sT_{1m}}$$

- In the same way, we can define b_t for the interconnected system.
- Observe that connecting several machines in parallel is “good” for the system as:

M increases $\rightarrow$ $\Delta\Omega$ decreases

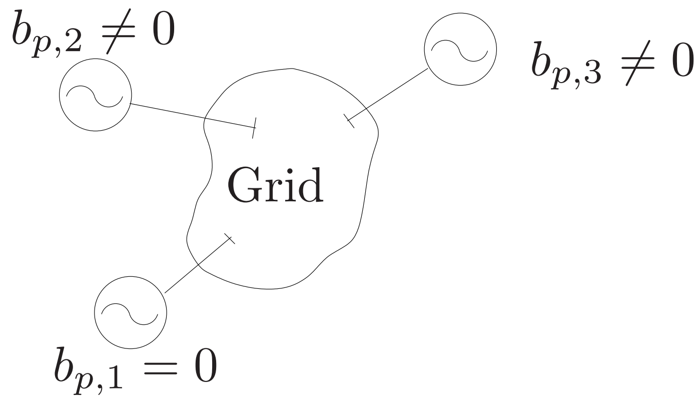
Parallel of Synchronous Machines

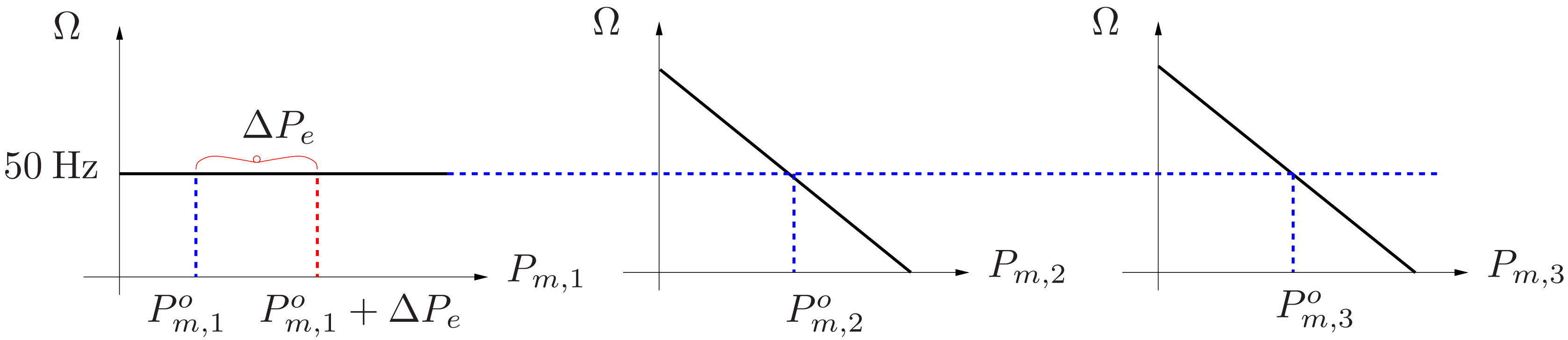
- Let's see the effect of the $b_{p,i}$ on the overall system.
- $b_{p,i}$ can be zero or not zero:

$$G_{f,i}(s) \begin{cases} K_I \dfrac{1 + sT_{2m}}{s} & \Rightarrow \quad b_{p,i} = 0 \\ \\ \dfrac{P_{n,i}}{\Omega_n} \cdot \dfrac{1}{b_{p,i}} \cdot \dfrac{1 + sT_{2m}}{1 + sT_{1m}} & \Rightarrow \quad b_{p,i} \neq 0 \end{cases}$$

Example of the Parallel of 3 Machines

- Let's consider the following example:





Example of the Parallel of 3 Machines

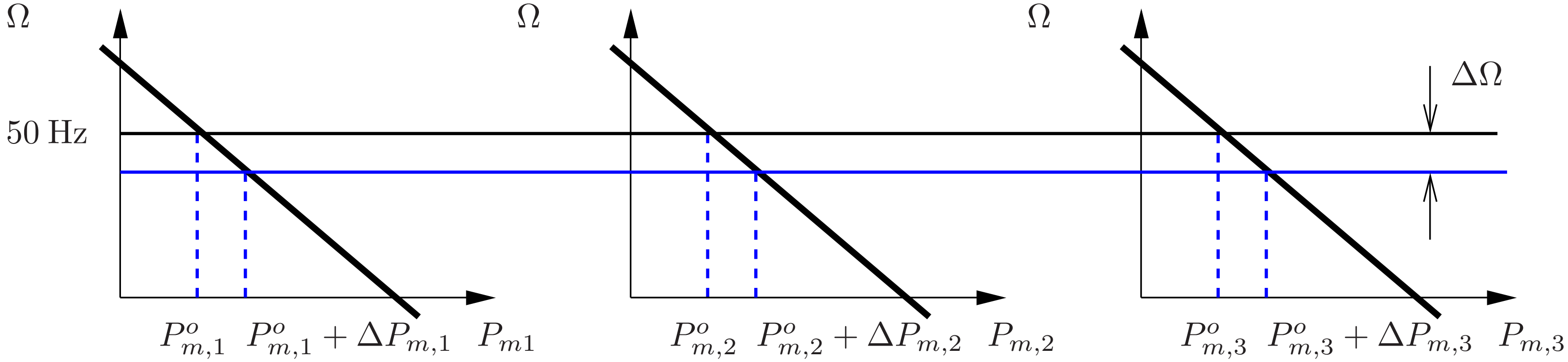
- To balance the power, we have

$$P_e^o = P_{m,1}^o + P_{m2}^o + P_{m,3}^o$$

- Since $b_{p,1} = 0$, any variation ΔP_e is provided by machine 1.
- This explains why no machine in an interconnected system can have $b_p = 0$.
- Actually, in interconnected systems, the $b_{p,i}$ are more or less all the same so that all machines contribute to ΔP_e proportionally to their power rate.
- ... but if all $b_{p,i} \neq 0$, then we have that $\Delta\Omega(t)\big|_{t\to\infty} \neq 0$.

Example of the Parallel of 3 Machines

- Let's now assume $b_{p,1} \neq 0$, $b_{p,2} \neq 0$, $b_{p,3} \neq 0$.



$$\Delta P_e = \Delta P_{m,1} + \Delta P_{m,2} + \Delta P_{m,3}$$

$$\Downarrow$$

$$P_e = P_{m,1} + P_{m,2} + P_{m,3}$$

but: $\Delta\Omega \neq 0$

Example of the Parallel of 3 Machines

- If $\Delta\Omega^{\mathrm{ref}} = 0$, then: $$\Delta P_{m,i} = -G_{f,i}(s)\Delta\Omega$$
- In steady-state: $$\Delta P_{m,i} = -\frac{P_{n,i}}{\Omega_n}\frac{1}{b_{p,i}}\Delta\Omega$$
- Moreover, since $\dfrac{1}{sM}$ is an integrator:

$$\sum_{i=1}^{n}\Delta P_{m,i} = \Delta P_e = -\frac{P_n}{\Omega_n}\frac{1}{b_p}\Delta\Omega$$

$\rightarrow$ $\boxed{\Delta\Omega = -b_p\dfrac{P_n}{\Omega_n}\Delta P_e}$ = static frequency error

Moreover: $$\Delta P_{m,i} = \frac{P_{n,i}}{P_n}\cdot\frac{b_p}{b_{p,i}}\cdot\Delta P_e \quad\Rightarrow\quad \begin{aligned}&\Delta P_{m,i} \propto P_{n,i}\\ &\Delta P_{m,i} \propto \frac{1}{b_{p,i}}\end{aligned}$$

Steady-State Error of the Primary Frequency Control

- Assume:

$$G_f(s) = \frac{P_n}{\Omega_n} \frac{1}{b_p} \frac{1 + sT_{2,m}}{1 + sT_{1,m}} = \frac{P_n}{\Omega_n} \frac{1 + sT_{2,m}}{b_p + sT_{2,m} b_t}$$

where we used the identity: $\dfrac{b_p}{b_t} = \dfrac{T_{2,m}}{T_{1,m}}$

- Then, in steady-state and assuming $\Delta\Omega^{\text{ref}} = 0$:

$$\Rightarrow \frac{\Delta\Omega/\Omega_n}{\Delta P_L/P_n}(0) = b_p$$

- Note that the total droop b_p is a measure of the steady-state frequency error.
  - $b_p \in [0, 5]\%$
  - $b_t \in [25, 40]\%$

Synthesis of the Primary Frequency control

- We have $G_f = G_r G_v G_a \qquad \Rightarrow \qquad G_r = \dfrac{G_f}{G_v G_a}$
- G_v can be assumed to be a constant (we neglect delays)
- G_a depends on the type of turbine:

$$\begin{cases} 1. \text{ thermoelectric} \\ 2. \text{ hydroelectric} \\ 3. \text{ gas} \end{cases}$$

- Once we know G_v and G_a, we impose the resulting $G_f(s)$ and determine the transfer function of $G_r(s)$

Model of $G_v(s)$

- The valve transfer function can be assumed as follows:

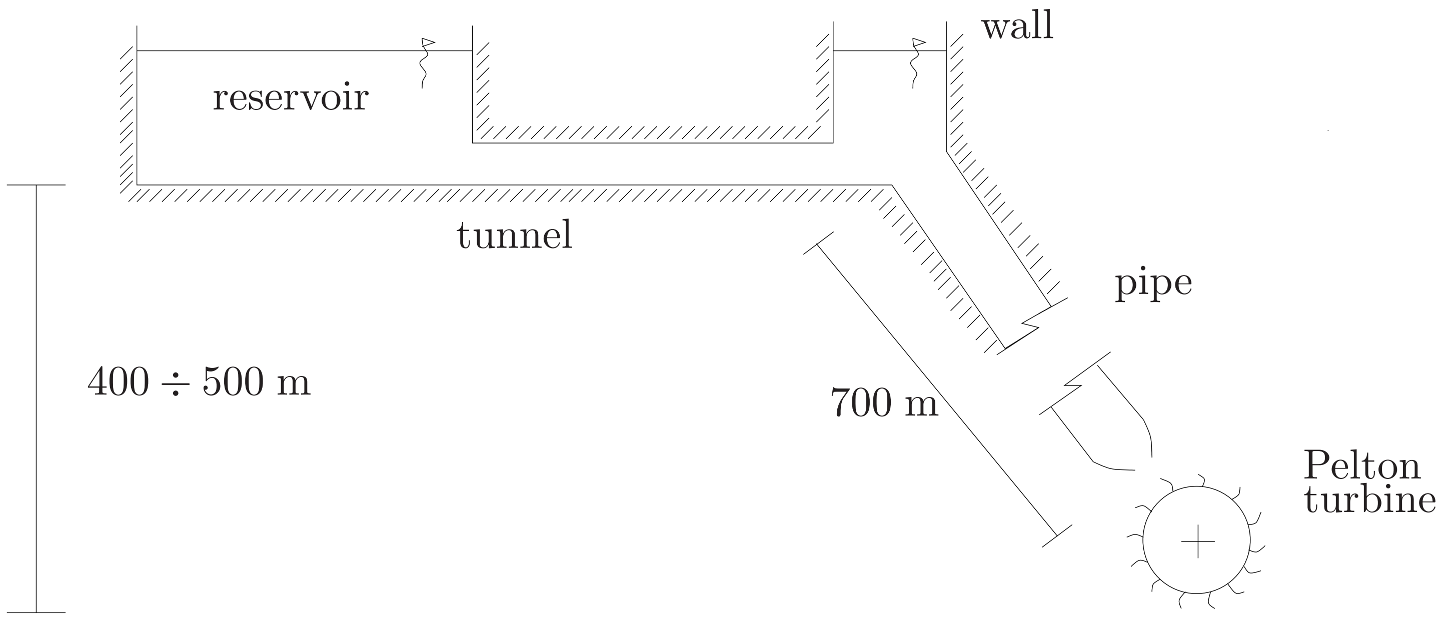
$$G_v(s) = \frac{\Delta A}{\Delta \beta} \approx \frac{K_v}{1 + sT_v} \approx K_v$$

where $T_v \in [0.3, 0.5]$ s.

