



# Voltage Regulation of Synchronous Machines

**POWER SYSTEM MODELLING AND CONTROL (EEEN40550)**

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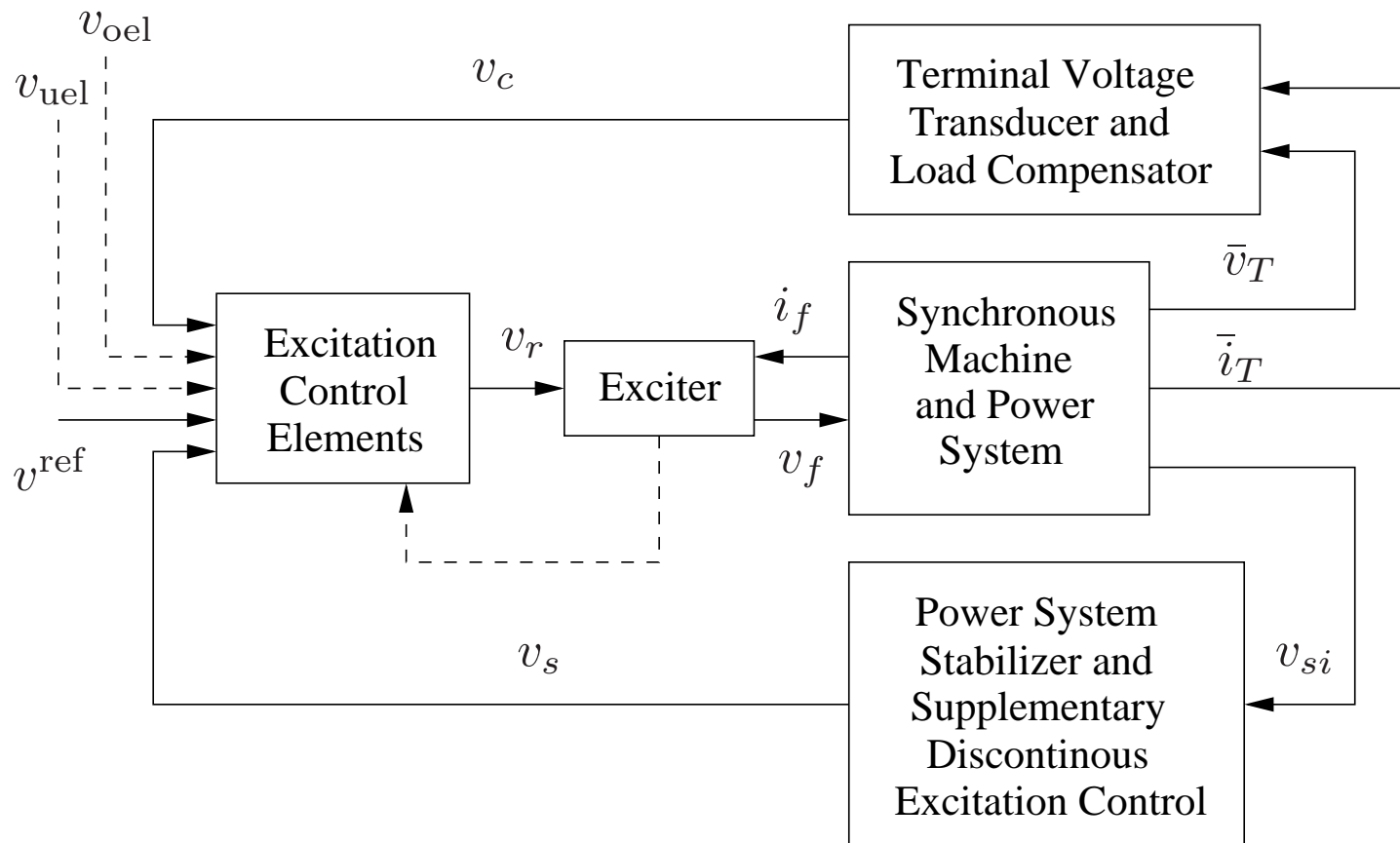
Dublin, Ireland



## Automatic Voltage Regulation

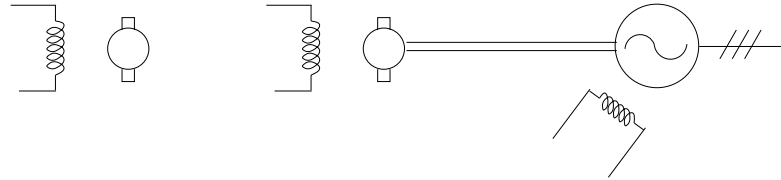
- Automatic Voltage Regulators measure the bus terminal voltage of the machine (or any relevant bus voltage) and compare it with a reference voltage. The error signal is used to modify the field voltage of the rotor of the synchronous machine.
- There are several models of AVR, depending on the technology:
  - based on DC machines (slow)
  - based on power electronics (fast)
  - brushless
  - permanent magnet (no voltage control!)

## Synoptic AVR Scheme

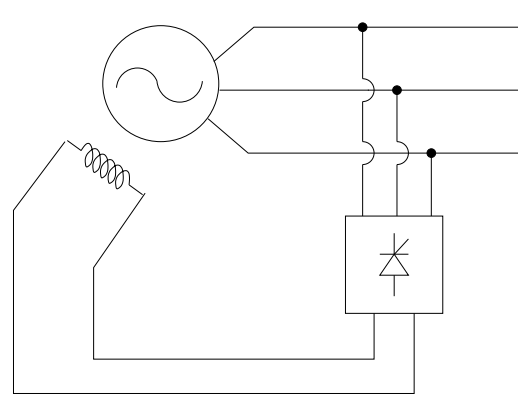


## AVR Types

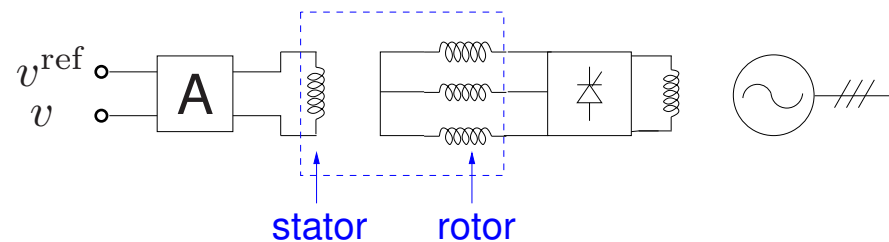
- DC machine:



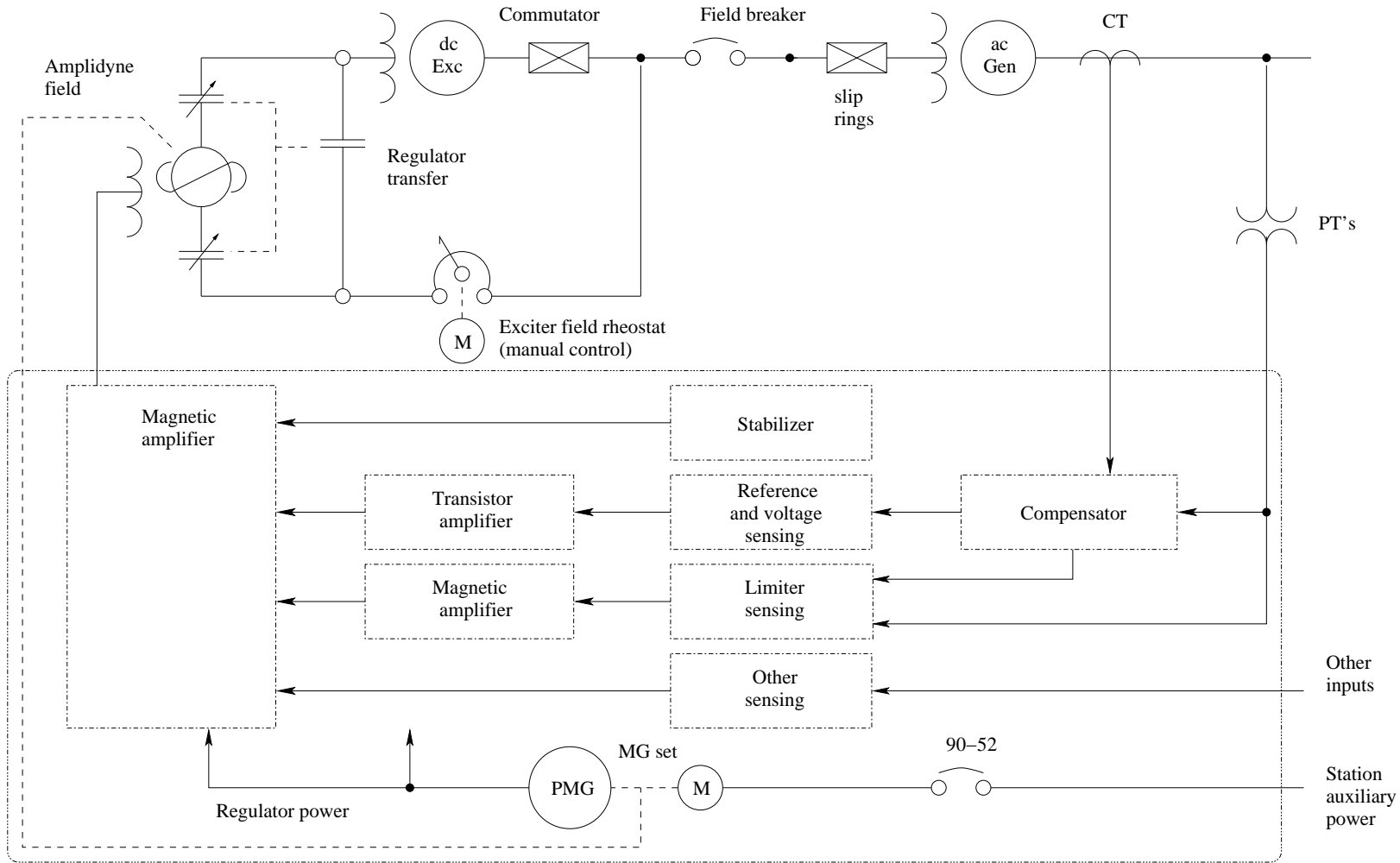
- Power Electronics Device:



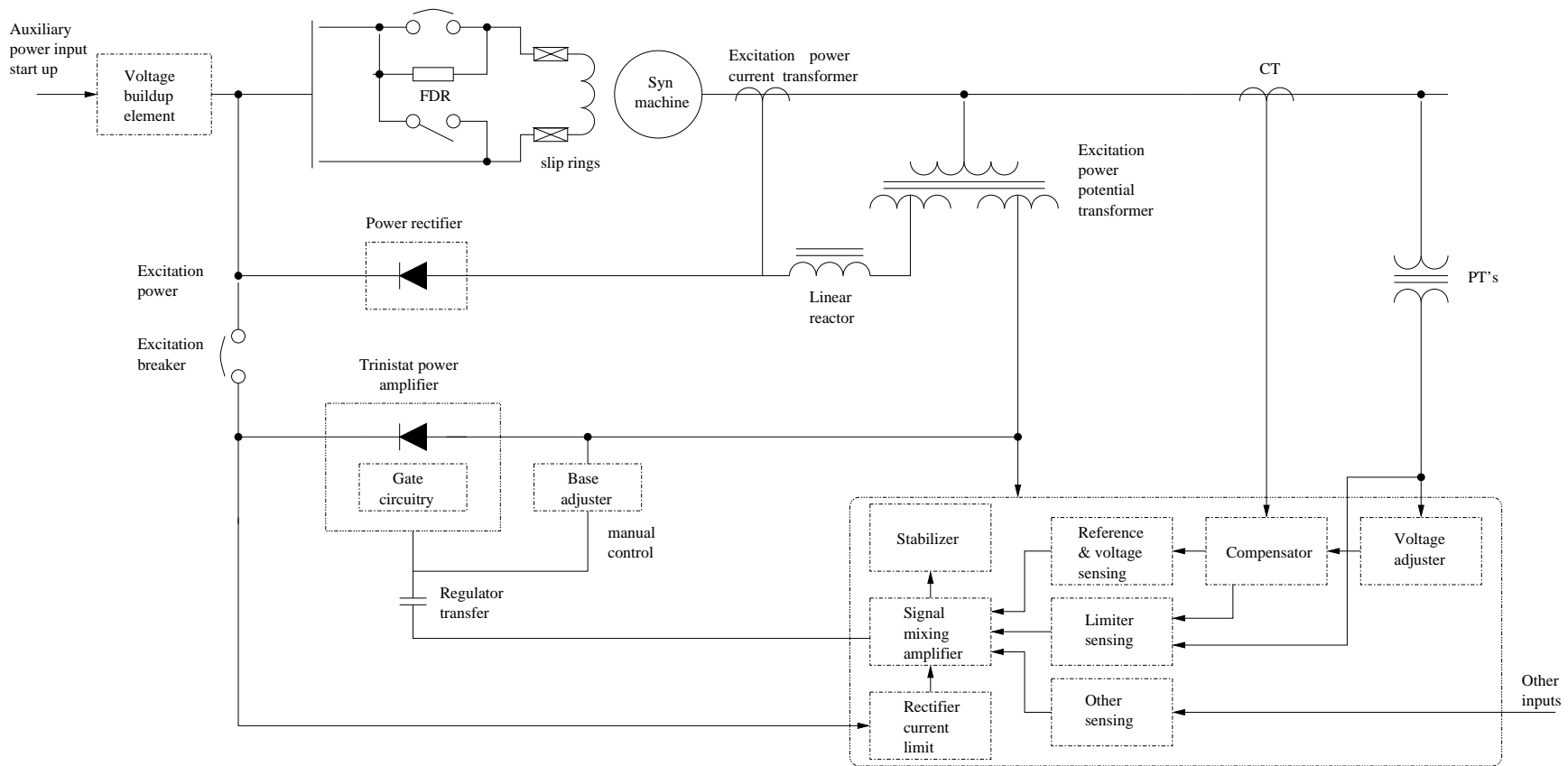
- Brushless: it includes a synchronous machine with a dual structure with respect to the main generator.



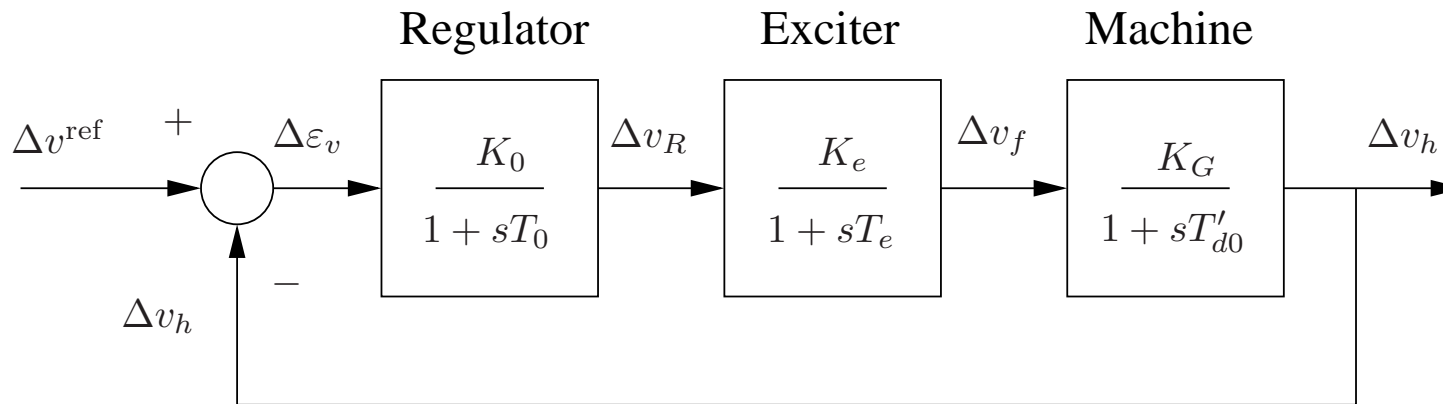
# DC-Machine-Based AVR



# Power-Electronics-Based AVR



## Basic Scheme of an AVR



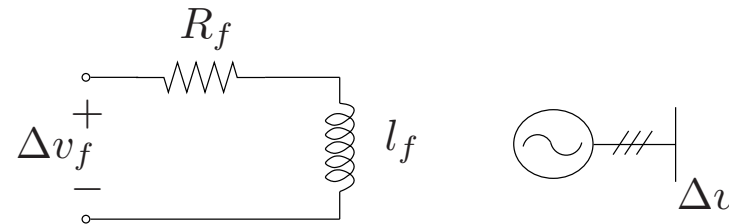
$$G_A = \frac{\Delta v_R}{\Delta \varepsilon_v} = \frac{K_0}{1 + sT_0} \quad \text{with} \quad T_0 \in [0.02, 0.08] \text{ s}$$

$$G_G = \frac{\Delta v}{\Delta v_f} = \frac{K_G}{1 + sT'_{d0}} \quad \text{with} \quad T'_{d0} \in [1, 5] \text{ s}$$

$$G_E = \frac{\Delta v_f}{\Delta v_R} = \frac{K_e}{1 + sT_e} \quad \text{with} \quad T_e \in [0.5, 0.8] \text{ s}$$

## Synthesis of the Transfer Function $G_G(s)$

- We have the field circuit of the machine:



$$\Delta v_f = R_f \Delta i_f + L_f \frac{d\Delta i_f}{dt} \Rightarrow \Delta v_f = (R_f + sL_f) \Delta i_f(s) = (R_f + sL_f) \frac{\Delta v}{K}$$

hence:

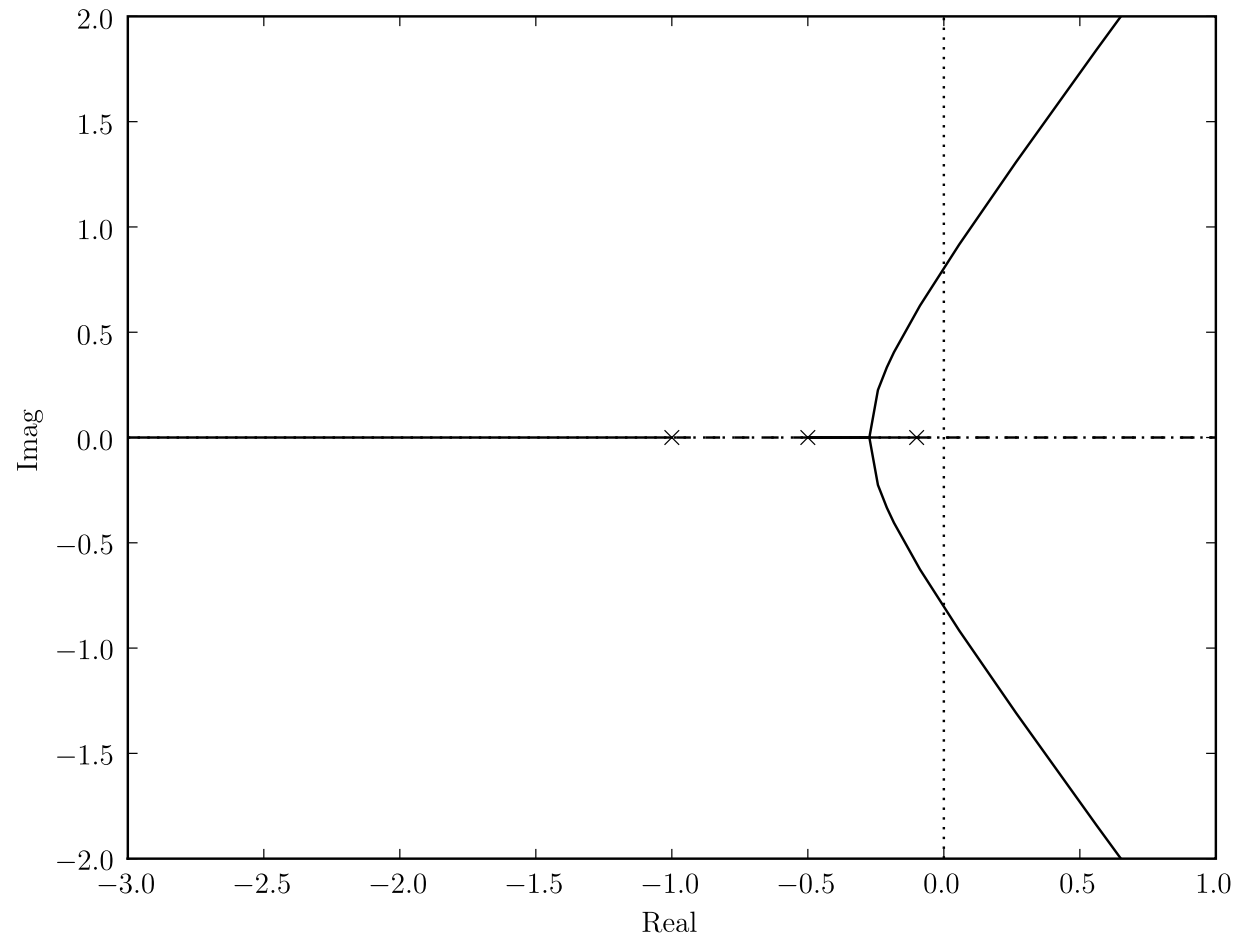
$$\frac{\Delta v}{\Delta v_f} = G_G = \frac{1}{K} \frac{1}{R_f + sL_f} = \frac{K_G}{1 + s \frac{L_f}{R_f}}$$

where

$$K_G = \frac{1}{K} \frac{1}{R_f} \quad \text{and} \quad T_G = \frac{L_f}{R_f} = T'_{d0} \quad \#$$

