Regulating Transformers

POWER SYSTEM MODELLING AND CONTROL (EEEN40550)

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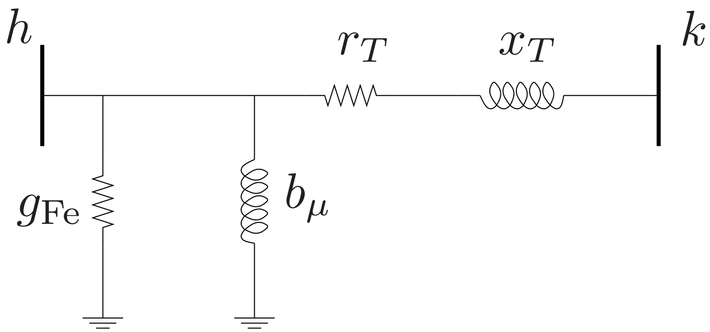
Transformer Model

- We consider the two-winding transformer first.
- Two-winding transformers can be modelled as transmission lines were:

$$
\begin{aligned}
r_L &= r_T & \qquad x_L &= x_T \\
b_{L,h} &= b_\mu & \qquad g_{L,h} &= g_{\mathrm{Fe}} \\
b_{L,k} &= 0 & \qquad g_{L,k} &= 0
\end{aligned}
$$

- The resulting circuit is:



- Observe that $b_\mu < 0$ since the iron case is inductive.

Off-Nominal Tap Ratio Transformers

- From the modelling viewpoint, the main difference between transmission lines and tranformers is that transformers can show a complex off-nominal tap ratio:

$$me^{j\phi}$$

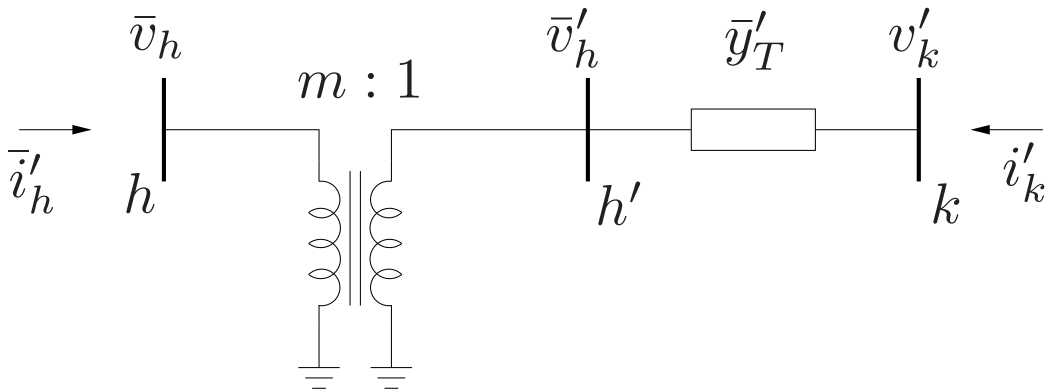
- Under load tap changers (ULTC) have $m \neq 1$ and $\phi = 0$
- Phase shifting transformers (PHS) have $m \approx 1$ and $\phi \neq 0$
- Even fixed-ratio transformers allow varying the tap ratio off-line, hence, in general $m \neq 1$ in most transformers.

Equivalent Circuit of a ULTC (I)

- The standard model of a single-phase transformer assumes a nominal ratio $1 : 1$.

- The off-nominal ratio can be modelled as $m : 1$, where $0.8 \leq m \leq 1.2$.

- Let's consider the following equivalent circuit:



Equivalent Circuit of a ULTC (II)

- Current injections are:

$$\bar{i}'_h = \frac{1}{m}\bar{y}'_T(\bar{v}'_h - \bar{v}'_k) = \frac{1}{m}\bar{y}'_T(\frac{\bar{v}_h}{m} - \bar{v}'_k)$$

$$\bar{i}'_k = \bar{y}'_T(\bar{v}'_k - \bar{v}'_h) = \bar{y}'_T(\bar{v}'_k - \frac{\bar{v}_h}{m})$$