- The time constant T_v is “small” with respect to the timer frame of the f/P regulator.
- Hence we will assume that

$$G_v(s) \approx K_v = \text{constant}$$

Model of $G_a(s)$ for Hydro Plants - I



$$P_m = \eta \rho H \Phi$$

η = efficiency

ρ = specific weight (of the water)

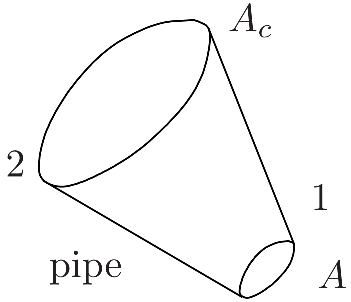
H = height (m)

$\Phi = \mathrm{m}^3/\mathrm{s}$ volumetric flow

Model of $G_a(s)$ for Hydro Plants - II

- If $\Delta\eta = \Delta\rho = 0 \quad \Rightarrow \quad \dfrac{\Delta P_m}{P_m^o} = \dfrac{\Delta H}{H^o} + \dfrac{\Delta \Phi}{\Phi^o}$

→ Injector model:



$$H_1 = 0 \quad , \quad H_2 = H$$

$$v = \frac{\Phi}{A} \left[\frac{\text{m}}{\text{s}}\right]$$

$$\cancel{m} g H_1 + \frac{1}{2} \cancel{m} v_1^2 = \cancel{m} g H_2 + \frac{1}{2} \cancel{m} v_2^2$$

$$\Rightarrow \qquad \frac{\Phi_1^2}{A^2} = \frac{\Phi_2^2}{A_c^2} + 2gH$$

since $\quad A_c \gg A \quad \Rightarrow \quad H = \dfrac{\left(\frac{\Phi}{A}\right)^2}{2g} \quad \Rightarrow \quad \Phi = A\sqrt{2gH}$

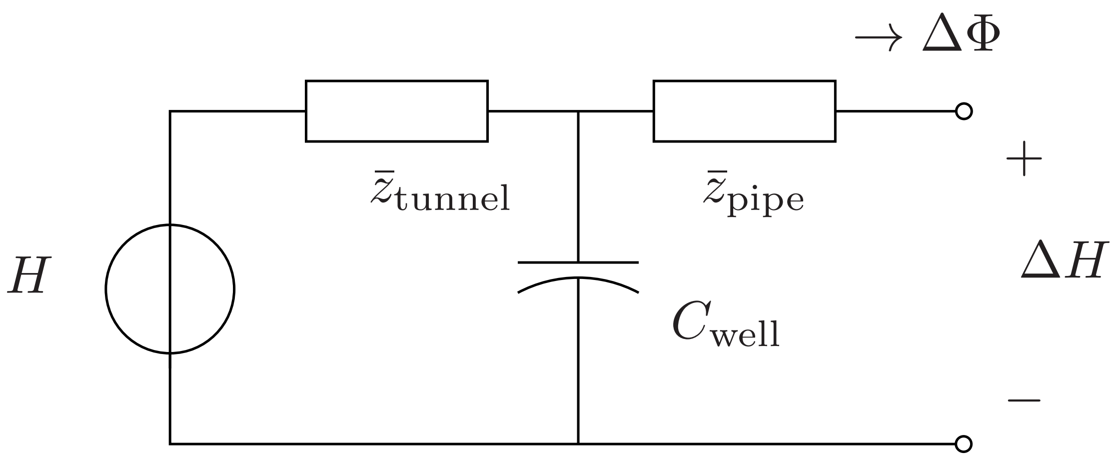
Model of $G_a(s)$ for Hydro Plants - III

- Hence we have:

$$\frac{\Delta\Phi}{\Phi^o} = \frac{\Delta A}{A^o} + \frac{1}{2}\frac{\Delta H}{H^o}$$

- We still need the relation between $\Delta\Phi$ and ΔH. It depends on the dynamics of the water in the tunnel, wall and pipe.

- Analogy with electrical systems:

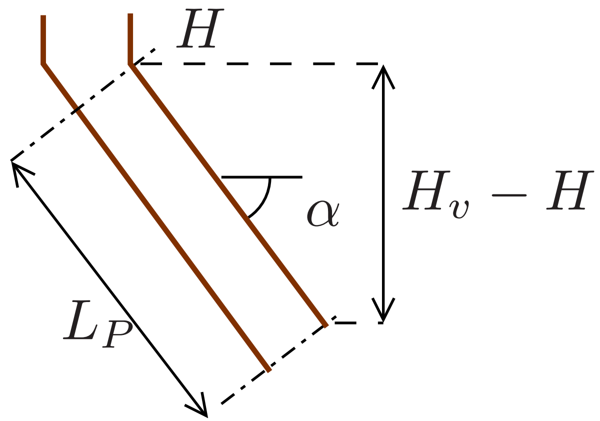


$\Phi \Rightarrow I$ (current)

$H \Rightarrow V$ (voltage)

Model of $G_a(s)$ for Hydro Plants - IV

- We neglect $\bar{z}_{\text{tunnel}}$ and C_{well}. Let's define $\bar{z}_{\text{pipe}}(s)$.



$$(*) \quad H_v - H = L_P \sin\alpha + \frac{\Pi_v - \Pi}{\rho}$$

water weight: $\rho A_P L_P \sin\alpha$

pressure: Π, Π_v

$\Rightarrow$ force balance: $$\rho A_P L_P \sin\alpha + (\Pi_v - \Pi)A_c = \frac{\rho A_P L_P}{g} \frac{\mathrm{d}\left(\frac{\Phi}{A_c}\right)}{\mathrm{d}t} \quad (**)$$

$\Rightarrow$ Let's divide by ρA_c and use $(*)$, hence:

$$\frac{L_P}{g} \frac{1}{A_c} \frac{\mathrm{d}\Phi}{\mathrm{d}t} = H_v - H$$

Model of $G_a(s)$ for Hydro-Plants - V

$\Rightarrow$ We define $J_P \triangleq \dfrac{L_P}{S_{AP}}$ = pipe **inertance**

$\Rightarrow$ So: $$-\Delta H = J_P \frac{d\Delta\Phi}{dt} \qquad \Rightarrow z_{\mathrm{pipe}}(s) = sJ_P$$

The pipe behaves similarly to a coil (inertance $\Rightarrow$inductance)

Inertance: *measure of the pressure gradient in a fluid to cause a change in flow rate with time* $[\mathrm{Pa\,m^{-3}\;s^2}]$

Model of $G_a(s)$ for Hydro-Plants - VI

- In summary we have:

$$\frac{\Delta P_m}{P_m^o} = \frac{\Delta A}{A^o} + \frac{\Delta H}{H^o}$$

$$\Delta H = -sJ_P\Delta\Phi$$

$$\frac{\Delta\Phi}{\Phi^o} = \frac{\Delta A}{A^o} + \frac{1}{2}\frac{\Delta H}{H^o}$$

$\Rightarrow$ Let's define $T_w = \dfrac{J_P\Phi^o}{H^o}$ [s] $\Rightarrow$

$$\begin{cases} \dfrac{\Delta H}{H^o} = \dfrac{-sT_w}{1+s\frac{Tw}{2}}\dfrac{\Delta A}{A^o} \\ \dfrac{\Delta\Phi}{\Phi^o} = \dfrac{1}{1+s\frac{Tw}{2}}\dfrac{\Delta A}{A^o} \end{cases}$$

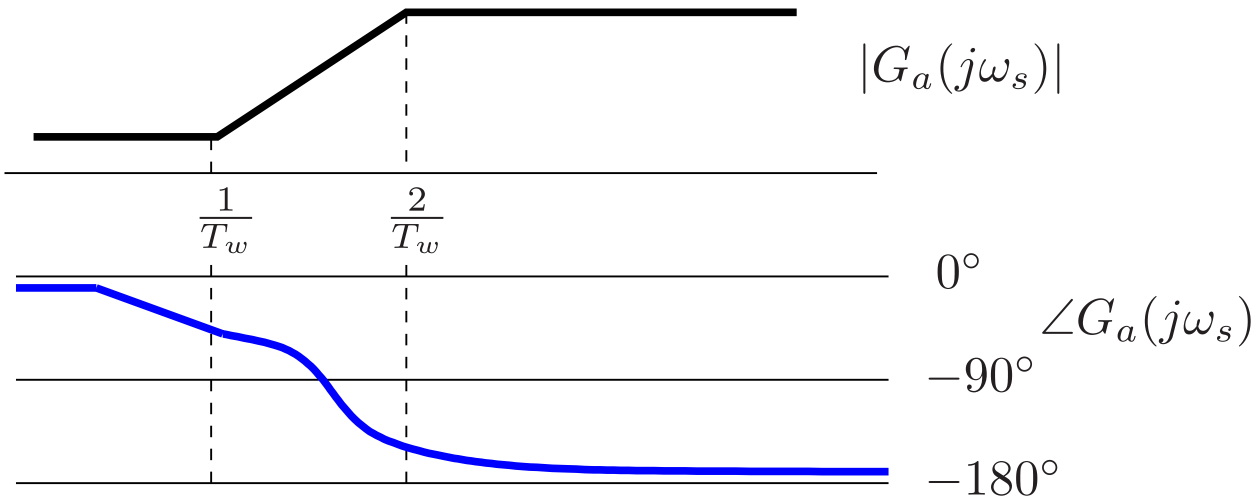
$$\Rightarrow G_a(s) = \frac{\Delta P_m}{\Delta A}$$

Model of $G_a(s)$ for Hydro Plants - VII

$$\Rightarrow \frac{\Delta P_m}{P_m^o} = \left(\frac{1}{1 + sT_{w/2}} - \frac{sT_w}{1 + sT_{w/2}} \right) \frac{\Delta A}{A^o}$$

$$\Rightarrow \frac{\Delta P_m}{\Delta A} = \frac{P_m^o}{A^o} \frac{1 + sT_w}{1 + sT_w/2} \equiv G_a(s)$$

$\Rightarrow$ Bode's Diagram:



Model of G_a(s) for Hydro-Plants - VIII

- Observe that, actually, the model of G_a(s) is much more complex:

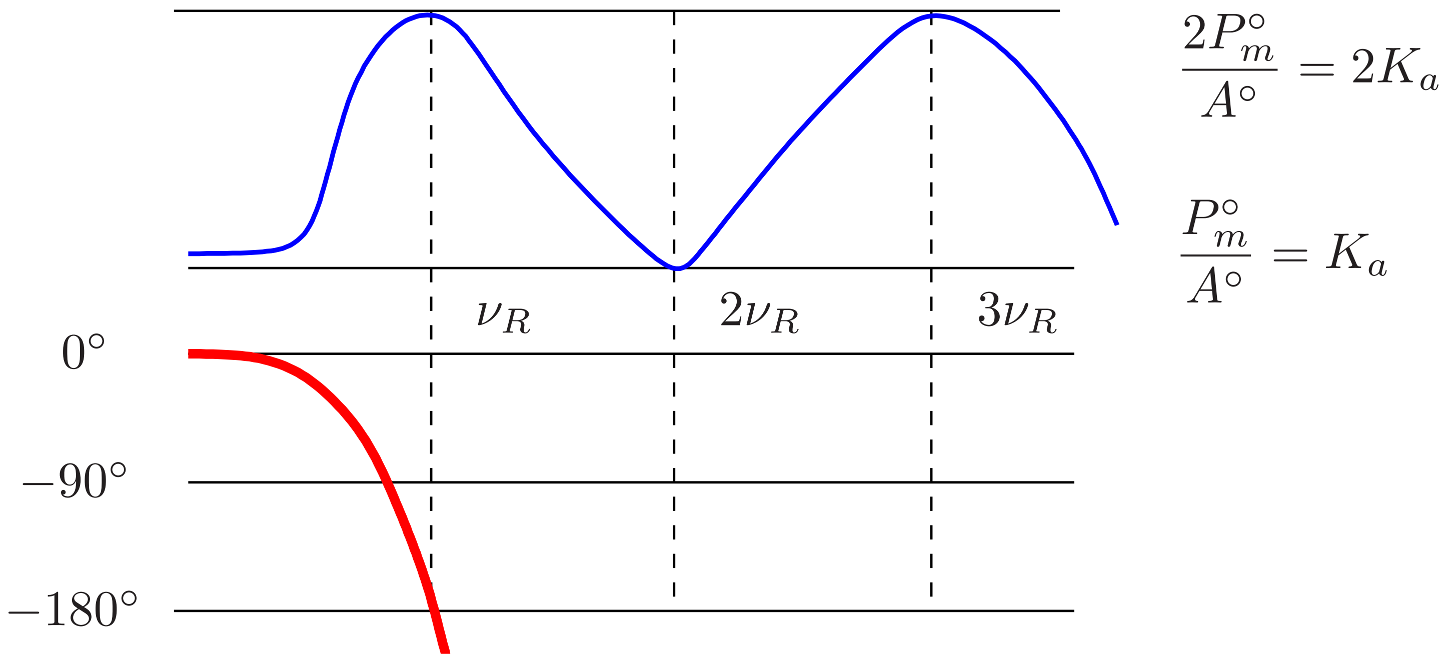
$$G_a(s) = \frac{P_m^o}{A^o} \frac{1 - \dfrac{a_P}{gA_P}\dfrac{\Phi^o}{H^o}\tanh\dfrac{sL_P}{a_P}}{1 + \dfrac{a_P}{gA_P}\dfrac{\Phi^o}{H^o}\dfrac{1}{2}\tanh\dfrac{sL_P}{a_P}}$$

where a_P = wave proportion speed within the pipe $\approx 1000\ \mathrm{m/s}$

$\Rightarrow$ the resonance frequency is: $\nu_r = \dfrac{\pi}{2\frac{L_P}{a_P}} \approx 3\ \mathrm{Hz}$

Model of $G_a(s)$ for Hydro Plants - IX

- Bode diagram of the *precise* $G_a(s)$



Synthesis of $G_r(s)$ for Hydro Plants - X

- Despite the complexity of the model of G_a, for the synthesis of G_r(s) we assume:
  $G_a(s) \approx K_a$ (and $G_v(s) \approx K_v$)