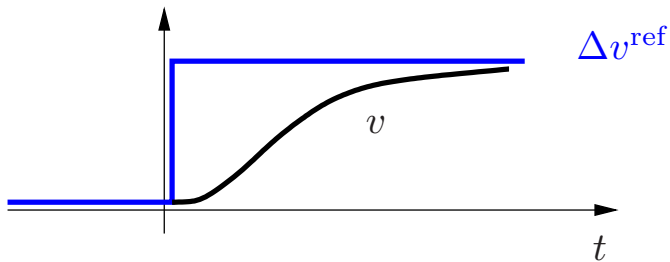


## Root Loci of a Basic AVR Scheme

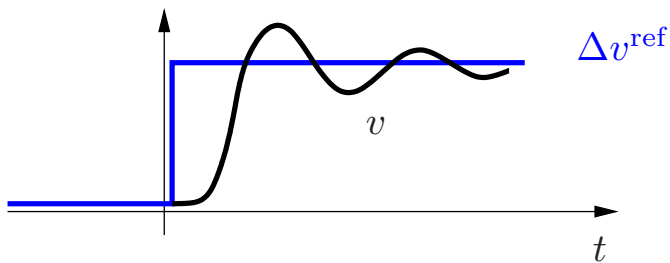


## Time Response of the AVR

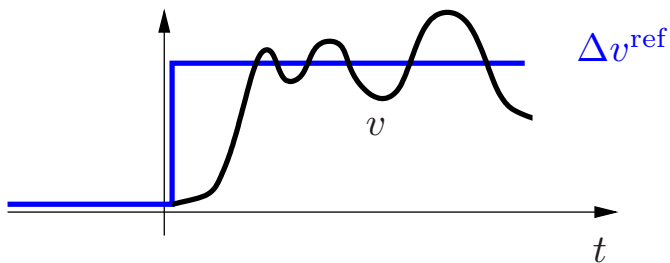
- Depending on the gain  $K_0$ , we can face three situations:



$\Rightarrow$  two real poles  $< 0$



$\Rightarrow$  complex eigenvalues with  $\mathbb{R}\{\lambda\} < 0$



$\Rightarrow$  complex eigenvalues with  $\mathbb{R}\{\lambda\} > 0$

## Basic AVR Scheme

- We have:

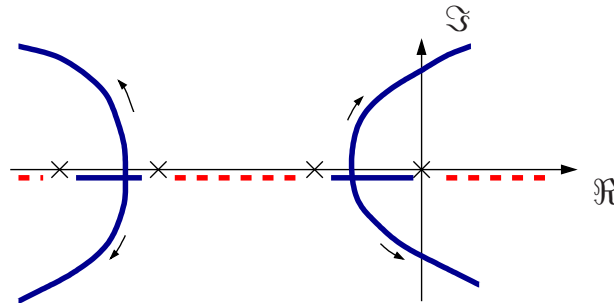
$$\frac{\varepsilon_v}{\Delta v^{\text{ref}}} = \frac{1}{1 + \frac{K_0 K_e K_G}{(1 + sT_0)(1 + sT_e)(1 + sT'_{d0})}}$$

→ Assume a step variation of  $\Delta v^{\text{ref}}(t) = \Delta(t) \Rightarrow \frac{1}{s}$

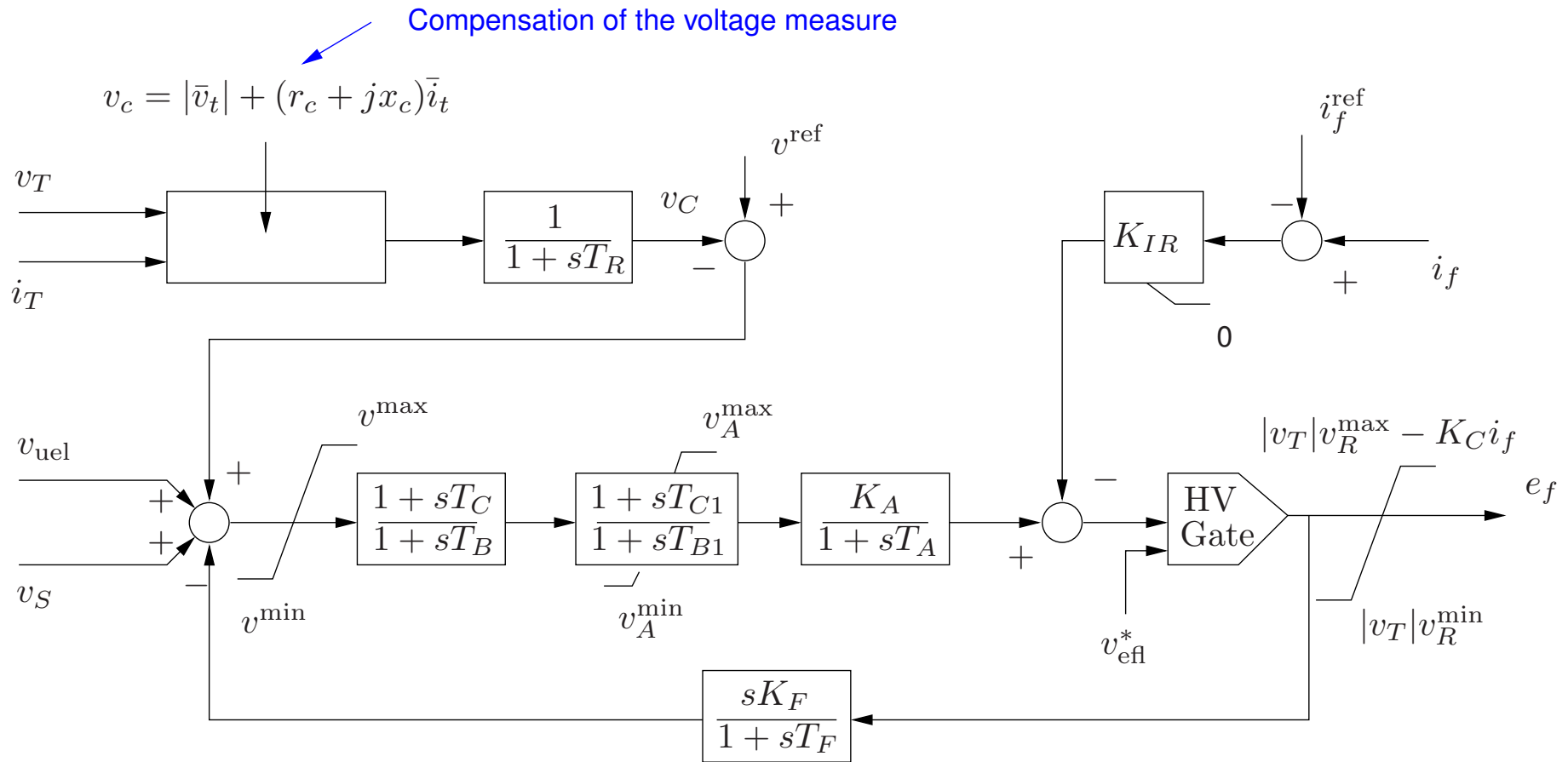
→ The static error is:  $\Delta \varepsilon_v(s \rightarrow 0) = \frac{1}{1 + K_0 K_e K_G}$

- If we do not want the static error, we have to include an integrator:

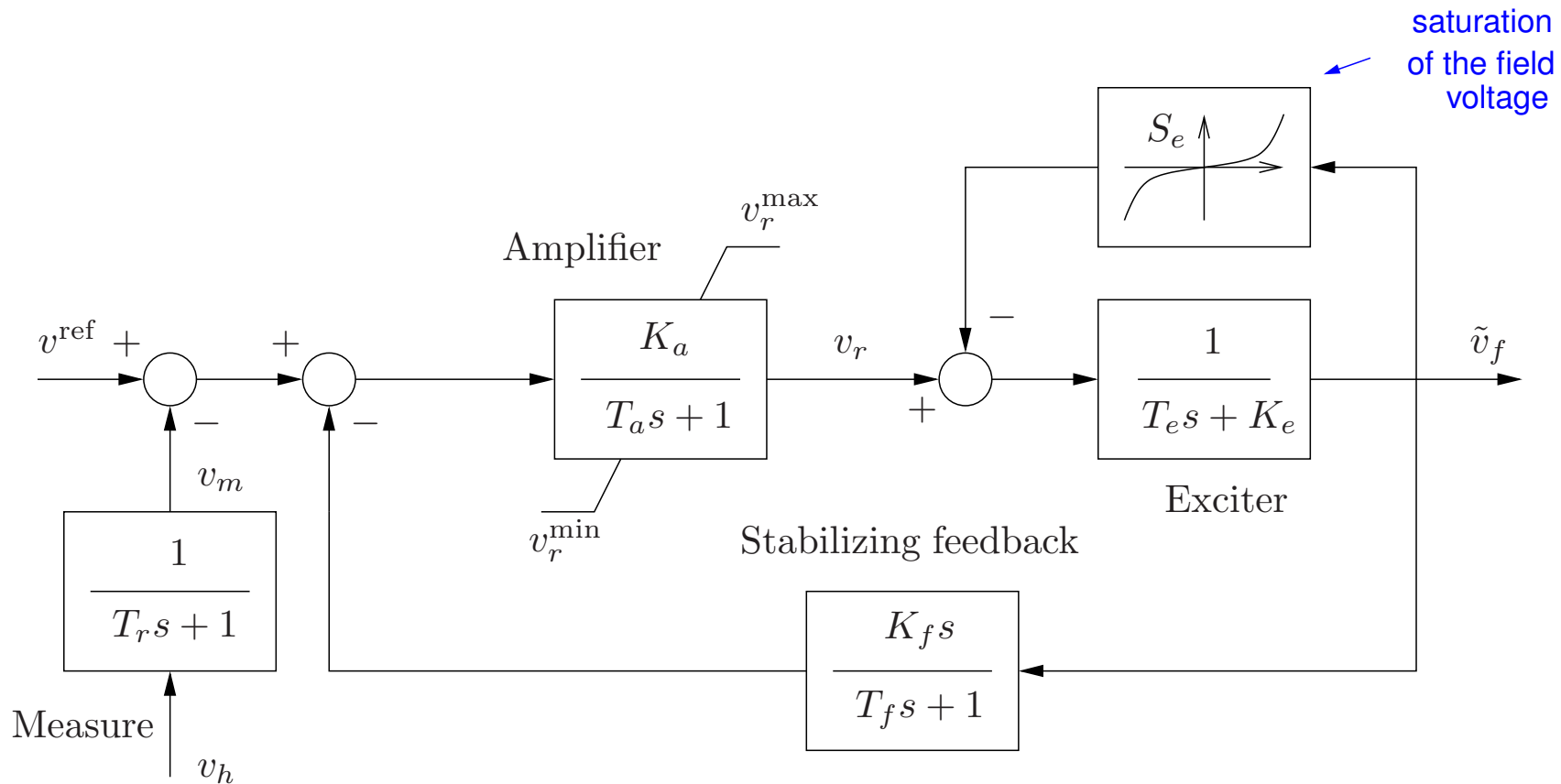
$$G_A(s) = \frac{1}{s} \frac{K_0}{(1 + sT_0)}$$