where

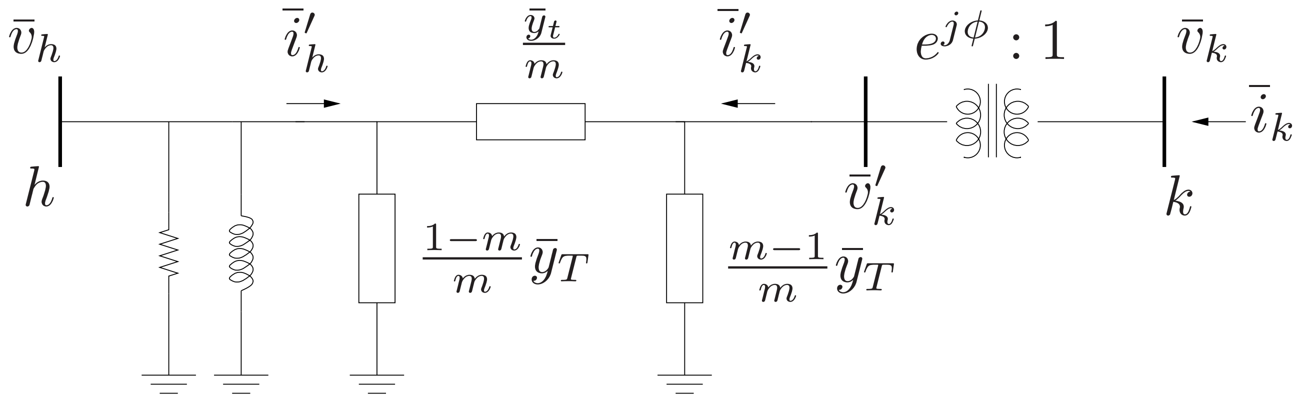
$$\bar{y}'_T = \bar{y}_T = (r_T + jx_T)^{-1} \quad , \quad \bar{v}'_h = \frac{\bar{v}_h}{m} \quad \text{and} \quad \bar{v}'_k = \bar{v}_k e^{j\phi}$$

($\phi = 0 \Rightarrow$ there is no phase shift)

- In vectorial form:

$$\begin{bmatrix} \bar{i}'_h \\ \bar{i}'_k \end{bmatrix} = \bar{y}'_T \begin{bmatrix} \frac{1}{m^2} & -\frac{1}{m} \\ -\frac{1}{m} & 1 \end{bmatrix} \begin{bmatrix} \bar{v}'_h \\ \bar{v}'_k \end{bmatrix}$$

Equivalent Circuit of a ULTC (III)



- Power injections:

$$p_h = v_h^2(g_{fe} + \frac{g_T}{m^2}) - v_h v_k(g_T \cos(\theta_{hk} - \phi) + b_T \sin(\theta_{hk} - \phi))/m$$

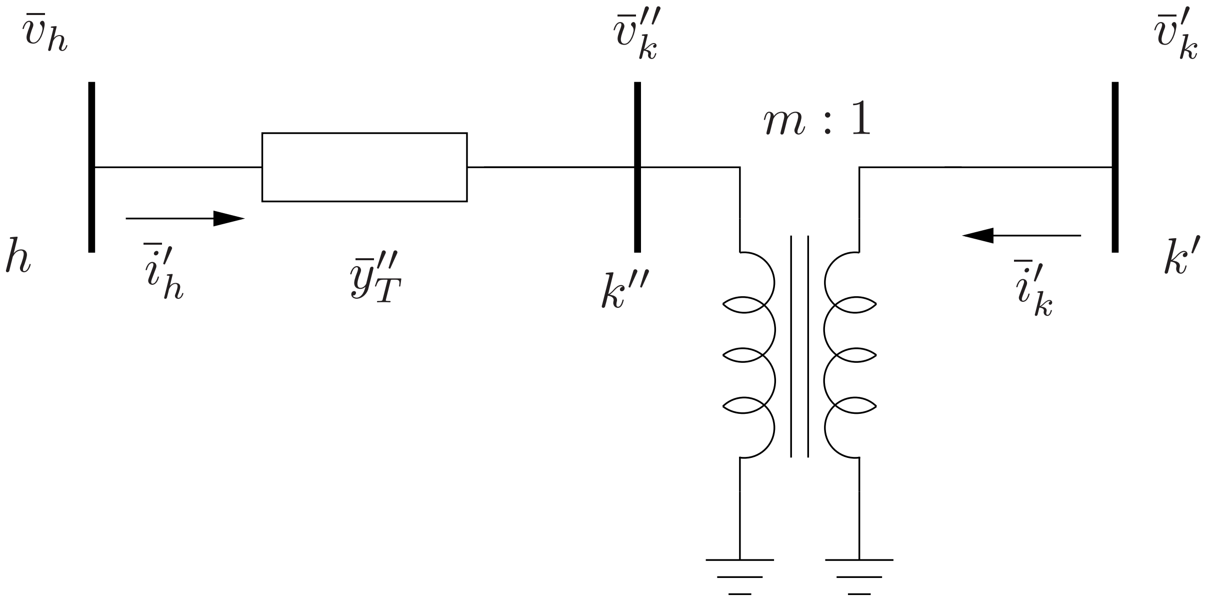
$$q_h = -v_h^2(b_\mu + \frac{b_T}{m^2}) - v_h v_k(g_T \sin(\theta_{hk} - \phi) - b_T \cos(\theta_{hk} - \phi))/m$$