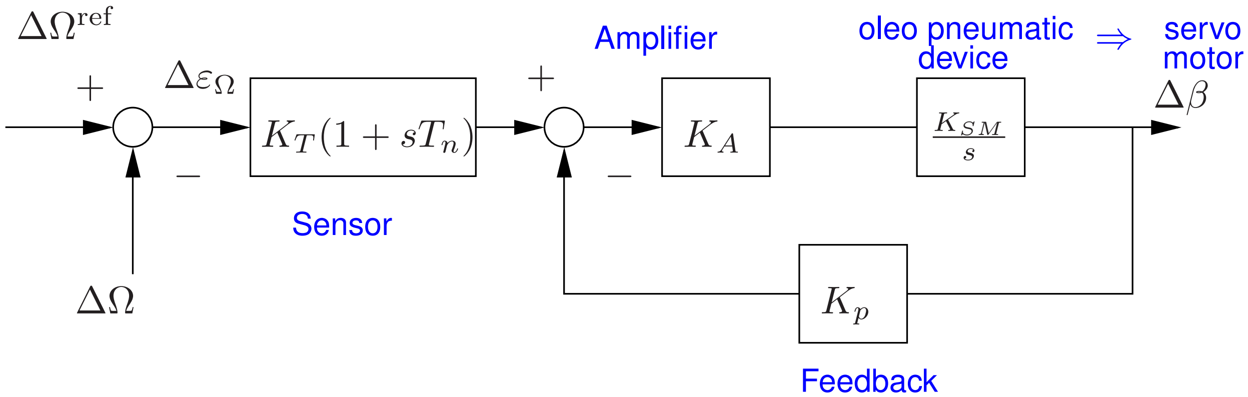
- Hence we have:

$$G_r(s) = \underbrace{\frac{P_n}{\Omega_n} \cdot \frac{1}{b_p} \frac{1 + sT_2}{1 + sT_1}}_{G_f(s)} \cdot \frac{1}{G_v(0) G_a(0)}$$

- There are two kinds of regulators:

  1. Accelerometer
  2. Transient Feedback

Accelerometer-based Regulator



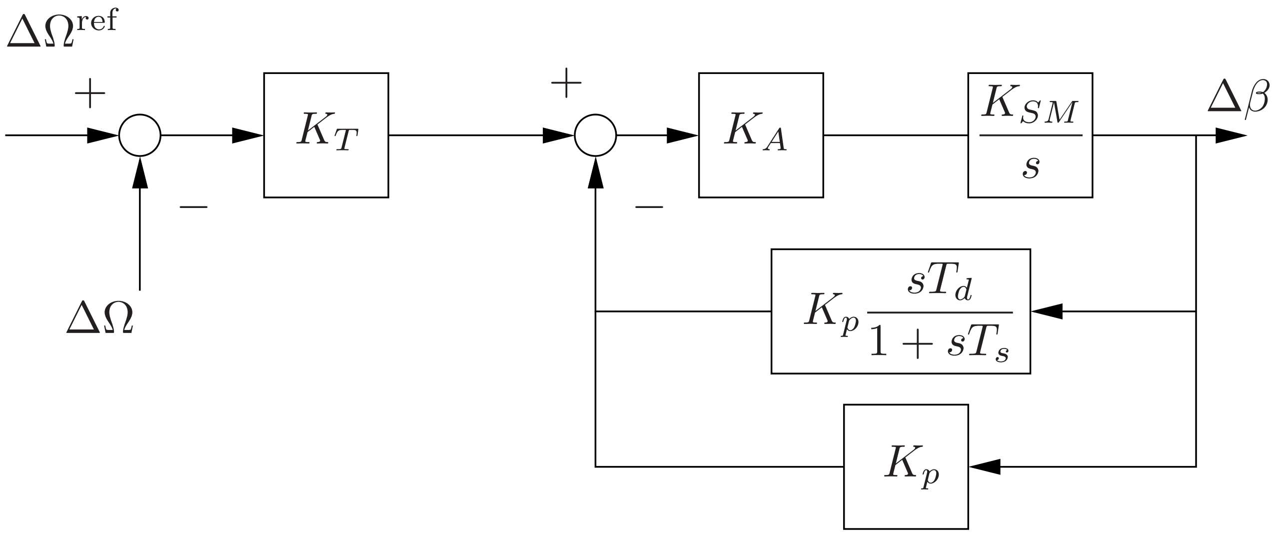
$$\Rightarrow \quad \frac{\Delta\beta}{\Delta\varepsilon_\Omega} = K_T(1+sT_n)\frac{K_A\frac{K_{SM}}{s}}{1+K_A\frac{K_{SM}}{s}K_p} = \frac{K_T}{K_p}\frac{1+sT_n}{\frac{s}{K_AK_pK_{SM}}+1} = \frac{K_T}{K_p}\frac{1+sT_n}{1+sT_{\text{Reg}}}$$

where

$$T_{\text{Reg}} = \frac{1}{K_AK_pK_{SM}} \quad \epsilon \quad [10 \div 15] \quad \text{s} \quad \Rightarrow \quad T_1$$

$$T_n \quad \in \quad [3,5] \quad \text{s} \quad \Rightarrow \quad T_2, \quad \frac{K_T}{K_p} = \frac{P_n}{\Omega_n}\frac{1}{b_p}\frac{1}{G_v(s)G_a(s)}$$

Transient Feedback Regulator



we have:

$$G_r(s) = \frac{K_T}{K_p} \frac{1+sT_d}{(1+sT'_{\text{Reg}})(1+sT_s)}$$

where

$$T_{\text{Reg}} = \frac{T_d(K_p+K_T)}{K_p}$$

T_s is very small $\sim$ 0.1 seconds.

$$T_2 \triangleq T_d$$

$$T_1 \triangleq T'_{\text{Reg}}$$

$$\frac{K_T}{K_p} \triangleq \frac{P_n}{\Omega_n} \frac{1}{b_p} \frac{1}{G_r(0)G_a(0)}$$

Behavior of the regulators of a Hydro Plant

- In steady state, t $\to \infty$, $s \to 0$, both regulators behave in the same way.

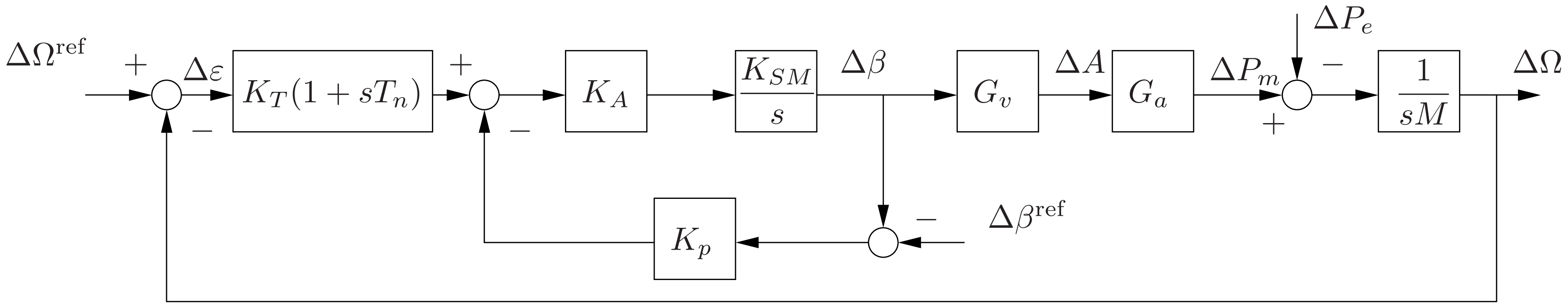
To obtain

$$b_p = 0 \quad \Rightarrow \quad K_P = 0$$
$$b_p \to \infty \quad \Rightarrow \quad K_T = 0$$

- During a transient, since T_s is "small", the behaviour of the two regulators is very similar.
- In practise, the two regulators are similar and both are rather slow.

Effect of Varying the Reference of β - I

- Let's assume we add an additional signal to vary the reference angle β.



- In steady state, we have:

$$\Delta\beta = \Delta\beta^{\text{ref}} + \frac{K_T}{K_p}\Delta\varepsilon = \Delta\beta^{\text{ref}} + G_R(0)\Delta\varepsilon$$

- Moreover:

$$\frac{\Delta\beta}{\Delta\beta^{\text{ref}}} = \frac{K_A\frac{K_{sM}}{s}K_p}{1+\frac{K_A K_{SM} K_p}{s}} = \frac{K_A K_{SM} K_p}{s + K_A K_{SM} K_p} \Rightarrow \left.\frac{\Delta\beta}{\Delta\beta^{\text{ref}}}\right|_{s=0} = 1$$

Effect of varying the reference of β - II

$$\Rightarrow \qquad \Delta\beta = \Delta\beta^{\text{ref}} + \frac{K_T}{K_P}(\Delta\Omega^{\text{ref}} - \Delta\Omega)$$

$\Delta P_m = G_a(0)G_v(0)\Delta\beta \Rightarrow$ in steady state:

$$\Delta P_m = G_a(0)G_v(0)\Delta\beta_{\text{ref}} + G_a(0)G_v(0)\frac{K_T}{K_P}(\Delta\Omega^{\text{ref}} - \Delta\Omega)$$

$\Rightarrow$ so we can vary the reference of the mechanical power by imposing:

$$\Delta P_m^{\text{ref}} = G_a(0)G_v(0)\Delta\beta^{\text{ref}}$$

Effect of varying the reference of β - III

- During a transient, we have:

$$\Delta\beta = \Delta\beta^{\text{ref}} + G_R(0)\Delta\varepsilon_\Omega$$

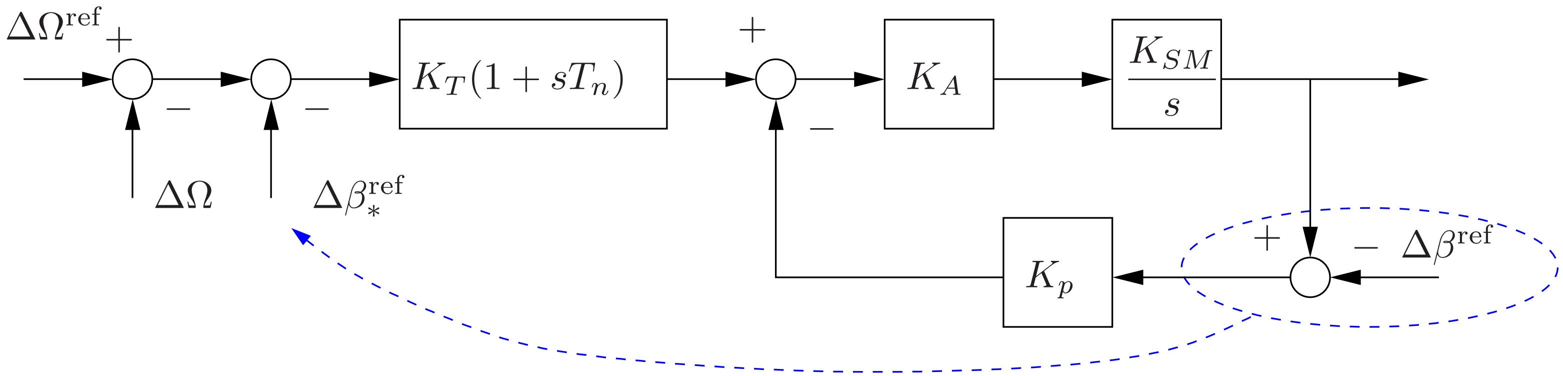
accelerometer: $\rightarrow \dfrac{\Delta\beta}{\Delta\beta^{\text{ref}}} \approx \dfrac{1}{1+sT_1}$ where $T_1 \in [10, 15]\text{s}$

transient feedback: $\rightarrow \dfrac{\Delta\beta}{\Delta\beta^{\text{ref}}} = \dfrac{K_P}{K_T} G_R(s) \approx \dfrac{1}{(1+sT_1)(1+sT_s)}, \quad T_1 > T_s$

$\rightarrow$ During the transient, the transient feedback regulation is faster, as its response depends on T_s.