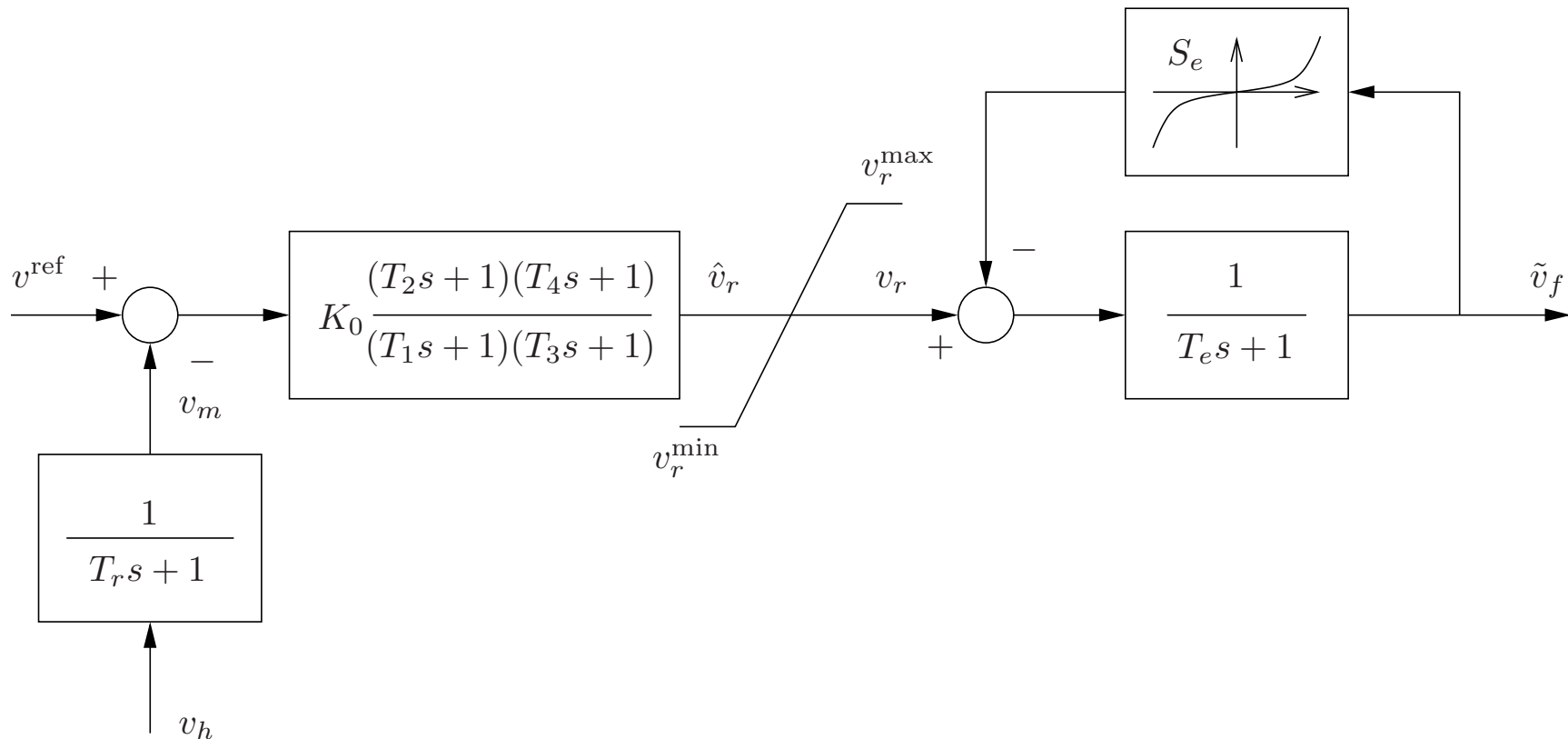
# Real-World AVR Schemes



## Standard IEEE Type 1 AVR Scheme



## Typical AVR Scheme for Power Electronics Exciter



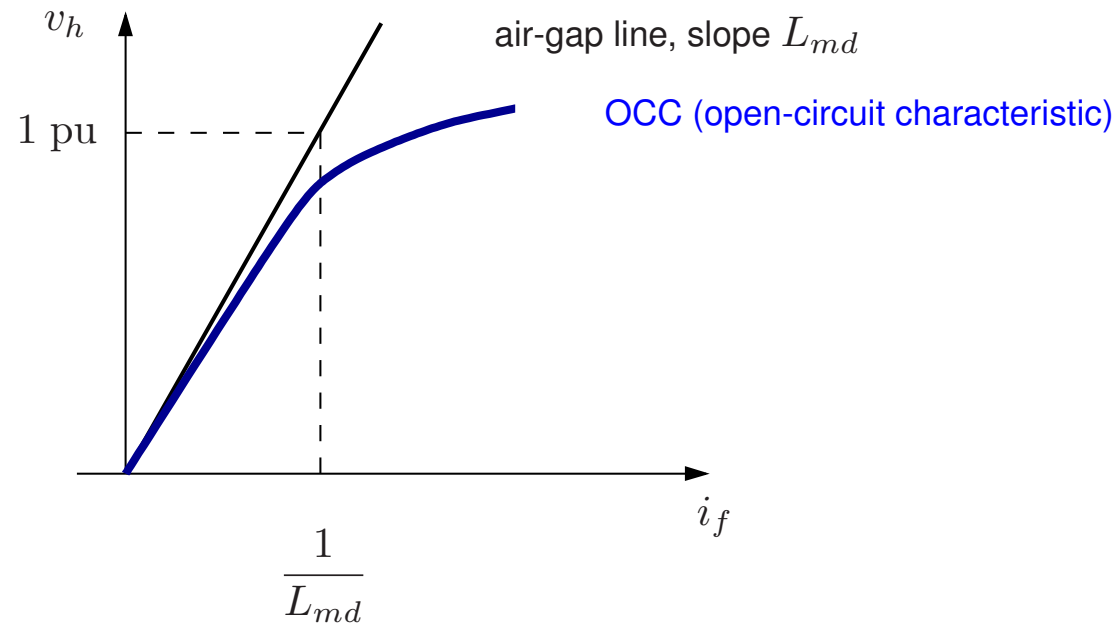
## Load Compensation

- Typically the AVR controls the generator terminal voltage.
- However, there are cases in which we need to compensate the measured signal:

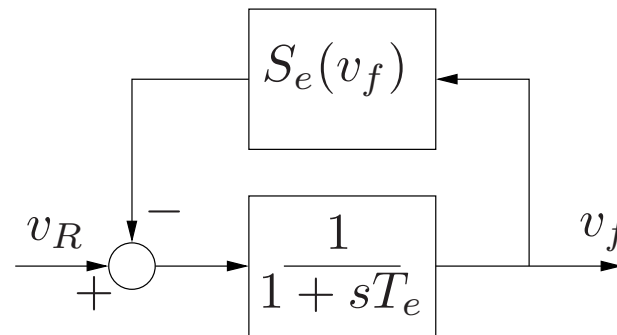
$$|\bar{v}_t| = |\bar{v}_{\text{bias}} + (r_c + jx_c)\bar{i}_{\text{bias}}|$$

- $r_c > 0$  and  $x_c > 0 \rightarrow$  regulate a voltage *within* the generator.  
used when several generators are in parallel with a unique step-up transformer.
- $r_c < 0$  and  $x_c < 0 \rightarrow$  regulate a point *beyond* the machine terminals.  
this compensates the voltage drop on the step-up transformer.