$$p_k = v_k^2 g_T - v_h v_k(g_T \cos(\theta_{hk} - \phi) - b_T \sin(\theta_{hk} - \phi))m$$

$$q_k = -v_k^2 b_T - v_h v_k(g_T \sin(\theta_{hk} - \phi) + b_T \cos(\theta_{hk} - \phi))m$$

Equivalent Circuit of a ULTC (IV)

- An alternative circuit model is the following:



- where:

$$\bar{i}'_h = \bar{y}''_T(\bar{v}_h - \bar{v}''_k) = \bar{y}''_T(\bar{v}_h - m\bar{v}'_k)$$

$$\bar{i}'_k = m\bar{y}''_T(\bar{v}''_k - \bar{v}_h) = m\bar{y}''_T(m\bar{v}'_k - \bar{v}_h)$$

Equivalent Circuit of a ULTC (V)

- where

$$\bar{y}_T'' = \frac{\bar{y}_T}{m^2} \text{ and } \bar{v}_k'' = m\bar{v}_k'$$

- so we have:

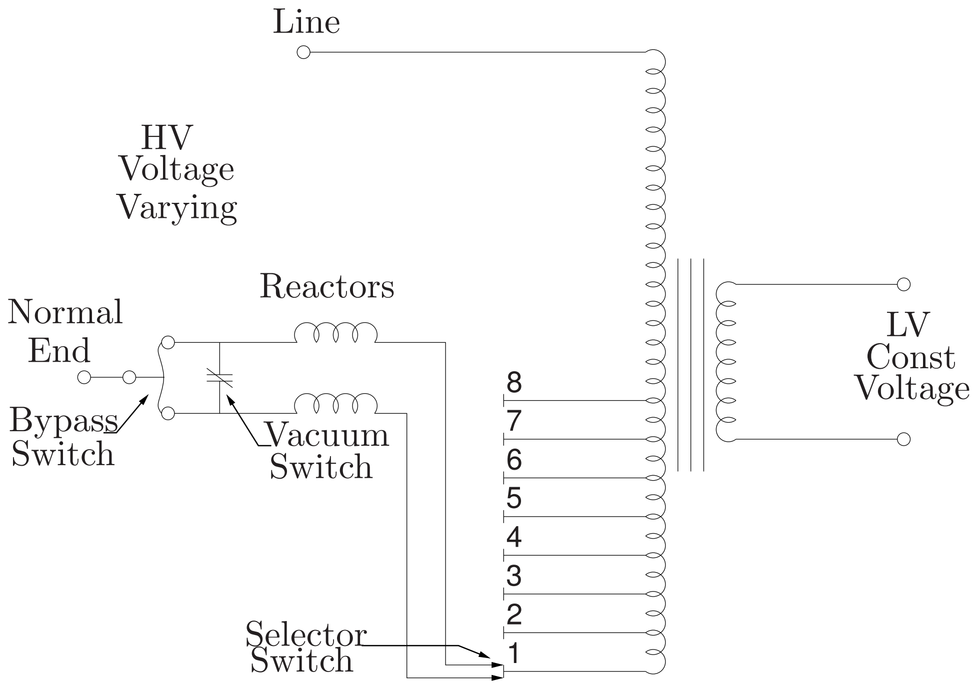
$$\begin{bmatrix} \bar{i}_h' \\ \bar{i}_k' \end{bmatrix} = \bar{y}_T' \begin{bmatrix} 1 & -m \\ -m & m^2 \end{bmatrix} \begin{bmatrix} \bar{v}_h \\ \bar{v}_k' \end{bmatrix}$$