Effect of varying the reference of β - IV

- If in the accelerometer-based regulator we have the signal $\Delta\beta^{\mathrm{ref}}$:

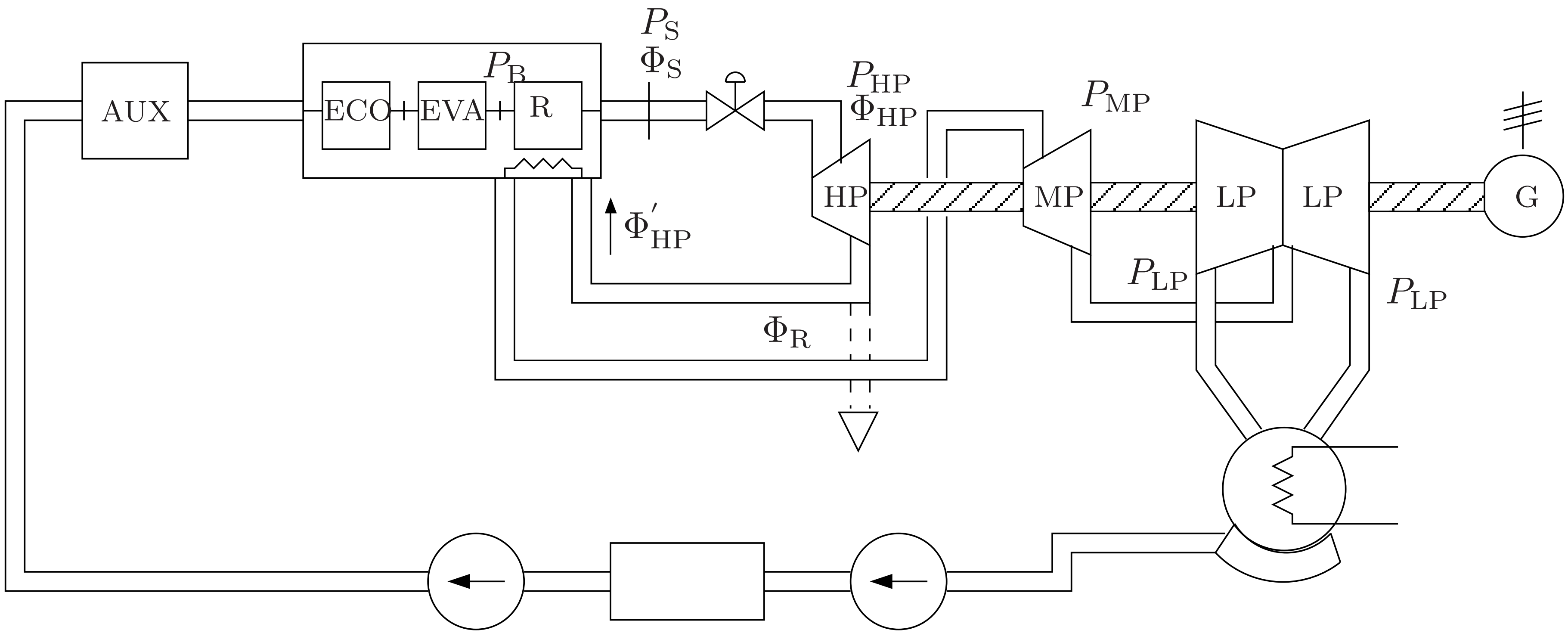


$$\Delta\beta_*^{\mathrm{ref}} = \Delta\beta^{\mathrm{ref}} \frac{K_p}{K_T(1+sT_n)}$$

hence: $$\frac{\Delta\beta}{\Delta\beta_*^{\mathrm{ref}}} = G_R(s)\frac{K_p}{K_T(1+sT_n)} = \frac{K_T}{K_p}\frac{1+sT_2}{1+sT_1}\frac{K_p}{K_T}\frac{1}{1+sT_n} = \frac{1}{1+sT_1}$$

for the transient feedback regulator we have: $$\frac{\Delta\beta}{\Delta\beta_*^{\mathrm{ref}}} \cong \frac{1}{1+sT_s}$$

Model of $G_a(s)$ for a Thermo Plant - I

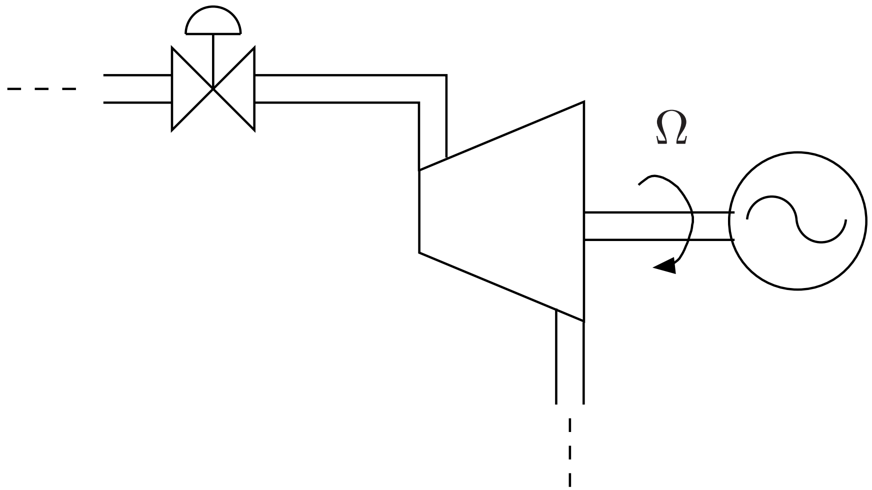


P: power, Φ: volumetric flow, R: reheater, S: steam, B: boiler

HP: high pressure, MP: medium pressure, LP: low pressure.

Model of $G_a(s)$ for a Thermo Plants - II

- If there was only the turbine:



$$\Delta P_{\mathrm{HP}} = \left(\frac{P_{\mathrm{HP}}}{\Phi_{\mathrm{S}}}\right)_{\mathrm{nom}} \Delta\Phi_{\mathrm{S}}$$

and

$$\Delta\Phi_{\mathrm{S}} = h'_A \Delta A$$

- The re-heater (R) introduces a delay in the fluid passing from HP and MP.

Model of $G_a(s)$ for thermo plants - III

- For the complete model, we have:

$$G_a(s) = \frac{\Delta P_m}{\Delta A} = \frac{\Delta P_m}{\Delta \Phi_\mathrm{S}} \frac{\Delta \Phi_\mathrm{S}}{\Delta A}$$

with:

$$\frac{\Delta P_m}{\Delta \Phi_\mathrm{S}} = \frac{\Delta P_\mathrm{HP} + \Delta P_\mathrm{MP} + \Delta P_\mathrm{LP}}{\Delta Q_\mathrm{S}}$$

assume:

$$\alpha = \frac{P_\mathrm{HP}}{P_m} \cong 0.3$$

and assume:

$$\Delta P_\mathrm{MP} + \Delta P_\mathrm{LP} = \left(\frac{P_\mathrm{MP} + P_\mathrm{LP}}{\Phi_\mathrm{S}} \right)_\mathrm{nom} \frac{1}{1 + s T_\mathrm{R}} \Delta \Phi_\mathrm{S}$$

with:

$$T_\mathrm{R} \in [10,\ 15]\ \mathrm{s}.$$

Model of $G_a(s)$ for Thermo plants - IV

- So, we obtain:

$$\frac{\Delta P_m}{\Delta \Phi_{\mathrm{S}}} = \left(\frac{P_m}{\Phi_{\mathrm{S}}}\right)_{\mathrm{nom}} \left(\alpha + \frac{1-\alpha}{1+sT_{\mathrm{R}}}\right)$$

- and, finally:

$$\frac{\Delta P_m}{\Delta \Phi_{\mathrm{S}}} = \frac{P_n}{\Phi_{\mathrm{S}n}} \frac{1+\alpha s T_{\mathrm{R}}}{1+sT_{\mathrm{R}}}$$

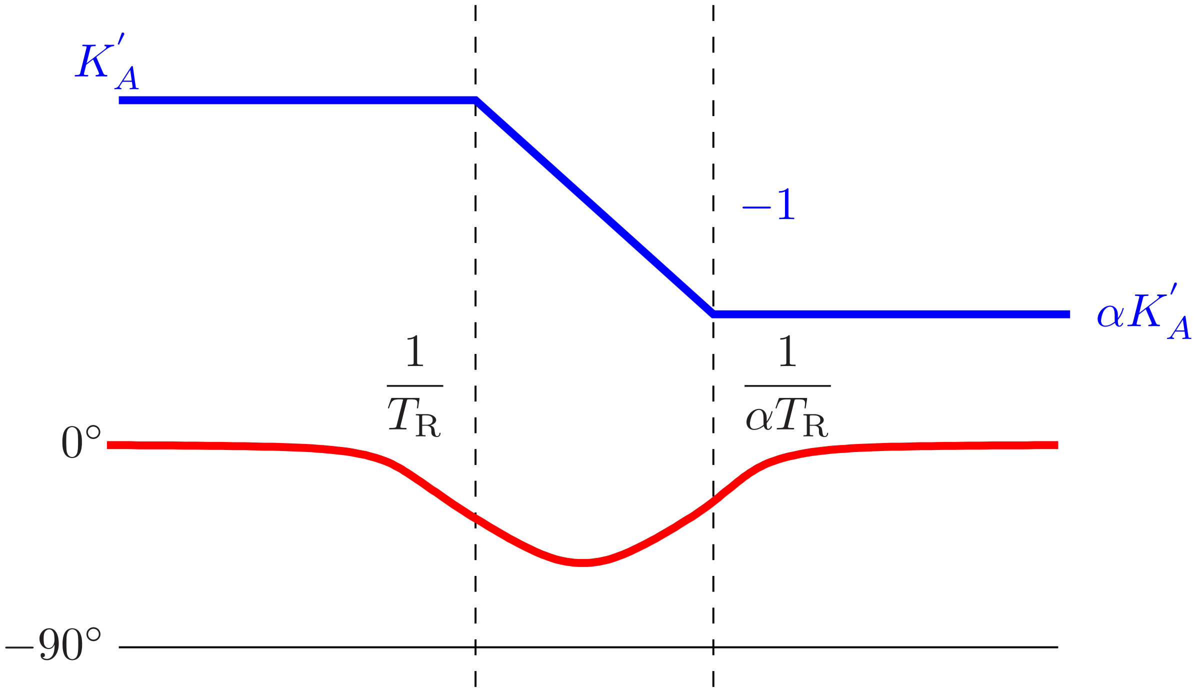
- So:

$$G_a(s) = \frac{\Delta P_m}{\Delta \Phi_{\mathrm{S}}} \cdot \frac{\Delta \Phi_{\mathrm{S}}}{\Delta A} = \frac{P_n}{\Phi_{\mathrm{S}n}} \frac{1+\alpha s T_{\mathrm{R}}}{1+sT_{\mathrm{R}}} h'_A = K'_A \frac{1+s\alpha T_{\mathrm{R}}}{1+sT_{\mathrm{R}}}$$

observe that $\alpha T_{\mathrm{R}} < T_{\mathrm{R}}$, so the situation is better than what we had for the hydro plant.

Model of $G_a(s)$ for a Thermo Plant - V

- If $T_\mathrm{R} = 15$ sec, $\alpha T_\mathrm{R} \cong 5$ sec, so we do not face too many control issues.



Model of $G_a(s)$ for Thermo plants - VI

- Actually assuming $\Delta P_{\rm HP} = \left(\frac{P_{\rm HP}}{\Phi_{\rm S}}\right)_{\rm nom} \Delta\Phi_{\rm S}$ is too simplistic.
- There are delays in the turbine:

$$\Delta P_{\rm HP} = \left(\frac{P_{\rm HP}}{\Phi_{\rm S}}\right)_{\rm nom} \frac{1}{1 + sT_{\rm HP}} \Delta\Phi_{\rm S} \qquad T_{\rm HP} \in [0.1,\ 0.5]\ {\rm s}$$

- Hence:

$$\Delta P_{\rm MP} + \Delta P_{\rm LP} = \left[\left(\frac{P_m}{\Phi_{\rm S}}\right)_{\text{nom}} + \left(\frac{P_{\rm LP}}{\Phi_{\rm S}}\right)_n \frac{1}{1 + sT_{\rm LP}}\right] \frac{1}{(1 + sT_{\rm R})(1 + sT_{\rm HP})} \Delta\Phi_{\rm S}$$

(We neglect the delay in MP because it is really small).

$$\Rightarrow \quad G_a(s) = K'_A \frac{1 + \alpha' T_{\rm R} s}{(1 + sT_{\rm R})(1 + sT_{\rm HP})}$$ where $\alpha' \cong 0.8\alpha$

Model of $G_a(s)$ for Thermo plants - VII

- For slow transient, we have to take into account the boiler dynamic.
- We have:

$$\Delta\Phi_{\mathrm{S}} = \frac{R_v h_A}{R_v + R_s}\Delta A + \Delta P_{\mathrm{B}}$$

$$\Delta\Phi_i - \Delta\Phi_{\mathrm{S}} = C_{\mathrm{B}}\frac{d}{dt}\Delta P_{\mathrm{B}}$$

(assume $\Delta\Phi_i = 0$ and obtain ΔP_{B})

$\rightarrow$ We obtain:

$$G_a(s) = \frac{K'_A s T_{\mathrm{B}}(1+\alpha T_{\mathrm{S}})}{(1+sT_{\mathrm{B}})(1+sT_{\mathrm{R}})}$$

where: $T_{\mathrm{B}} = C_{\mathrm{B}}(R_v + R_s) \in [50, 500]\ \mathrm{s}$ $\begin{cases} 50\ \mathrm{s}\text{: once through boiler} \\ 500\ \mathrm{s}\text{: drum boiler} \end{cases}$

Synthesis of the Primary Frequency Regulator

- Again, we want: $$G_f(s) = \frac{P_n}{\Omega_n} \frac{1}{b_p} \frac{1+sT_2}{1+sT_1} = G_r(s)G_v(s)G_a(s)$$
- Since we know $G_v(s)$ and $G_a(s)$, we have:

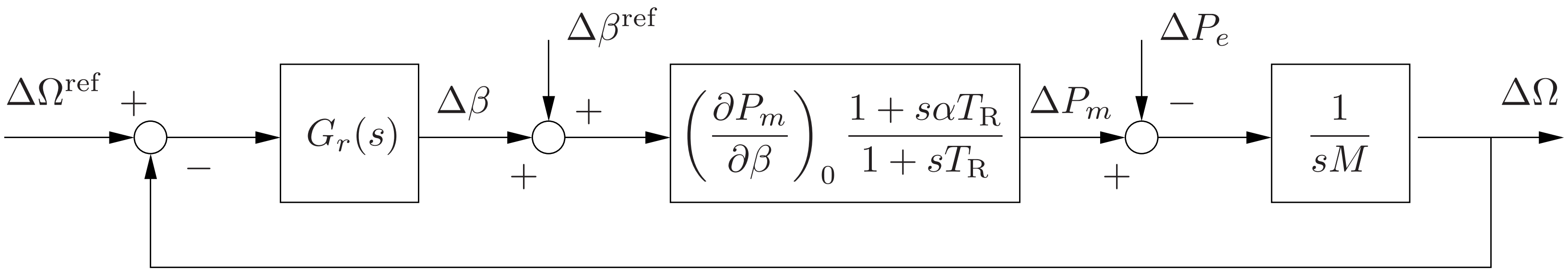
$$G_r(s) = \frac{P_n}{\Omega_n} \frac{1}{b_p} \frac{1+sT_2}{1+sT_1} \frac{1+sT_\mathrm{R}}{1+s\alpha T_\mathrm{R}} \frac{1}{G_v(0)G_a(0)}$$

where: $T_1 = 15\ \mathrm{s}$, $T_2 = 5\ \mathrm{s}$, $\alpha T_\mathrm{R} \approx T_2$ and $T_\mathrm{R} \approx T_1$

hence: $$G_r(s) = \frac{P_n}{\Omega_n} \frac{1}{b_p} \frac{1}{G_v(0)G_a(0)} \approx \mathrm{constant}$$

- Observe that $G_r(s)$ depends on the operating point!