## Effect of Saturation of the Field Current



Proposed saturation model





## IEEE Saturation Function (Ceiling)

- A common saturation function has been proposed by the IEEE:

$$S_e(v_f) = A_e e^{B_e |v_f|}$$

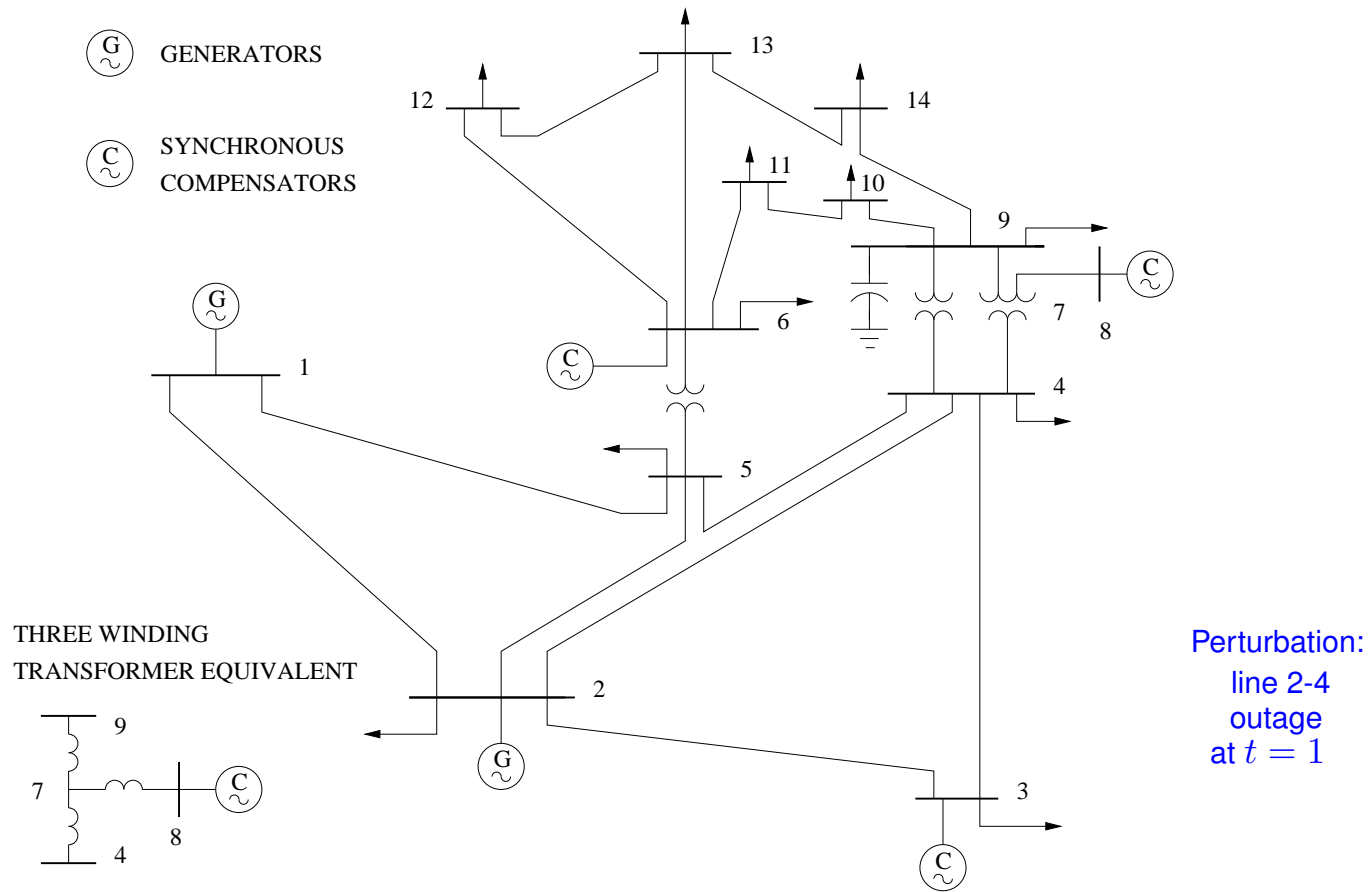
- Coefficients  $A_e$  and  $B_e$  are determined from two points of the ceiling function:

$$\begin{pmatrix} S_e^{\max} \\ v_f^{\max} \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} S_e^{0.75 \max} \\ v_f^{0.75 \max} \end{pmatrix}$$

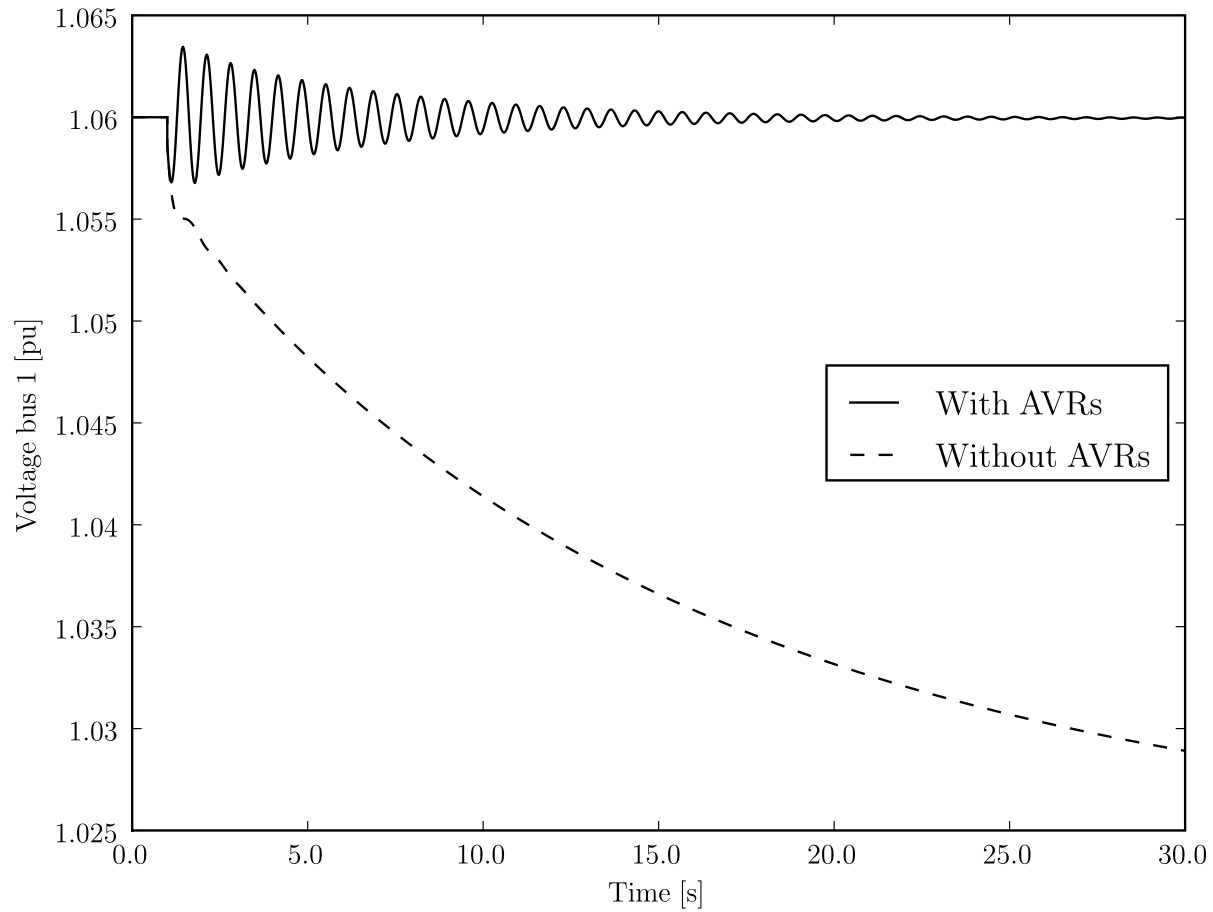
- Typical values are:

$$A_e = 0.0006 = 6 \times 10^{-4} \quad \text{and} \quad B_e = 0.9$$

# Example: IEEE 14-bus System

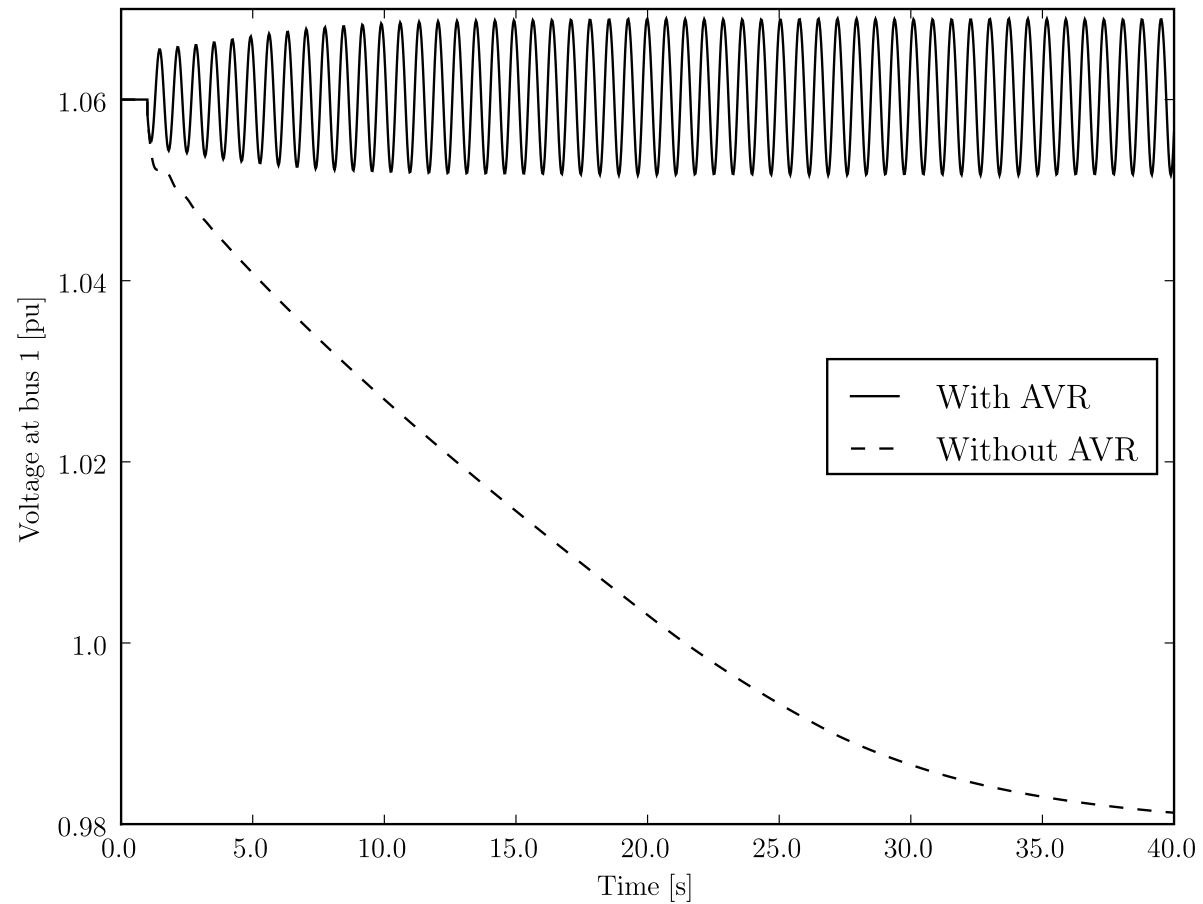


## IEEE 14-bus System - Base Case Loading

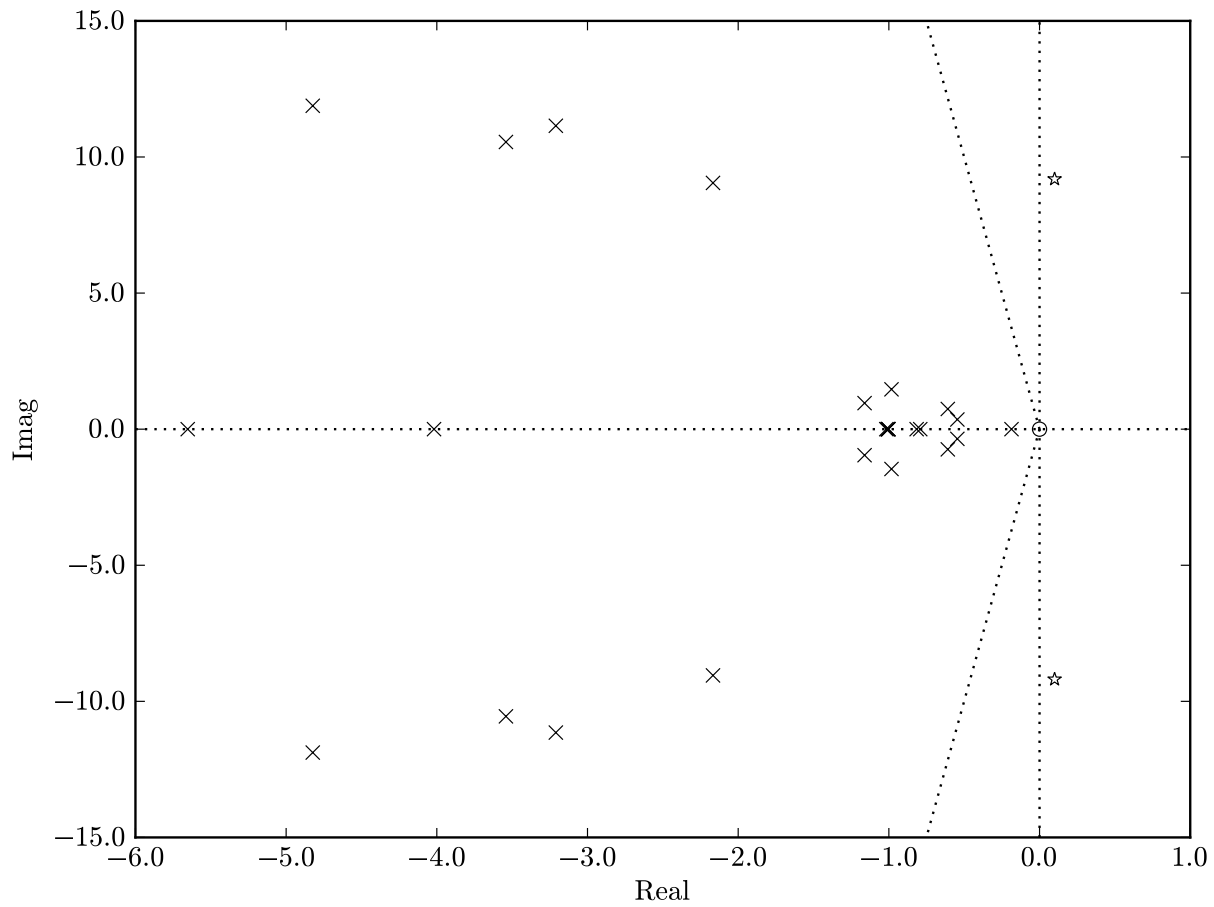




## IEEE 14-bus System - 120% of Base Case Loading



## IEEE 14-bus System - Root Loci 120% of Base Case Loading



## Power System Stabiliser (PSS)

- Let's consider the electro-mechanical model of the machine:

$$2H \frac{d\omega}{dt} = p_m - p_e(\delta)$$

where

$$p_e(\delta) = \frac{e'_q v_h}{x'_d} \sin(\delta - \theta_h)$$

and differentiating:

$$2H s \Delta\omega = -\frac{\partial p_e}{\partial \delta} \Delta\delta - \frac{\partial p_e}{\partial e'_q} \Delta e'_q - \frac{\partial p_e}{\partial v_h} \Delta v_h$$

if  $e'_q$  and  $v_h$  are constant, we have:

$$2H s \Delta\omega = -\frac{\partial p_e}{\partial \delta} \Delta\delta = -K \Delta\delta$$

## Power System Stabiliser (PSS)

- where

$$K = \frac{e'_q v_h}{x'_d} \cos(\delta_0 - \theta_0)$$

- Since  $\Delta\omega = s\Delta\delta$ , we have:

$$2Hs^2\Delta\delta + K\Delta\delta = 0$$

- which has a pair of pure complex eigenvalues (if there is no damping):

$$\lambda_{1,2} = \pm j\sqrt{\frac{K}{2H}}$$

## Power System Stabiliser (PSS)

- The PSS allows imposing:

$$\Delta e'_q = K_1 s \Delta \delta$$

- Hence, we have:

$$2Hs\Delta\omega = -K\Delta\delta - K_\omega s\Delta\delta$$

where:

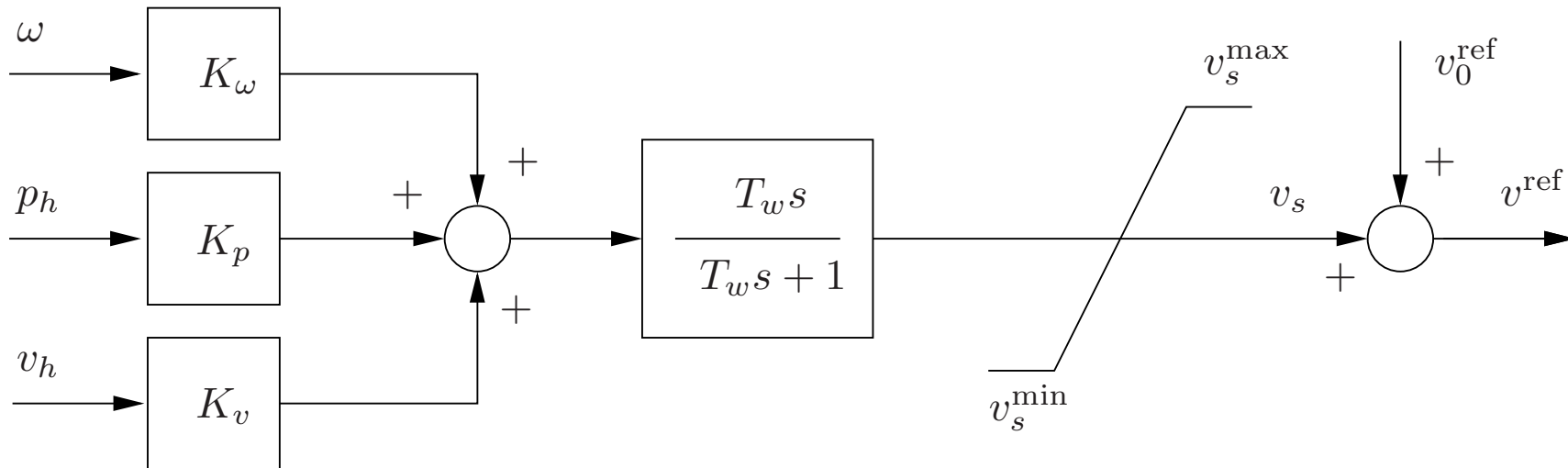
$$K_\omega = K_1 \frac{\partial p_e}{\partial e'_q}$$

so:

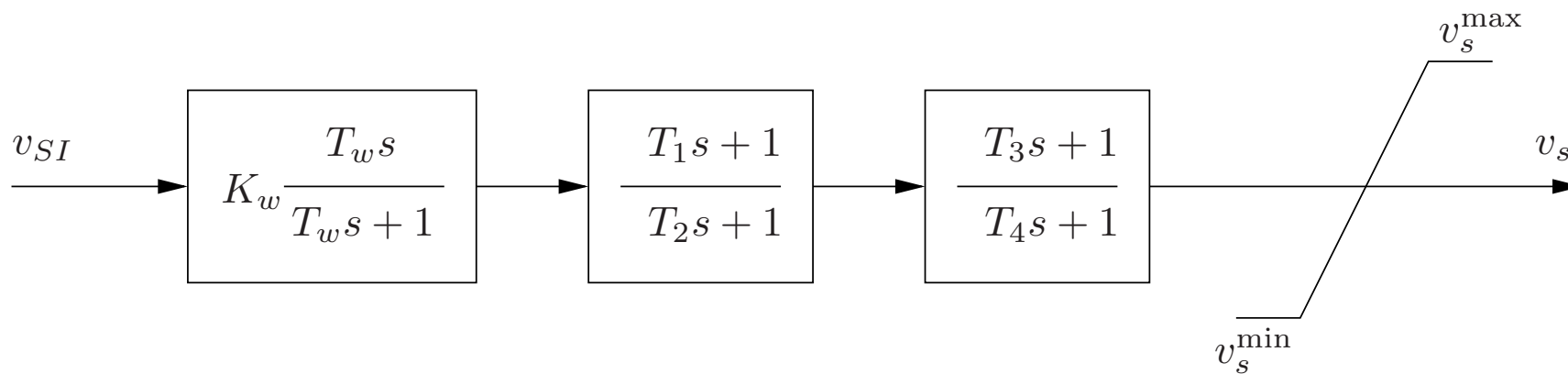
$$2Hs^2\Delta\delta + K_\omega s\Delta\delta + K\Delta\delta = 0 \quad \Rightarrow \quad \lambda_{1,2} = -\frac{K_\omega}{4H} \pm j \frac{\sqrt{8KH - K_\omega^2}}{4H}$$