- Observe that we assumed that the tap is on the primary winding.
- If the tap is on the secondary winding: $\tilde{m} = \frac{1}{m}$

  then we have:

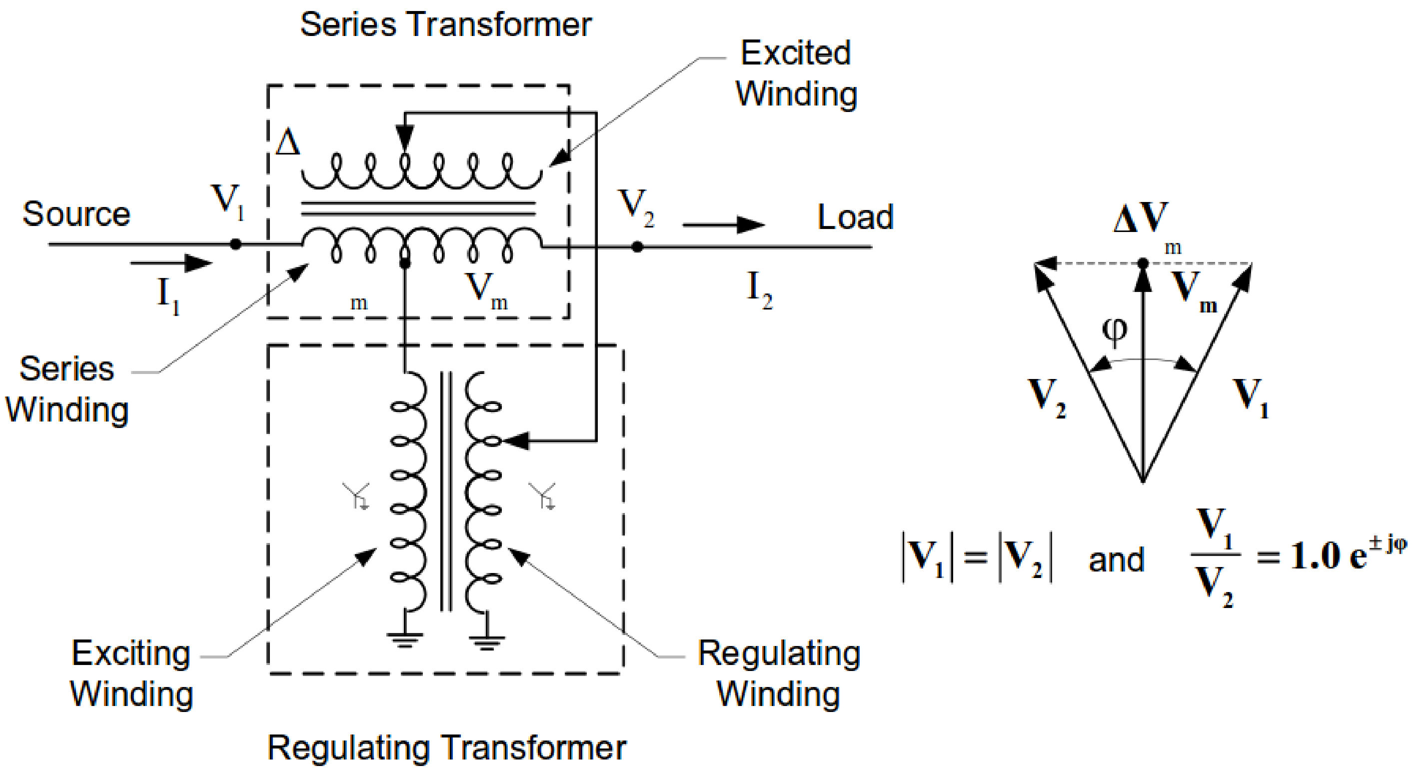
$$\bar{y}_T' = \frac{\bar{y}_T}{\tilde{m}^2} \quad \text{and} \quad \bar{y}_T'' = \bar{y}_T \qquad \#$$

Technical Insight of a ULTC



- Switches can be mechanical (driven by a motor) or based on power electronic devices.