Effect of the Reference Signal $\Delta\beta^{\mathrm{ref}}$



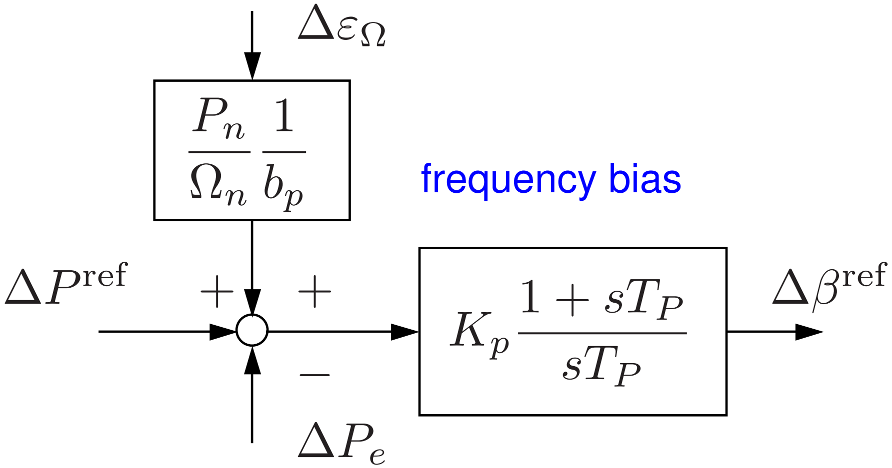
→ $\Delta\beta^{\mathrm{ref}}$ comes from the secondary frequency control.

→ In steady state, we have:

$$\Delta P_m = \left(\frac{\partial P_m}{\partial\beta}\right)_0 \Delta\beta^{\mathrm{ref}} + \left(\frac{\partial P_m}{\partial\beta}\right)_0 \frac{P_n}{\Omega_n}\frac{1}{b_p}\frac{1}{\left(\frac{\partial P_m}{\partial\beta}\right)_0}(\Delta\Omega^{\mathrm{ref}} - \Delta\Omega)$$

Auxiliary Power Regulator

- With the current regulator scheme, we canot obtain a constant frequency regulator.
- Let's add a PI controller before $\Delta\beta^{\text{ref}}$:



where

ΔP^{ref} comes from the secondary f/P regulator

ΔP_e is the power produced by the plant ($=\Delta P_L$)

Auxiliary Power Regulator

$\Rightarrow$ hence we have $\quad \Delta P_e = \Delta P_m - sM\Delta\Omega$

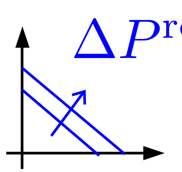
$\Rightarrow$ In steady state:

$$\Delta P_m = \left(\frac{\partial P_m}{\partial \beta}\right)_0 \Delta\beta^{\mathrm{ref}} + \frac{P_n}{\Omega_n}\frac{1}{b_p}(\Delta\Omega^{\mathrm{ref}} - \Delta\Omega)$$

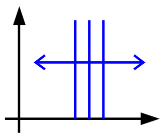
$$\Rightarrow \quad -\Delta P_e + \Delta P^{\mathrm{ref}} + \frac{P_n}{\Omega_n}\frac{1}{b_p}(\Delta\Omega^{\mathrm{ref}} - \Delta\Omega) = 0$$

$$\Rightarrow \quad \Delta P_e = \Delta P^{\mathrm{ref}} + \frac{P_n}{\Omega_n}\frac{1}{b_p}(\Delta\Omega^{\mathrm{ref}} - \Delta\Omega) = \Delta P_m$$

$\Rightarrow$ In such a way, ΔP^{ref} forces to produce the required power.



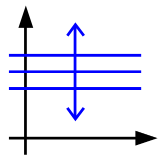
· to obtain



$\Rightarrow$ $G_R(0) = 0$

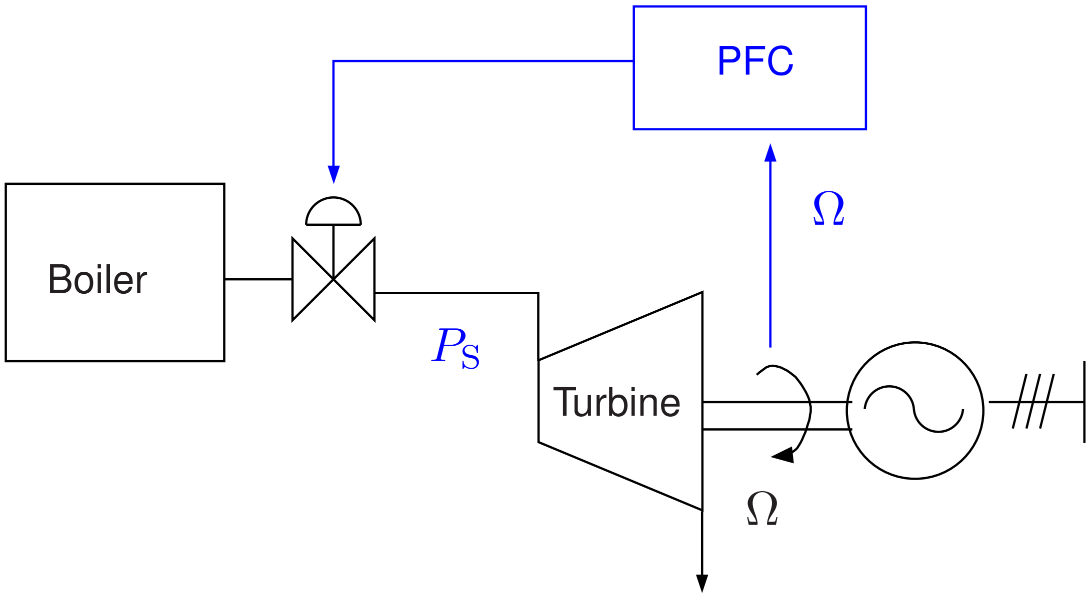
frequency bias = 0

· to obtain



$\Rightarrow$ disconnect ΔP^{ref} ΔP_e

Boiler-Follows-Turbine Regulator

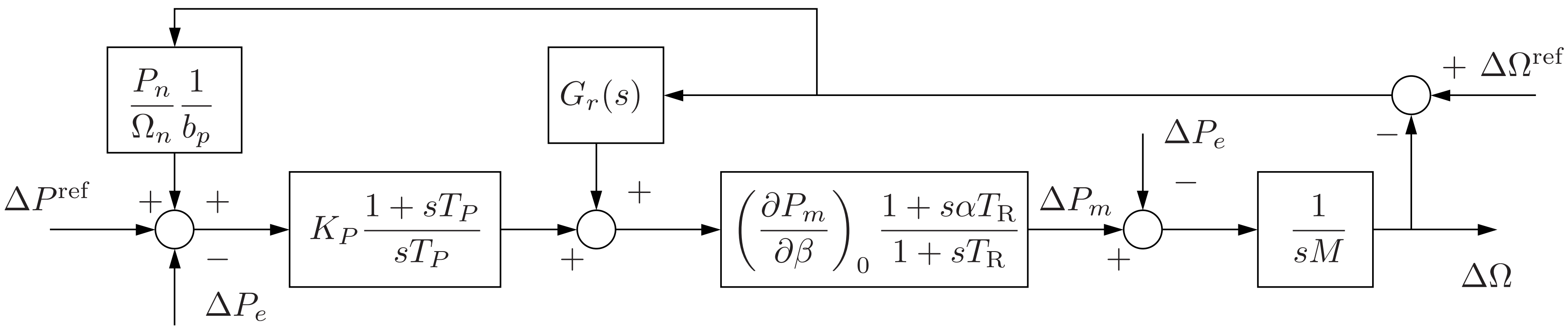


⇒ In this way the boiler follows the control on the turbine. It is fast but I have to open the valve several times.

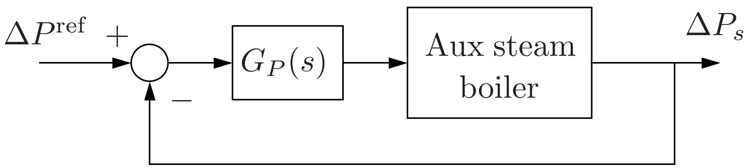
⇒ There is also a control of the type: turbine-follows-boiler. It is slow but does not stimulate too much the valve.

Co-ordinated Control

- This control has all the positives of the “boiler-follows-turbine” and the “turbine-follows-boiler”.



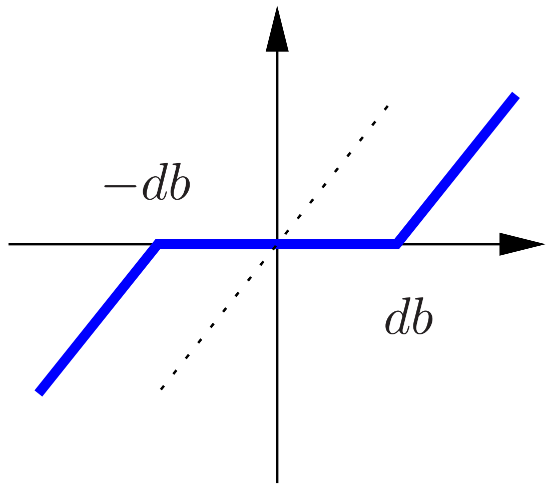
$\Rightarrow$



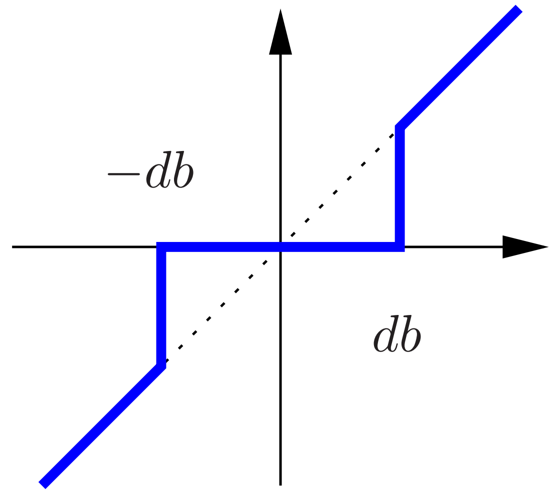
This is the boiler-follows-turbine. To obtain the turbine-follows-boiler $\rightarrow$ disconnect the feedback on $\Delta\Omega$

Effect of the Deadband on Primary Frequency Control - I

- It is a generalized industry practice to add a small deadband (DB) to the calibration of the governor speed error bias to reduce the movement for very small speed deviations.
- The selection of the DB affects the fidelity of the regulation.
- There are two types of deadbands.