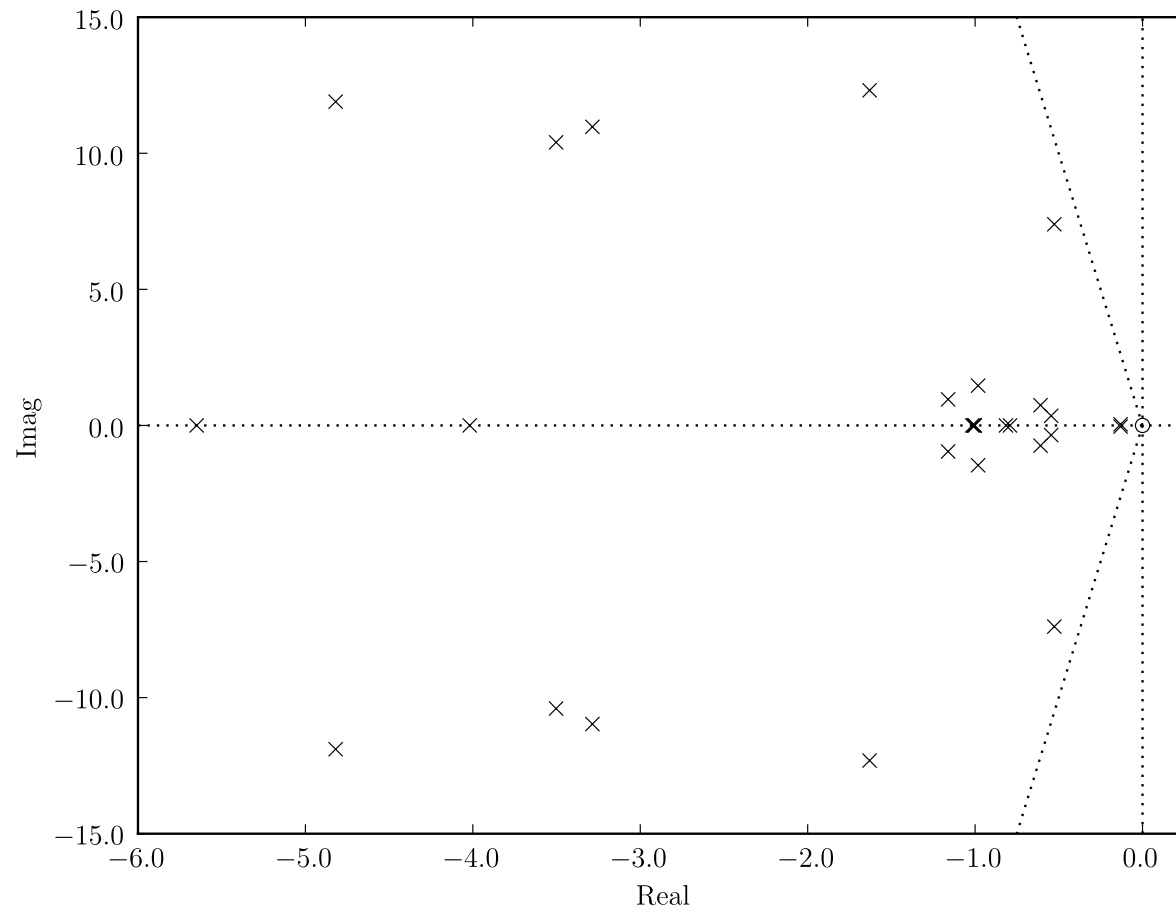
## Typical PSS Scheme (I)



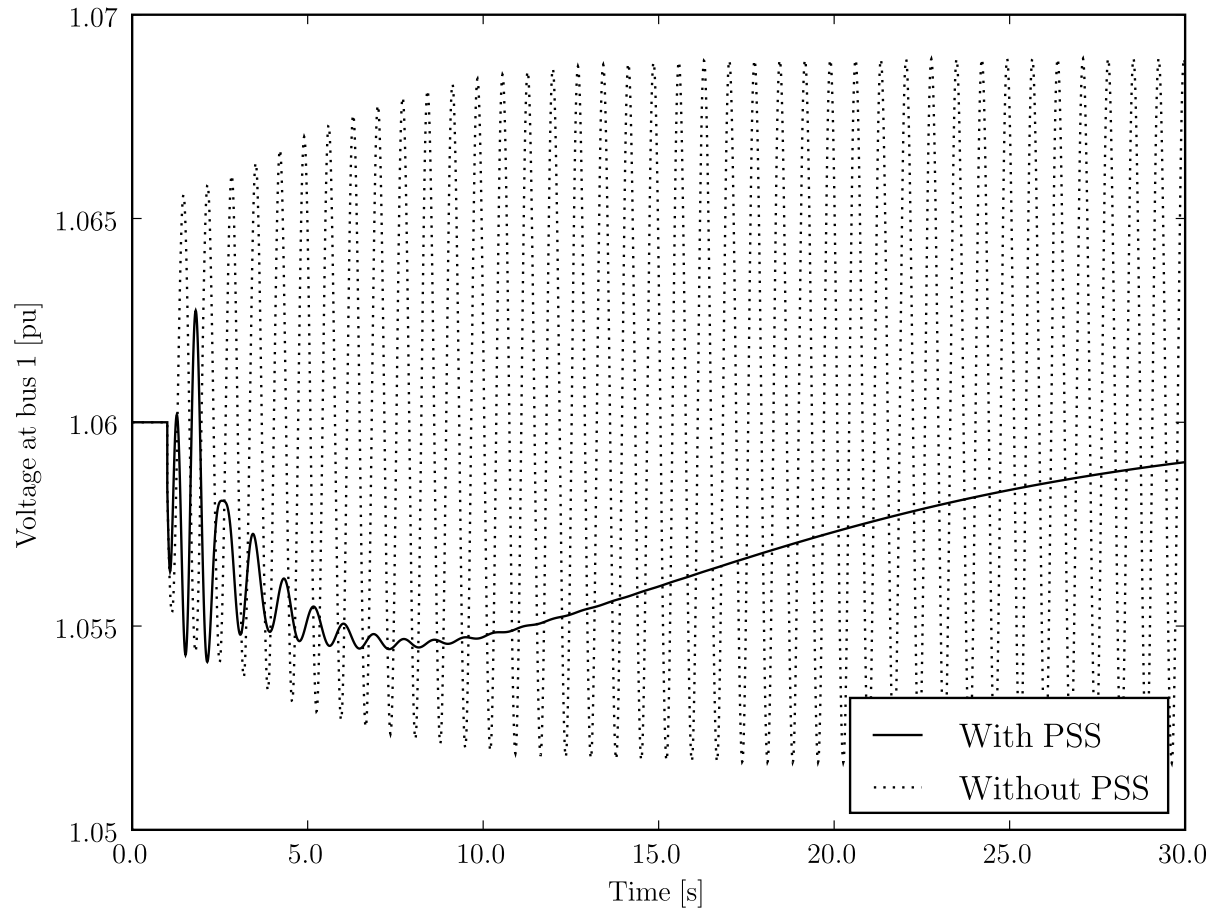
## Typical PSS Scheme (II)



## IEEE 14-bus System - Root Loci 120% Loading + PSS

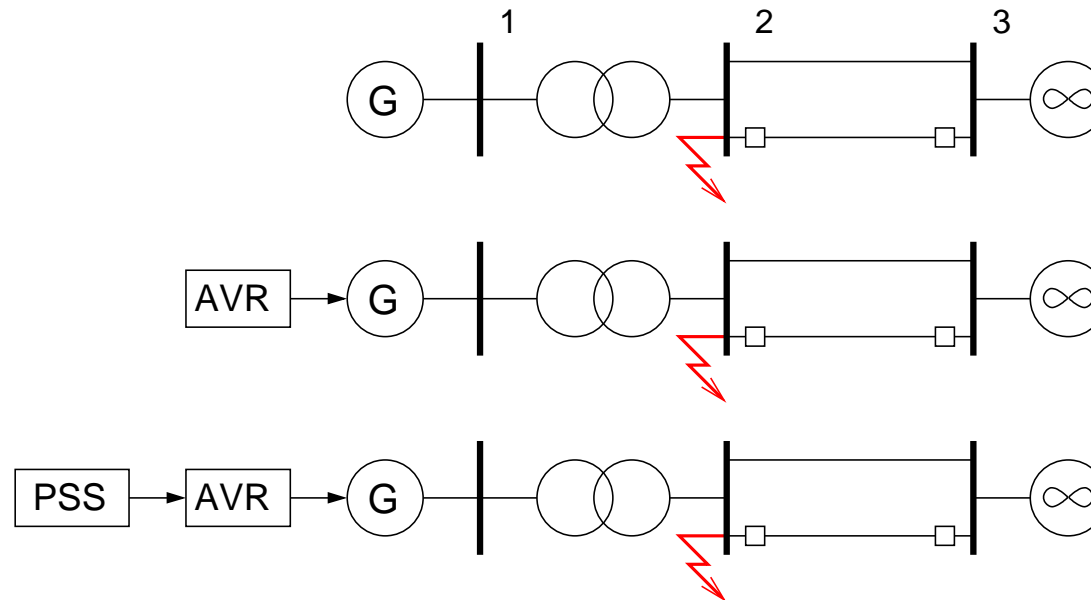


## IEEE 14-bus System - Line 2-4 Outage + PSS



## Example: OMIB with Different Control Schemes (I)

- Let consider the following one-machine infinite-bus system with different control schemes.