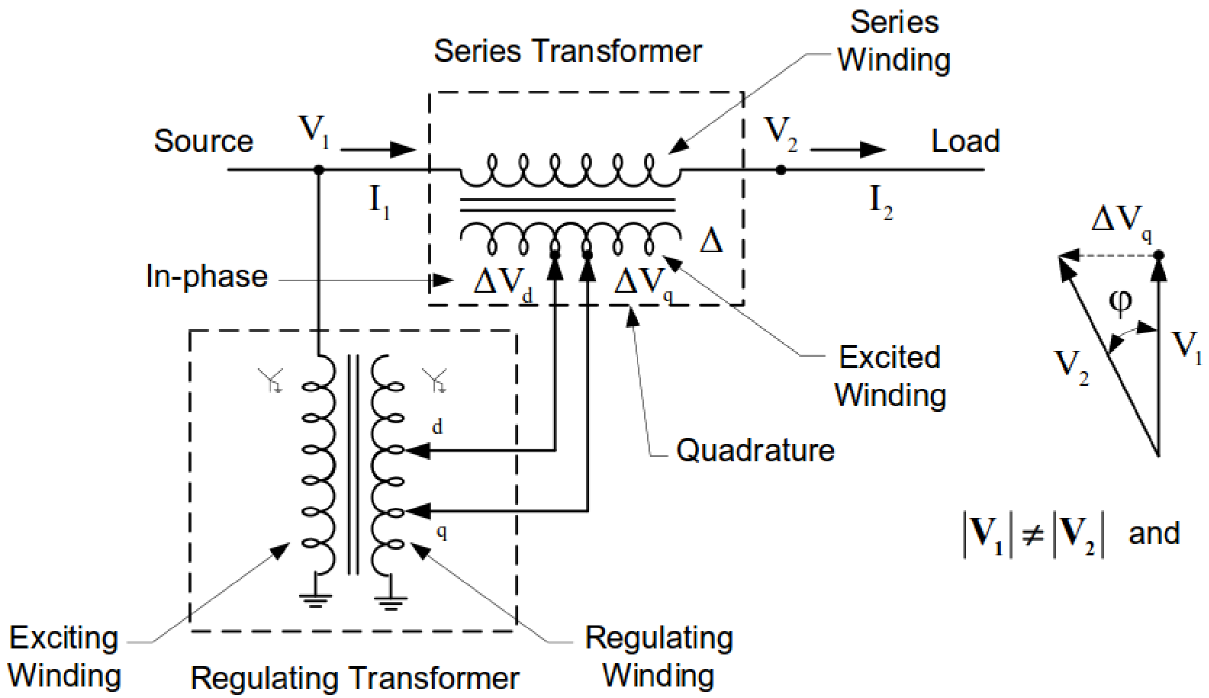
Symmetric Phase-Shifting Transformer



$$|V_1| = |V_2| \quad \text{and} \quad \frac{V_1}{V_2} = 1.0\, e^{\pm j\varphi}$$

Figure 1 - Symmetric Phase-shifting Transformer

Asymmetric Phase-Shifting Transformer



$$|\mathbf{V_1}| \neq |\mathbf{V_2}| \text{ and } \frac{\mathbf{V_1}}{\mathbf{V_2}} = \mathbf{t}\, \mathbf{e}^{\pm j\varphi}$$

Figure 2 - Asymmetric Phase-shifting Transformer

Control Models of ULTC

- The control is necessarily discrete as the tap ratio cannot catch fractional turns of the windings.
- However, if the number of tap positions is ”high” and the Δv due to a tap variation of one turn is ”small”, we can use a continuous model.
- We can find, in the literature, two kinds of control models:
  - continuous
  - discrete

Simplified Discrete Control Model of a ULTC

- Neglecting time delays, we have:

$$\alpha^{(i)} = \alpha(\Delta v^{(i)}, m^{(i-1)}, \mathrm{db}_v, m^{\max}, m^{\min}) = \begin{cases} 1, & \text{if} \quad \Delta v^{(i)} > \mathrm{db}_v \quad \text{and} \quad m^{(i-1)} < m^{\max} \\ -1, & \text{if} \quad \Delta v^{(i)} < -\mathrm{db}_v \quad \text{and} \quad m^{(i-1)} > m^{\min} \\ 0, & \text{otherwise,} \end{cases}$$

where i stands for a generic i-th iteration of a Newton-Raphson method and

$$\Delta v^{(i)} = v_k^{(i)} - v^{\mathrm{ref}}.$$

The tap ratio is updated as follows:

$$m^{(i)} = m^{(i-1)} + \alpha^{(i)} \Delta m.$$

and db_v is a *dead band* to avoid unnecessary fluctuations of the tap.