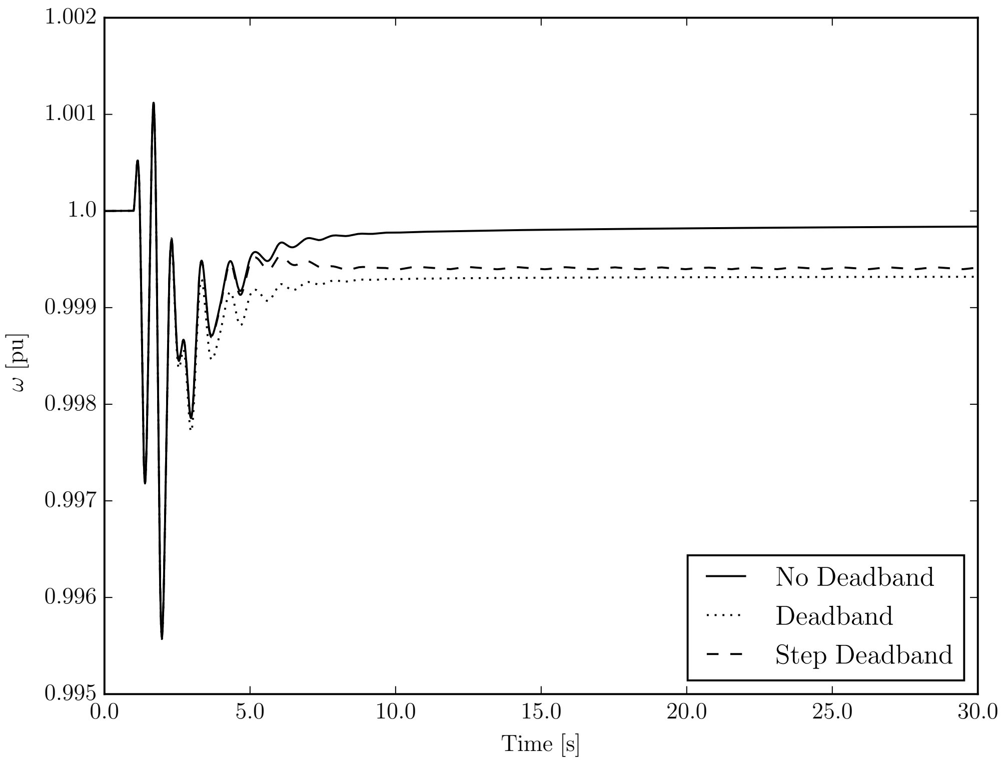
Deadband



Step Deadband

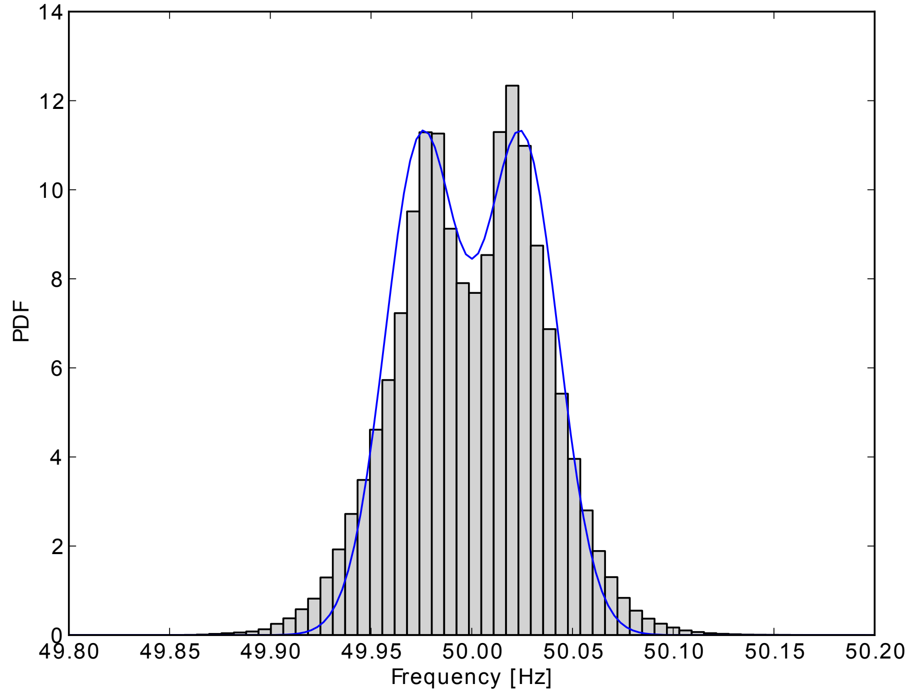
Effect of the Deadband on Primary Frequency Control - II

- Examples of trajectories of the frequency of a synchronous machine using a deadband in the primary frequency control.



Effect of the Deadband on Primary Frequency Control - III

- Another consequence of the deadband is that the distribution of the frequency is bimodal, not normal, as commonly expected.



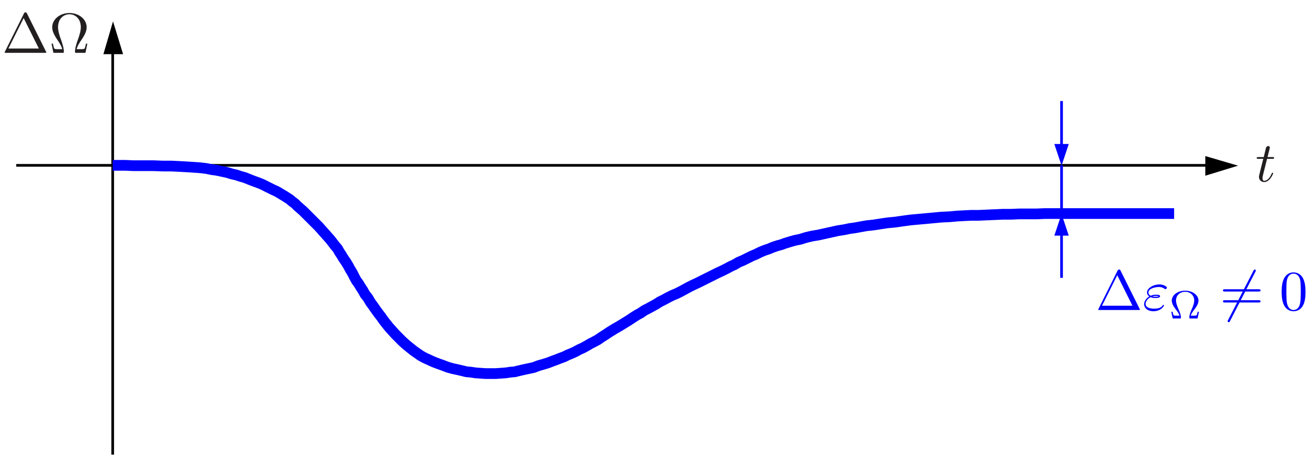
PDF distribution of the frequency of the Irish system (~ 6M samples, one week).

Secondary Frequency Control

- We have seen that using the primary frequency control, the frequency steady-state error is not zero.

- We used an external signal ($\Delta\beta^{\mathrm{ref}}$ or ΔP^{ref}) to obtian $\Delta\Omega = 0$ in steady state.

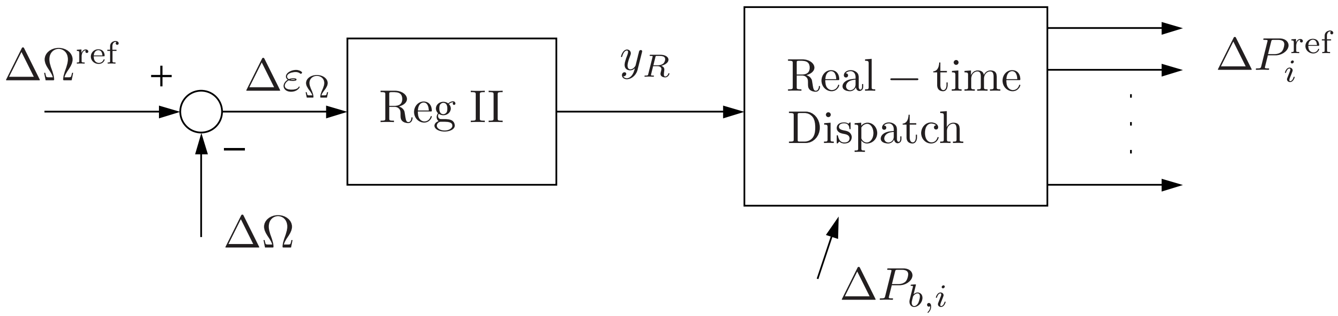
- For example, if the load increases:



Secondary Frequency Control

- The secondary frequency control is “unique” (or centralized) for the system (or area).

$\longrightarrow$ The National Control Center co-ordinates the Regional Control Centers.

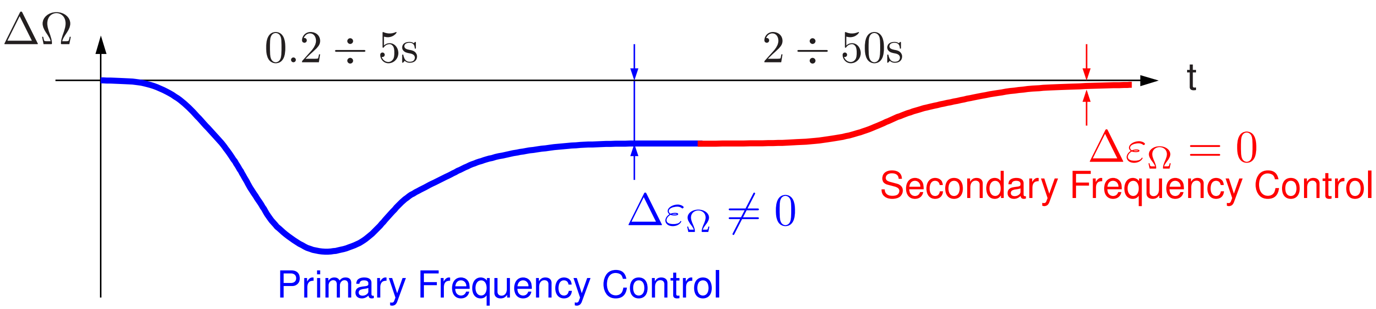


- ΔP_i^{ref} are the signals that go to each power plant (center into the primary frequency control of each generator).

- $\Delta P_{b,i}$ are the real-time power dispatches $\longrightarrow$ comes from short-term market (or from tertiary frequency control).

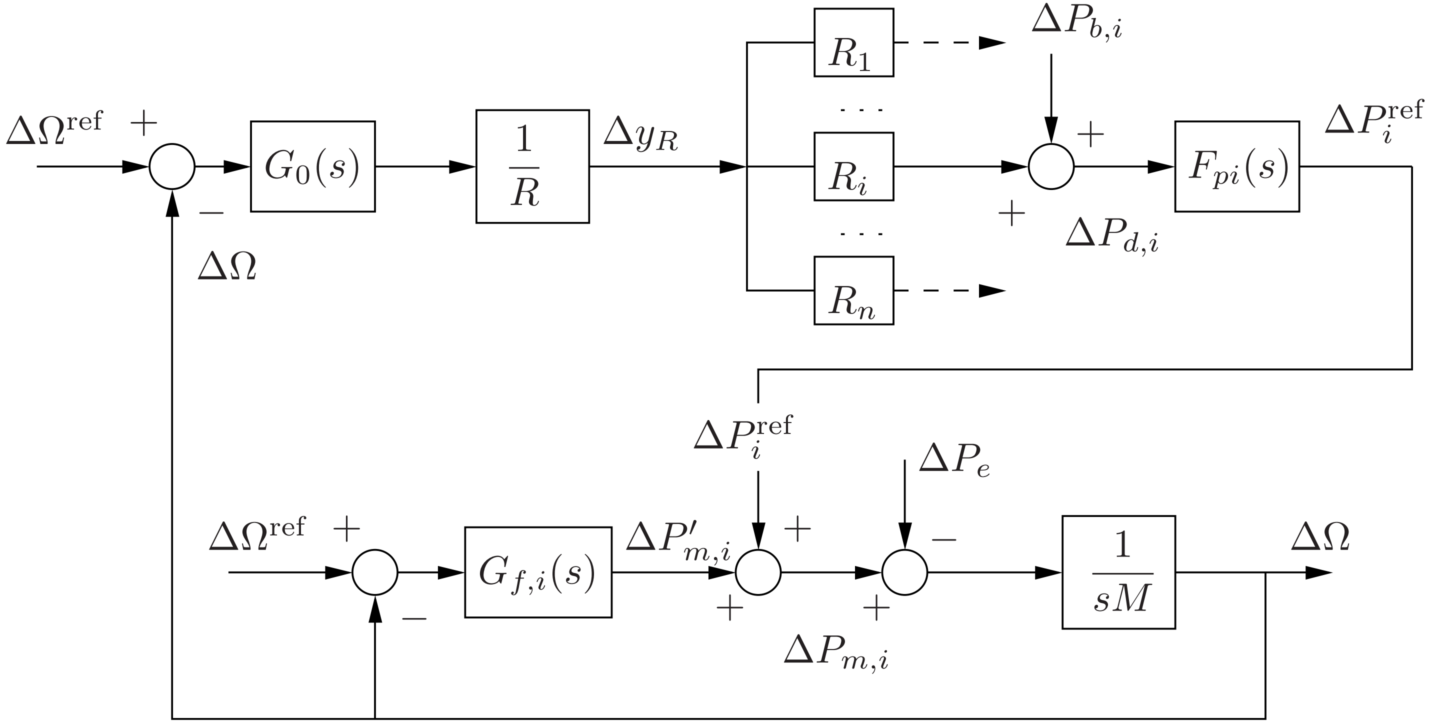
Effect of Secondary Frequency Control

$\Rightarrow$ The static error of the secondary frequency control must be zero:



$\rightarrow$ In steady-state, we have: $\Delta\varepsilon_\Omega = 0, \quad \Delta\Omega^{\mathrm{ref}} = 0, \quad \Delta\Omega = 0$

Scheme of the Seconday Frequency Control - I



$$\Delta P_{m,i} = \Delta P'_{m,i} + \Delta P_i^{\text{ref}}$$

PFC SFC

Scheme of the Secondary Frequency Control - II

- We have:

$$\Delta P_i^{\mathrm{ref}} = R_i \Delta y_R + \Delta P_{b,i} \qquad (*)$$

$$\Delta P_{d,i} = \Delta P_{b,i} + \hat{R}_i (\Delta P_d - \Delta P_b)$$

- Where $\hat{R}_i = R_i / R, \quad R = \sum_{i=1}^{n} R_i$
- $\Delta P_d - \Delta P_b$ is the power imbalance due to a wrong load forecast.
- $\hat{R}_i$ can depend also on economical factors (market dispatch).
- Then we have: $\Delta P_i^{\mathrm{ref}} = \Delta P_{b,i} + \hat{R}_i (\Delta P_d - \Delta P_b) \qquad (**)$