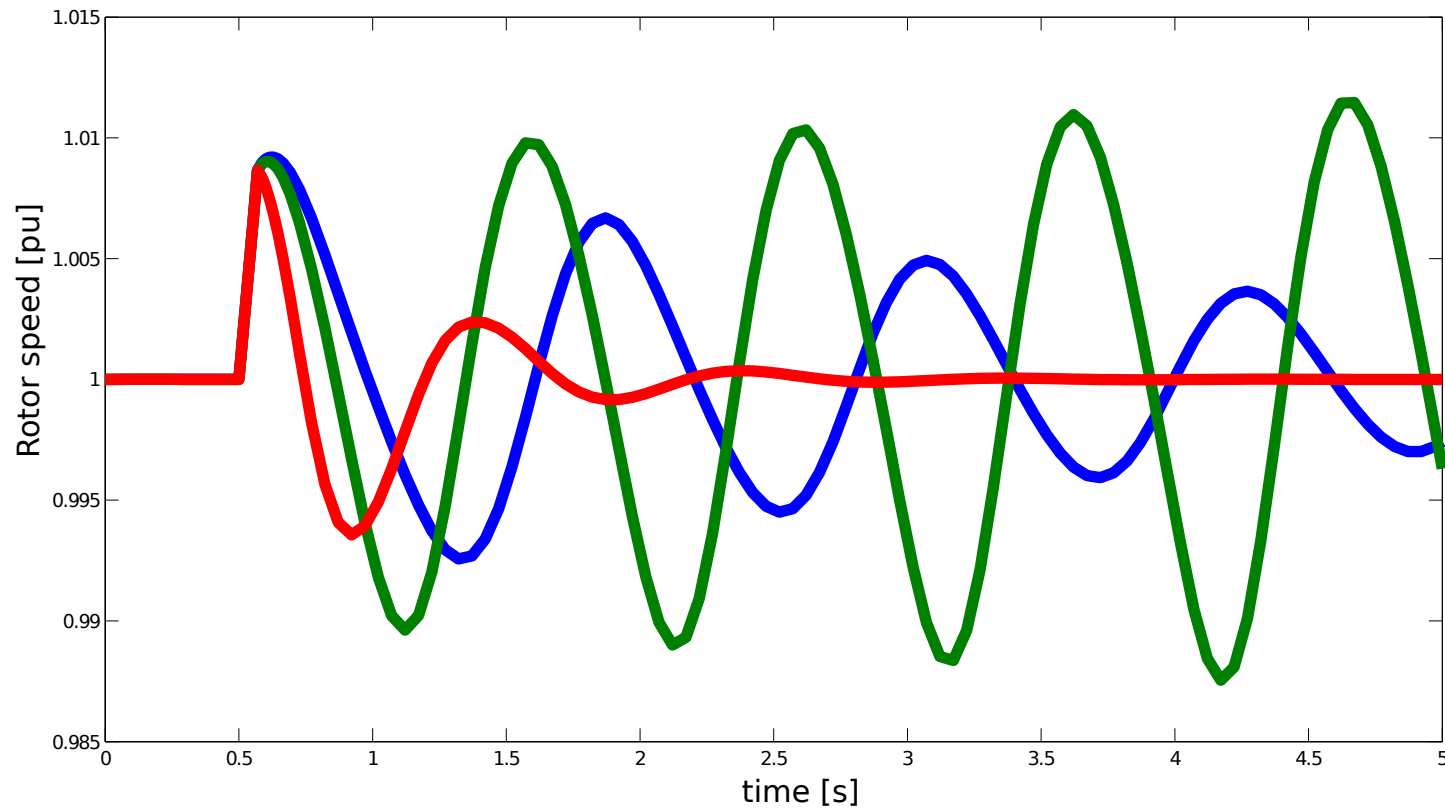


P. Kundur, "Power System Stability and Control," Example 13.2, pp. 864-869.

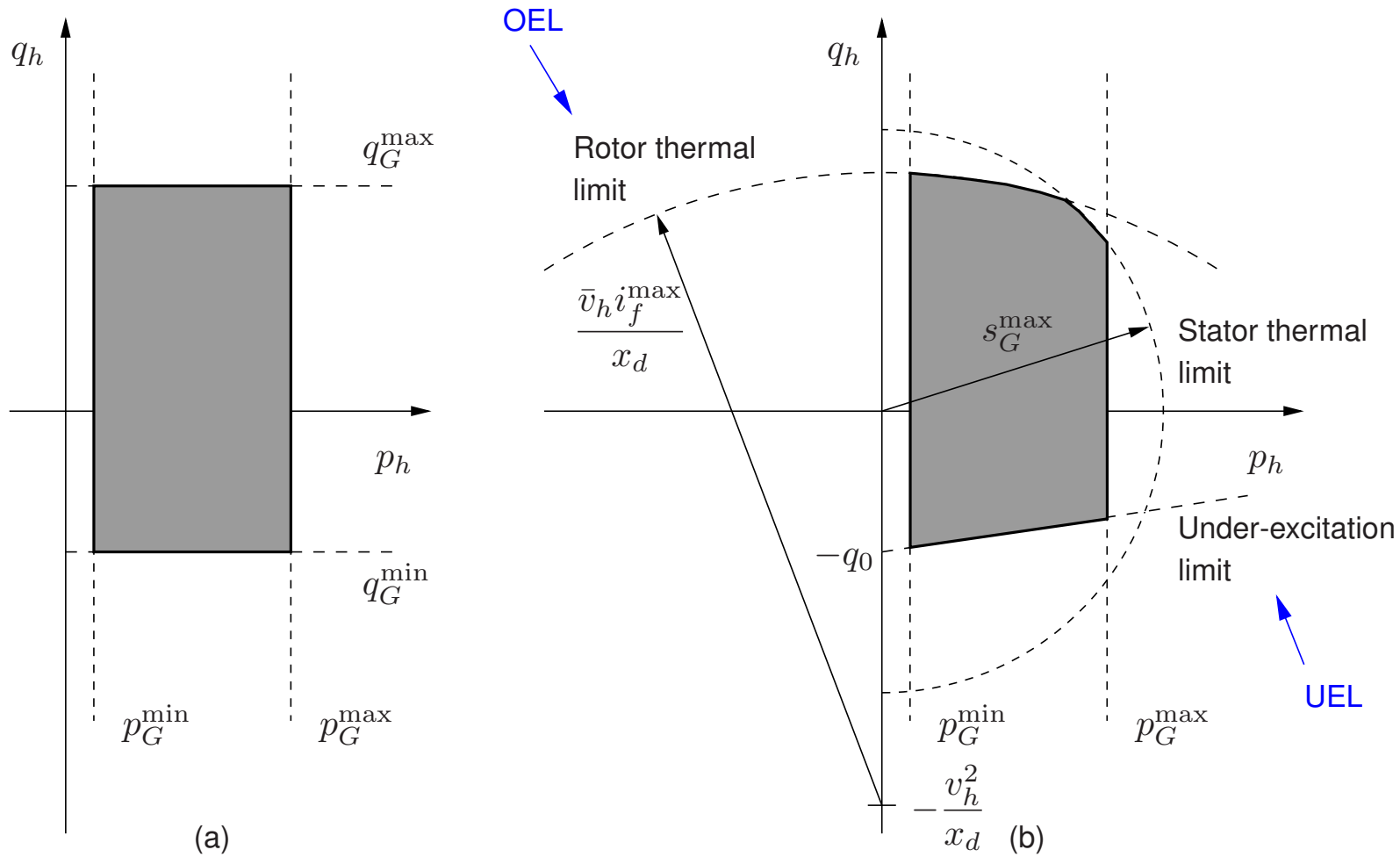
- A three-phase fault occurs close to bus 2 at  $t = 0.5$  s and is cleared at  $t = 0.57$  s.
- The fault is cleared by tripping one of the two lines that connect buses 2 and 3.

## Example: OMIB with Different Control Schemes (II)

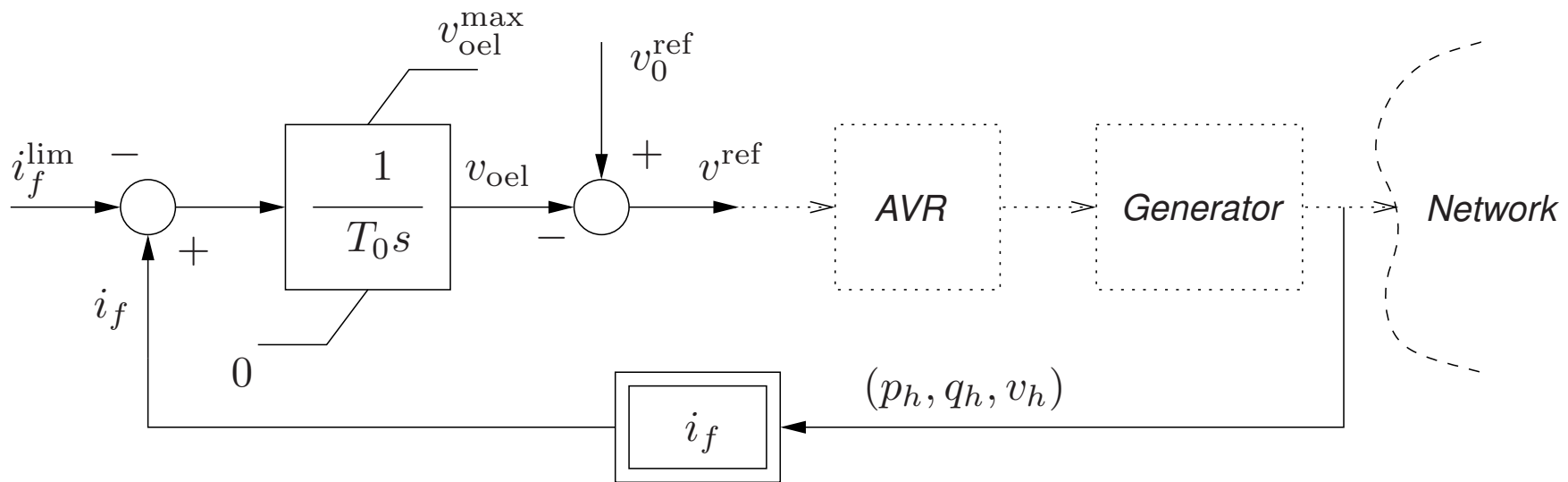
- Identify to which control scheme each rotor speed trajectory corresponds.



# Generator Capability Curve



## Over-Excitation Limiter (OEL)





## General Expression for the Field Current

- In principle the expression of the field current depends on the machine model.
- However, if one utilizes an implicit formulation, it is possible to derive a model-independent expression, as follows:

$$T'_{d0} \dot{e}'_q = v_f - \hat{x}_d i_f$$

where  $\hat{x}_d = x_d - x_\ell$ , and  $x_\ell$  is the leakage reactance.

- The expression above is *implicit* as the field current  $i_f$  depends on the first time derivative of  $e'_q$ .
- Note that the expression above is *always* valid, thus even in case of saturation of the machine fluxes.

## Estimation of the Field Current

- For DC-machine exciters the  $i_f$  can be measured using an ammeter.
- However, other exciter technologies prevent measuring  $i_f$  (e.g. brushless).
- An approximated formula to estimate  $i_f$  is:

$$i_f = \sqrt{(v_h + \gamma_q)^2 + p_h^2} + \left( \frac{x_d}{x_q} + 1 \right) \frac{\gamma_q(v_h + \gamma_q) + \gamma_p^2}{\sqrt{(v_h + \gamma_q)^2 + p_h^2}}$$

where:

$$\gamma_p = \frac{x_q p_h}{v_h} \quad \text{and} \quad \gamma_q = \frac{x_q q_h}{v_h}$$

## Under-Excitation Limiter (UEL)