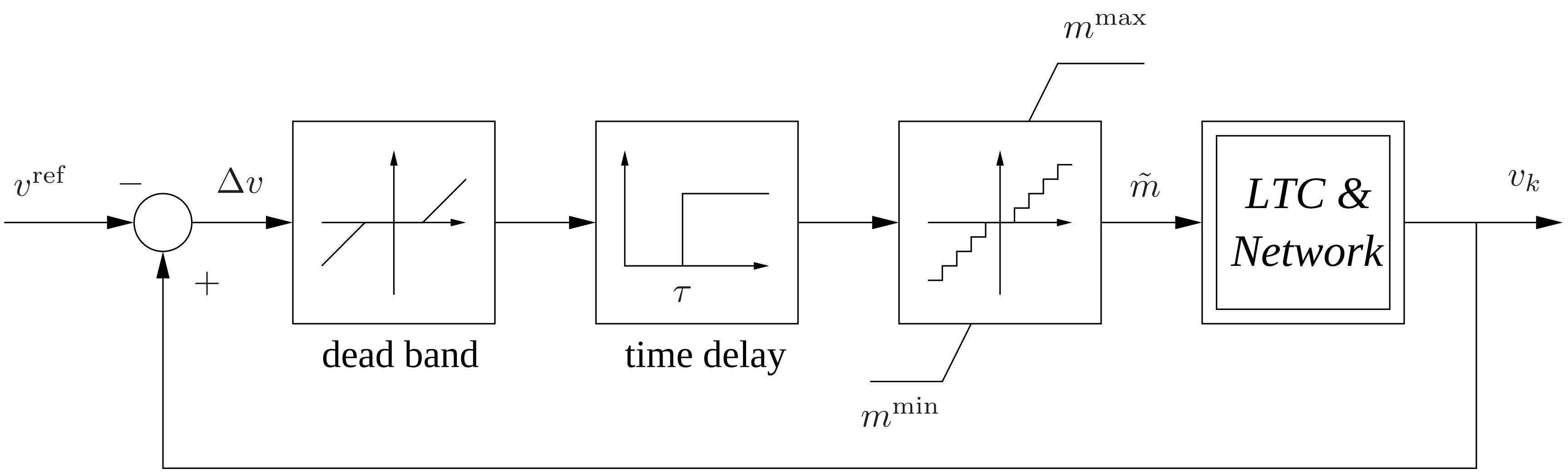
Detailed Discrete Control Model of a ULTC

$$
\begin{aligned}
&e(\Delta v(t), m(t-\Delta t), \mathrm{db}_v, m^{\max}, m^{\min}) \\
&\qquad = \begin{cases} 1, & \text{if} \quad \Delta v(t) > \mathrm{db}_v \quad \text{and} \quad m(t-\Delta t) < m^{\max} \\ -1, & \text{if} \quad \Delta v(t) < -\mathrm{db}_v \quad \text{and} \quad m(t-\Delta t) > m^{\min} \\ 0, & \text{otherwise,} \end{cases}
\end{aligned}
$$

$$
c(e(t), c(t-\Delta t)) = \begin{cases} c(t-\Delta t) + \Delta t, & \text{if} \quad e(t) = 1 \quad \text{and} \quad c(t-\Delta t) \geq 0 \\ c(t-\Delta t) - \Delta t, & \text{if} \quad e(t) = -1 \quad \text{and} \quad c(t-\Delta t) \leq 0 \\ 0, & \text{otherwise,} \end{cases}
$$

$$
f(e(t), c(t), \tau(t)) = \begin{cases} 1, & \text{if} \quad e(t) = 1 \quad \text{and} \quad c(t) > \tau(t) \\ -1, & \text{if} \quad e(t) = -1 \quad \text{and} \quad c(t) < \tau(t) \\ 0, & \text{otherwise.} \end{cases}
$$

Detailed Control Scheme of a ULTC (Discrete Model)



About the Time Delay in the Discrete Model

- In some ULTC models, the time delay is constant:

$$\tau(t) = \tau_0 = \text{constant}$$

- However, in most practical implementations, the higher Δv, the faster the tap ratio changes.