Scheme of the Secondary Frequency Control - III

- Imposing that $(*)$ and $(**)$ are equal, we obtain:

$$\Delta P_{b,i} + \hat{R}_i(\Delta P_d - \Delta P_b) = R_i \Delta y_R + \Delta P_{b,i}$$

$$\begin{aligned} \Rightarrow & \hat{R}_1(\Delta P_d - \Delta P_b) = R_1 \Delta y_R \\ & \hat{R}_2(\Delta P_d - \Delta P_b) = R_2 \Delta y_R \\ & \dots. \end{aligned}$$

$$\Rightarrow \left(\sum \hat{R}_i\right)(\Delta P_d - \Delta P_b) = \left(\sum R_i\right)\Delta y_R$$

but, by definition, $\sum \hat{R}_i = 1$ and $\sum R_i = R \quad \Rightarrow \quad \boxed{\Delta y_R = \frac{\Delta P_d - \Delta P_b}{R}}$

Scheme of the Secondary Frequency Control - IV

- From ($*$) we obtain:

$$\Delta y_R = \frac{\Delta P_{d,i} - \Delta P_{b,i}}{R_i}$$

- hence:

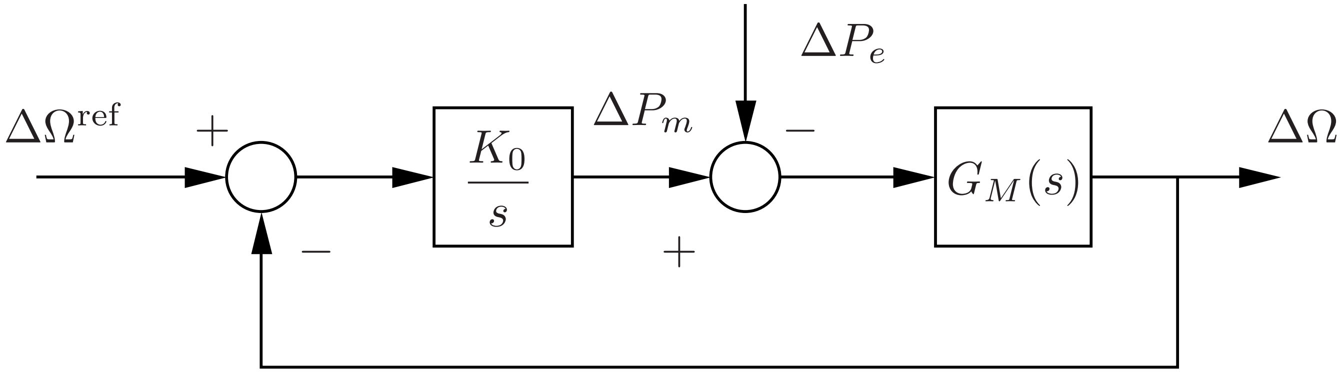
$$\frac{\Delta P_d - \Delta P_b}{R} = \frac{\Delta P_{d,i} - \Delta P_{b,i}}{R_i}$$

- and finally:

$$\frac{R_i}{R} = \hat{R}_i = \frac{\Delta P_{d,i} - \Delta P_{b,i}}{\Delta P_d - \Delta P_b}$$

- Conclusion: The frequency error is shared proportionally by all power plants involved in the secondary f/P control.

Simplified Secondary Frequency Control Scheme



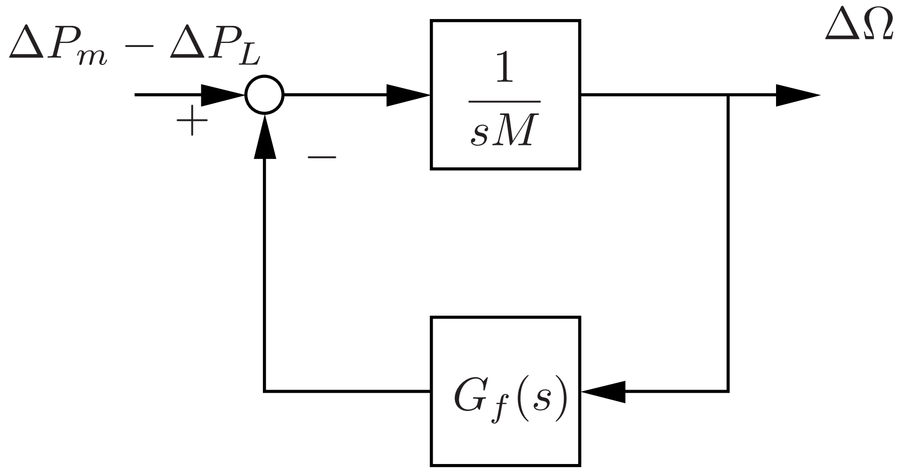
where

$$G_M(s) = \frac{\Delta\Omega}{\Delta P_m - \Delta P_e} = \frac{1}{E}\frac{1+sT_1}{\frac{s^2}{\nu_0^2} + 2\xi\frac{s}{\nu_0} + 1} \Rightarrow \text{synthesis of the I } f/P \text{ control}$$

and where we assumed $\hat{R}_i F_{p,i}(s) = 1$

Synthesis of $G_M(s)$ - I

$\longrightarrow$ Observe that:



where $$G_f(s) = \sum G_{fi}(s) \quad , \quad M = \sum M_i$$

$$G_f(s) = E\frac{1+sT_2}{1+sT_1}$$

$$\Rightarrow \quad G_M(s) = \frac{\frac{1}{sM}}{1+\frac{1}{sM}G_f(s)} = \frac{1+sT_1}{sM + s^2MT_1 + E(1+sT_2)}$$

Synthesis of $G_M(s)$ - II

- Hence, the closed-loop transfer function of the machine with its primary frequency regulation becomes:

$$G_M(s) = \frac{1 + sT_1}{sM + s^2 MT_1 + E(1 + sT_2)}$$

$$= \frac{1}{E} \frac{1 + sT_1}{s(\frac{M}{E} + T_2) + s^2 \frac{MT_1}{E} + 1}$$

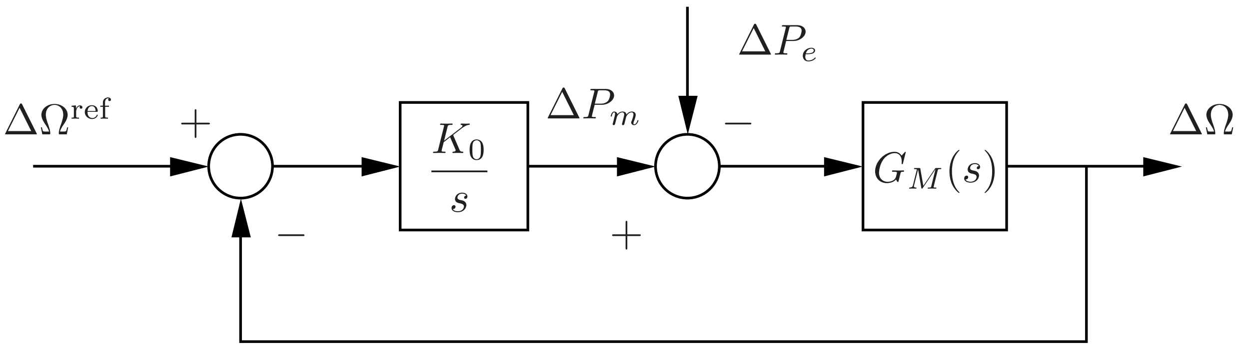
$$= \frac{1}{E} \frac{1 + sT_1}{\frac{s^2}{\nu_0^2} + 2\zeta \frac{s}{\nu_0} + 1}$$

$\Rightarrow$ hence:

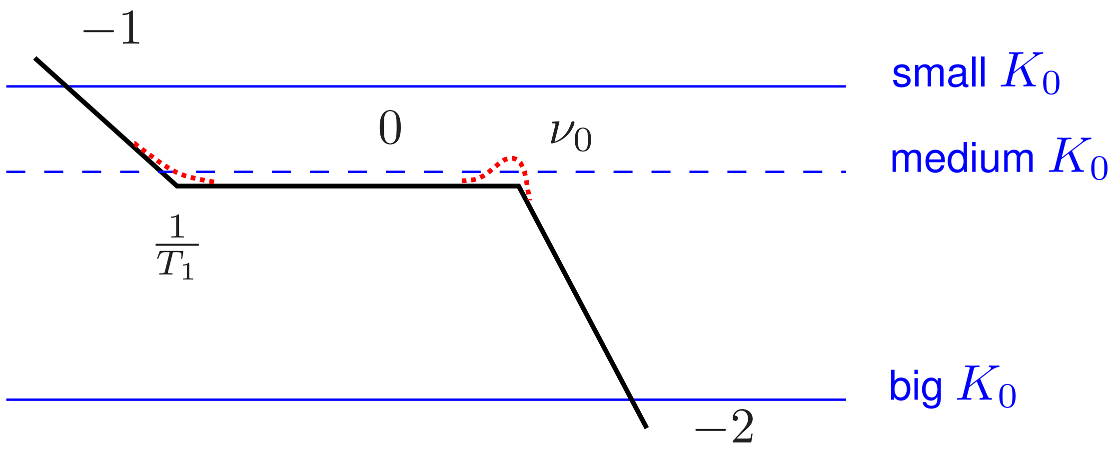
$$\begin{cases} \nu_0^2 = \frac{E}{MT_1} \\ \frac{2\zeta}{\nu_0} = \frac{M}{E} + T_2 \end{cases}$$

Stability of the Secondary Frequency Control Scheme - I

- Let's consider:



- We can expect that $\frac{1}{T_1} < \nu_0$, hence:



Stability of the Secondary Frequency Control Scheme - II

- So we cannot use “big” K_0
- The Bode cut-frequency must be $\nu_c'' < \nu_0$
- Observe that the cut-frequency of the primary f/P control is $\nu_c' \approx \nu_0$

  so we have:

  - $\nu_c' \approx 0.3\ \mathrm{Hz}$
  - $\nu_c'' \approx 0.03\ \mathrm{Hz}$
- The secondary f/P control must be slower than the primary f/P control.

Regulation Band

- Even though the secondary f/P control is relatively slow, some kinds of plants can be even slower and cannot follow the ΔP^{ref}.

- Thus we define a "regulation band":

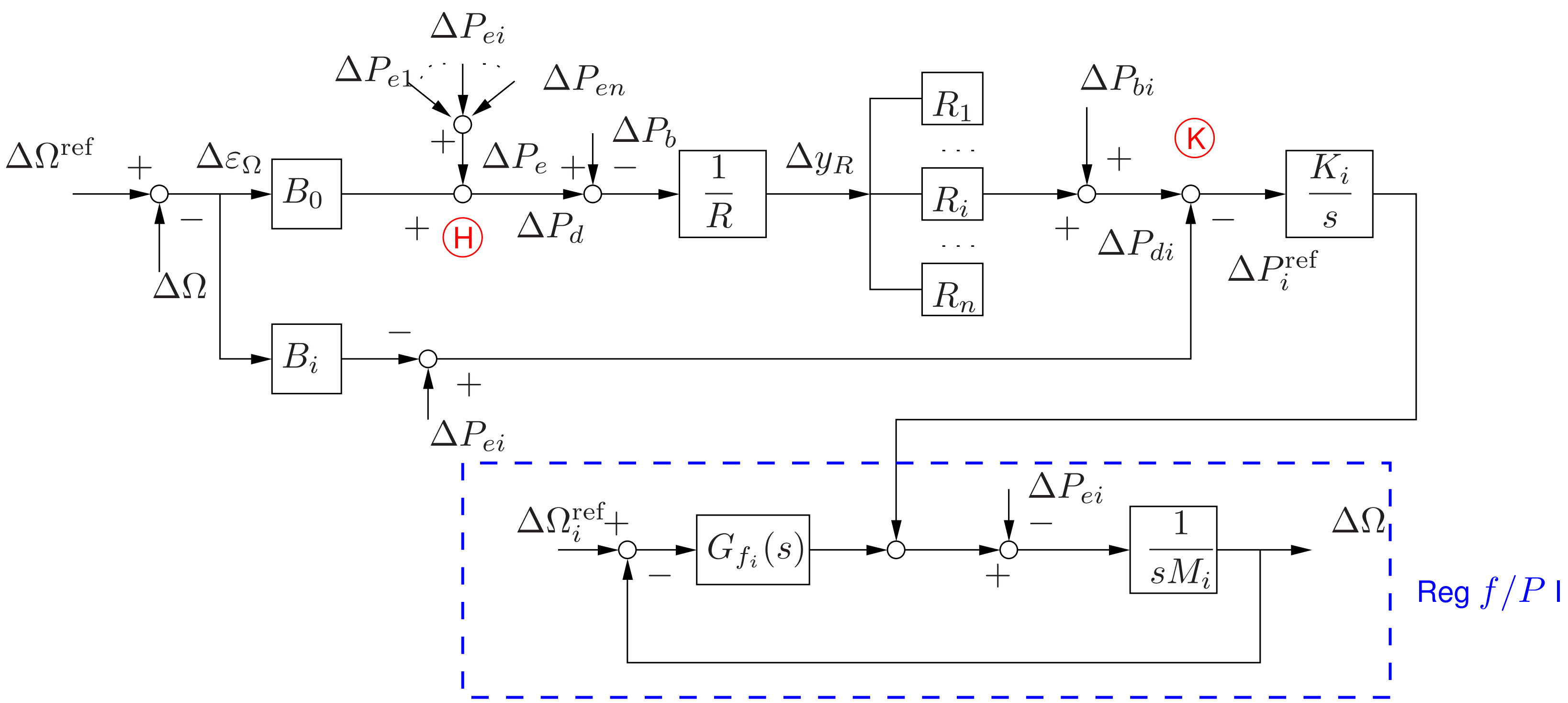
$$\begin{cases} 0 \div 100\% \to \text{hydro} \\ 10\% \to \text{thermo} \\ 0\% \to \text{nuclear} \end{cases}$$

(Pelton $\to 100\%$
Francis $\to 50\%$)

- So far most renewable energy sources do not participate to the f/P control $\to$ This must change in the near future.

Alternative Secondary Frequency Control Scheme: AGC

- The AGC (Automatic Generation Control) is used in USA
- It is heavily based on telecommunication systems.
- It co-ordinates power exchanges between different system operators.



AGC Scheme

- We have:

$$\text{Ⓗ} \quad \Delta P_d = \Delta P_e + B_0 \Delta \varepsilon_\Omega$$

$$\text{Ⓚ} \quad \Delta P_{di} - \Delta P_{ei} + B_i \Delta \epsilon_\Omega = 0 \quad (\Rightarrow \text{in steady state})$$

$$\Rightarrow \sum \Delta P_{di} = \Delta P_d$$
$$\Rightarrow \sum \Delta P_{ei} = \Delta P_e$$

$$\Rightarrow \Delta P_d - \Delta P_e + \sum_i^n B_i \Delta \varepsilon_\Omega = 0$$

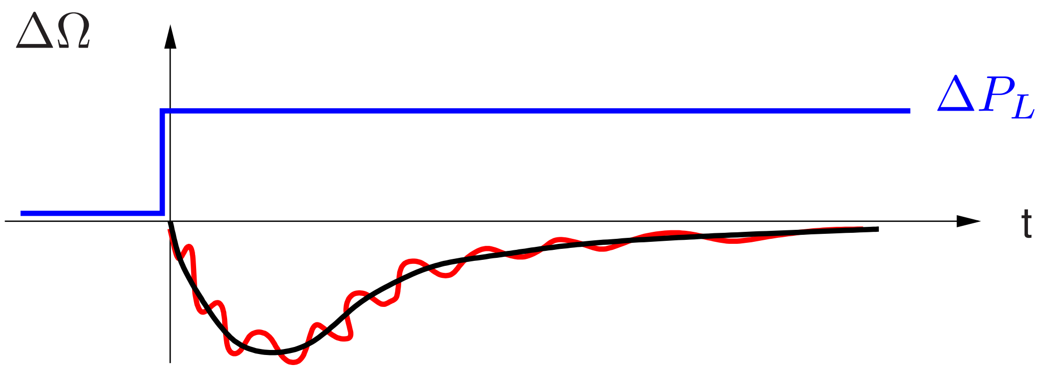
$$\Rightarrow 0 = (B_0 + \sum_i^n B_i) \Delta \varepsilon_\Omega$$

Since $B_0 + \sum B_i \neq 0 \quad \Rightarrow \quad \Delta \varepsilon_\Omega = 0$ in steady state

Moreover if $\Delta \varepsilon_\Omega = 0 \ \Rightarrow \ \Delta P_{di} - \Delta P_{ei} = 0$ $\Rightarrow$ Each generator produces exactly its reference power

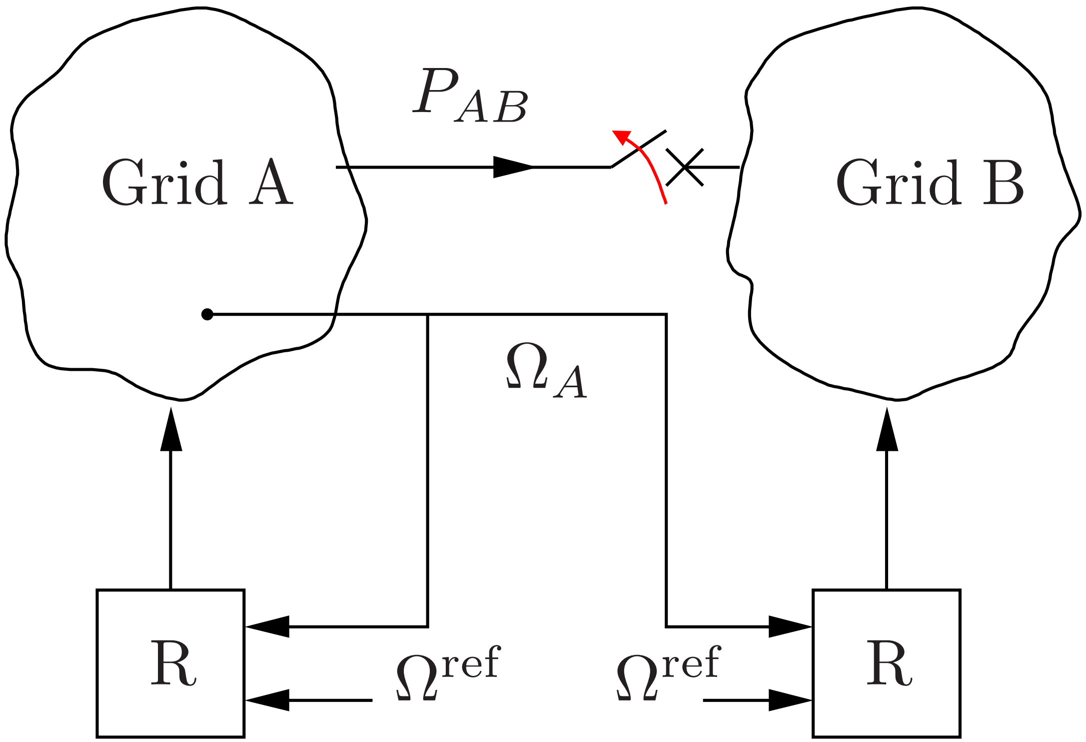
Effect of the Transient Response of Synchronous Machines

- Even though the system frequency in steady-state is unique, during a transient, each machine has its own oscillations.



- Hence, we need to measure several frequencies in the system and compute a mean value ($\Rightarrow$ COI), otherwise the frequency control could fail.

Effect of Local Frequency Measurements

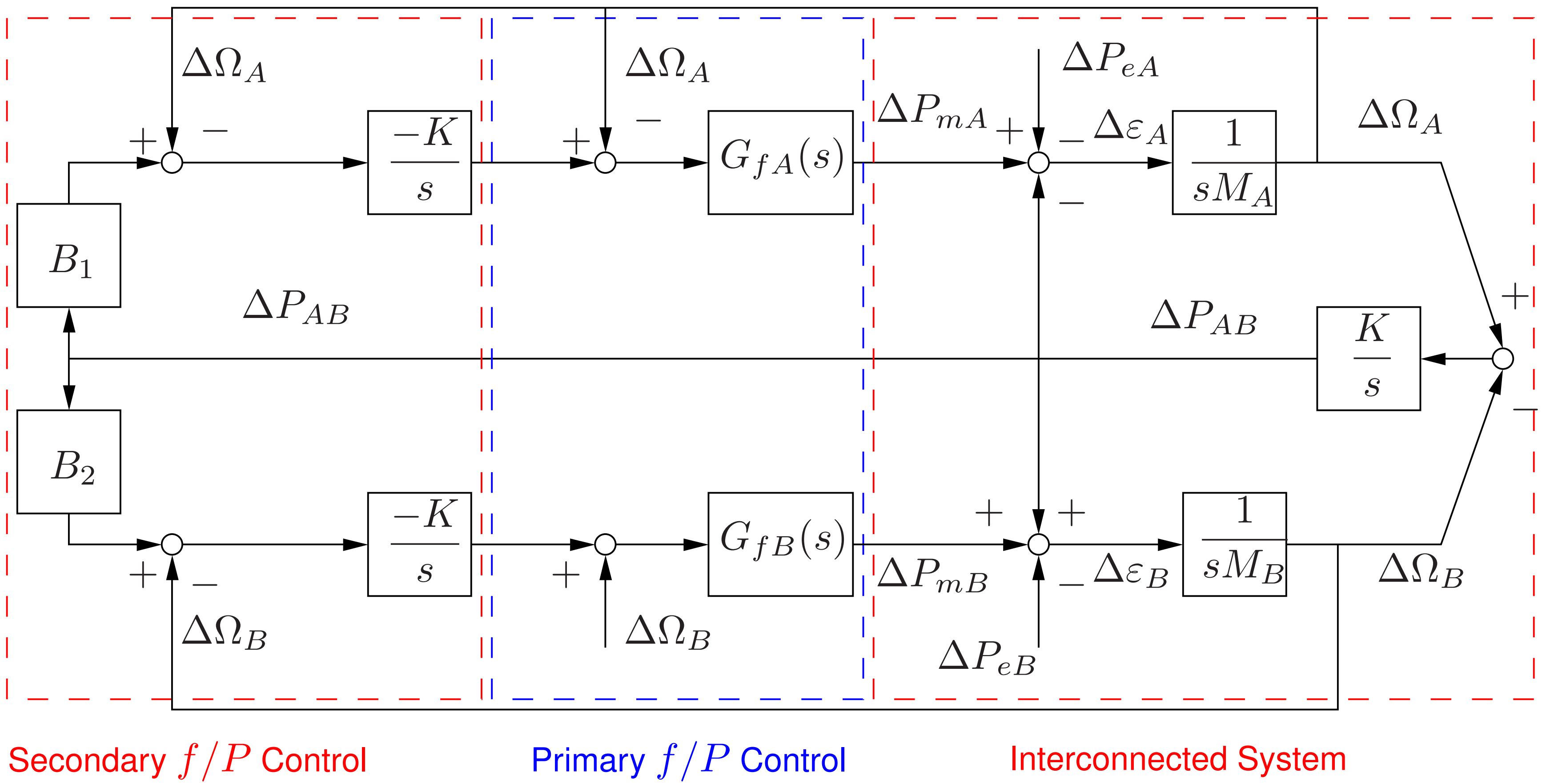


- In this case, if we measure only Ω_A and the two systems seperate ($P_{AB} = 0$)

$\rightarrow$ the regulator on Grid B does not operate properly.

Frequency Regulation of Interconnected Systems

- Let's consider the two systems above and their primary and secondary frequency control.



Comparison of Frequency Control and System Modes

$\Rightarrow$ To define the oscillation mode of the system, let's put $\Delta P_{mA} = \Delta P_{eA} = \Delta P_{mB} = \Delta P_{eB} = 0$

$$\Delta\varepsilon_A = -\frac{K}{s}(\Delta\Omega_A - \Delta\Omega_B) \quad (*)$$

$$\Delta\varepsilon_B = +\frac{K}{s}(\Delta\Omega_A - \Delta\Omega_B) \quad (**)$$

$$s^2 M_A M_B \quad \otimes \quad \Delta\Omega_A - \Delta\Omega_B = \frac{1}{sM_A}\Delta\varepsilon_A - \frac{1}{sM_B}\Delta\varepsilon_B$$

$$\Rightarrow (\Delta\Omega_A - \Delta\Omega_B)s^2 M_A M_B = sM_B \underbrace{\left(-\frac{K}{s}(\Delta\Omega_A - \Delta\Omega_B)\right)}_{*} - sM_A \underbrace{\left(+\frac{K}{s}(\Delta\Omega_A - \Delta\Omega_B)\right)}_{**}$$

$\Rightarrow \lambda_{1,2} = \pm j\sqrt{\frac{K}{M_{AB}}}$, with $M_{AB} = \frac{M_A M_B}{M_A + M_B}$

$\lambda_{1,2} \approx 1\ \text{Hz} \quad \Rightarrow \quad \nu_c' \sim 0.3\ \text{Hz} \quad \Rightarrow \quad \nu_c'' \sim 0.03\ \text{Hz} \quad \Rightarrow$ fully decoupled!

Frequency Variability and Time Error Regulation

- The variability of the frequency is very limited.
- For the continental European power system, the variability range is within the range 49.9 and 50.1 Hz.
- For islanded systems, the accepted variability is higher, e.g., 49.5 and 50.5 Hz.
- In addition to frequency and exchanged powers, the power system also regulates the *time error* – this is related to the problem of electric clocks.
- The time error is fixed by varying the reference frequency of primary and secondary frequency regulators.
- For example, in the continental European power system, the procedure is to set $\Omega^{\mathrm{ref}} = 49.99$ (or 50.01) Hz for 24 hours.

Tertiary Power Control

- The purpose of the tertiary power control is to avoid saturations of primary and secondary control units.
- This is obtained by changing the *power bases* (and, in turn, coefficients R_i) of each units participating to the secondary frequency control.
- The update of power bases is done according to economic considerations (UC problem).
- It can be off-line or on-line.
- If it is on-line, the cut-off frequency of the tertiary power control has to be smaller than that of the AGC, say $\nu_c''' \sim 0.003 \text{ Hz}$
- A typical time constant of the on-line tertiary power control loop is $T''' = 250 \text{ s}$.