- The most accepted model is thus with a time varying time delay:

$$\tau(t) = \tau_0 \frac{\mathrm{db}_v}{|\Delta v|}$$

About the Time Delay in the Discrete Model

- To avoid numerical issues in the case of $\Delta v = 0$

$$\tau(t) = \begin{cases} \tau_0 \dfrac{\mathrm{db}_v}{|\Delta v|}, & \text{if } |\Delta v| > \mathrm{db}_v \\ \tau_0, & \text{otherwise.} \end{cases}$$

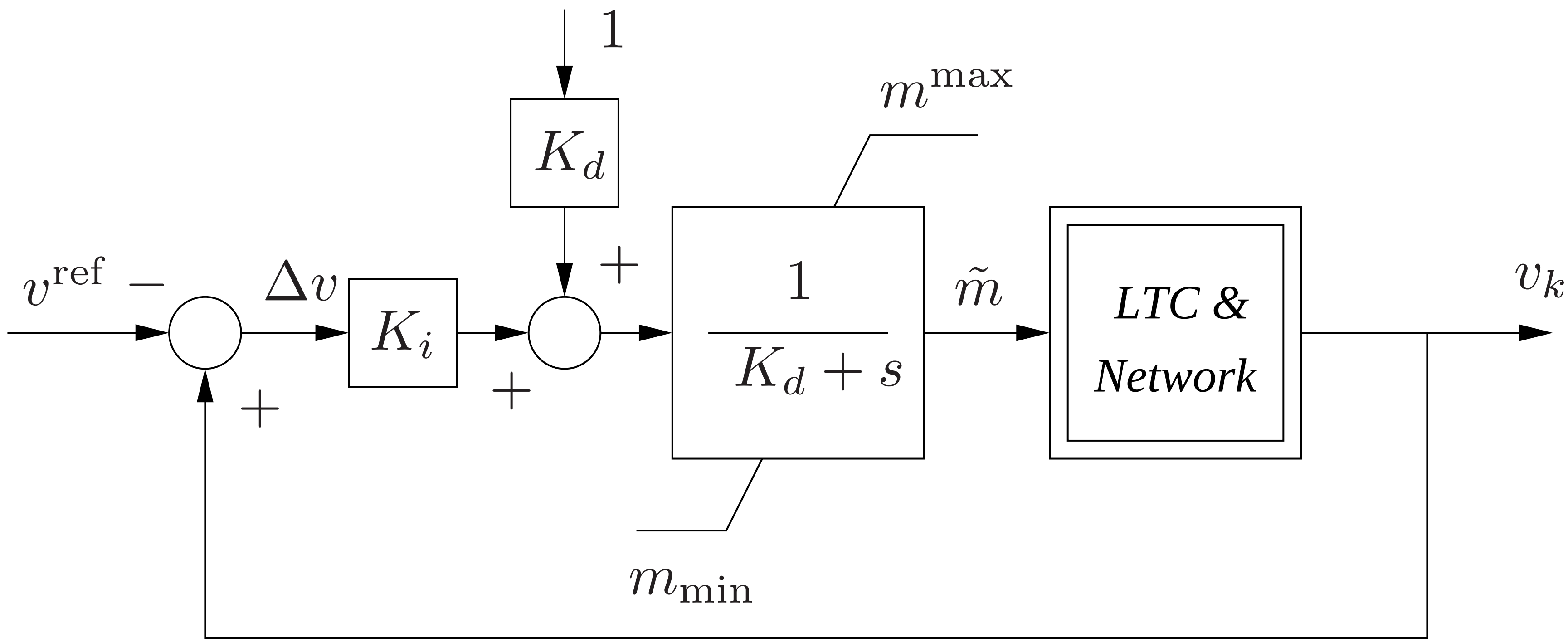
- However, observe that since no action is taken for $|\Delta v| < \mathrm{db}_v$, the value of τ is constant in the case $|\Delta v| \leq \mathrm{db}_v$

Alternative Discrete Control Model

- An alternative control model varies the tap ratio in order to maintain the voltage within a certain range, say $v^{\min} \leq v \leq v^{\max}$:

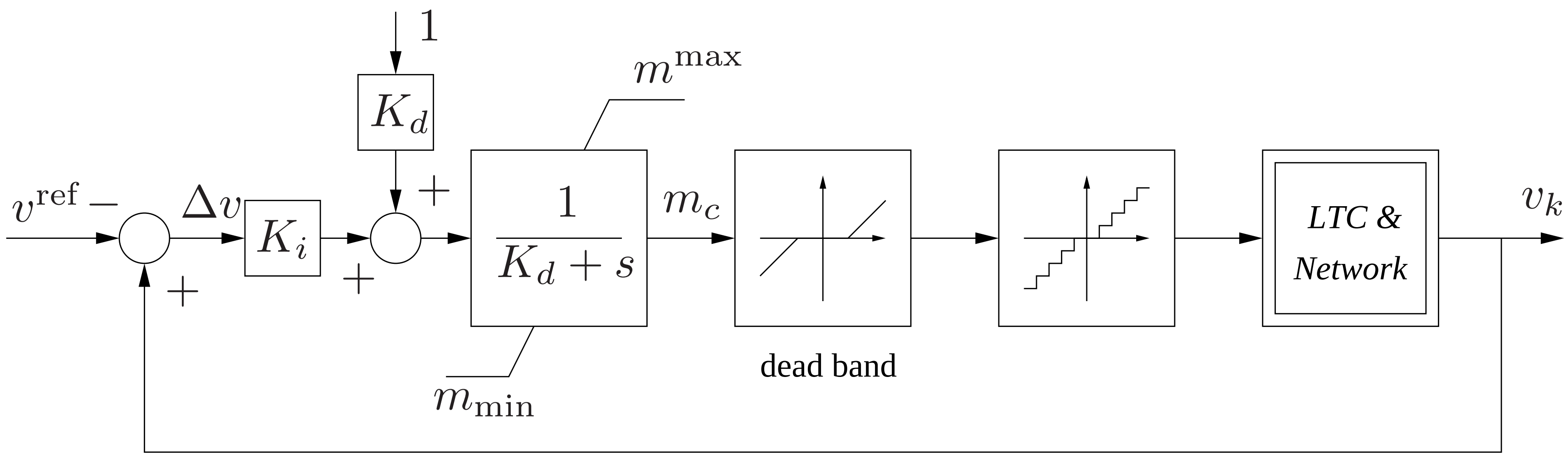
$$\hat{\alpha}^{(i)} = \hat{\alpha}(\Delta v^{(i)}, m^{(i-1)}, \mathrm{db}_v, v^{\max}, v^{\min})$$
$$= \begin{cases} 1, & \text{if} \quad \Delta v_k - v^{\max} > \mathrm{db}_v \quad \text{and} \quad m^{(i-1)} < m^{\max} \\ -1, & \text{if} \quad \Delta v_k - m^{\min} < -\mathrm{db}_v \quad \text{and} \quad m^{(i-1)} > m^{\min} \\ 0, & \text{otherwise.} \end{cases}$$

Continuous Control Model of a ULTC



$$\dot{\tilde{m}} = -K_d(\tilde{m} - 1) + K_i \Delta v$$

Hybrid Control Model of a ULTC



→ m_c = continuous tap ratio

→ m_d = discrete tap ratio

The differential equation is:

$$\dot{m}_c = -K_d(m_c - 1) + K_i \Delta v$$

Hybrid Control Model of a ULTC

- The switching logic of the tap ratio is as follows:

$$\beta(t) = \beta(m_c(t), m_d(t-\Delta t), \mathrm{db}_m)$$
$$= \begin{cases} 1, & \text{if} \quad m_c(t) - m_d(t-\Delta t) > \mathrm{db}_m \\ -1, & \text{if} \quad m_c(t) - m_d(t-\Delta t) < -\mathrm{db}_m \\ 0, & \text{otherwise,} \end{cases}$$

and:

$$m_d(t) = m_d(t-\Delta t) + \beta(t)\Delta m$$

- The actual tap ratio of the transformer is $m = m_d$.

Remarks on the Hybrid Control Model

- The differential equation accounts for the delay. A heuristic rule gives: $\tau_0 \approx \frac{3}{K_i}$.

- The switching logic works for power flow as well as for time domain analysis.

- The dead band db_m has a different meaning as db_v. In fact db_m allows mapping m_c into m_d.

- Observe that m_c is an auxiliary variable (not used outside the ULTC).

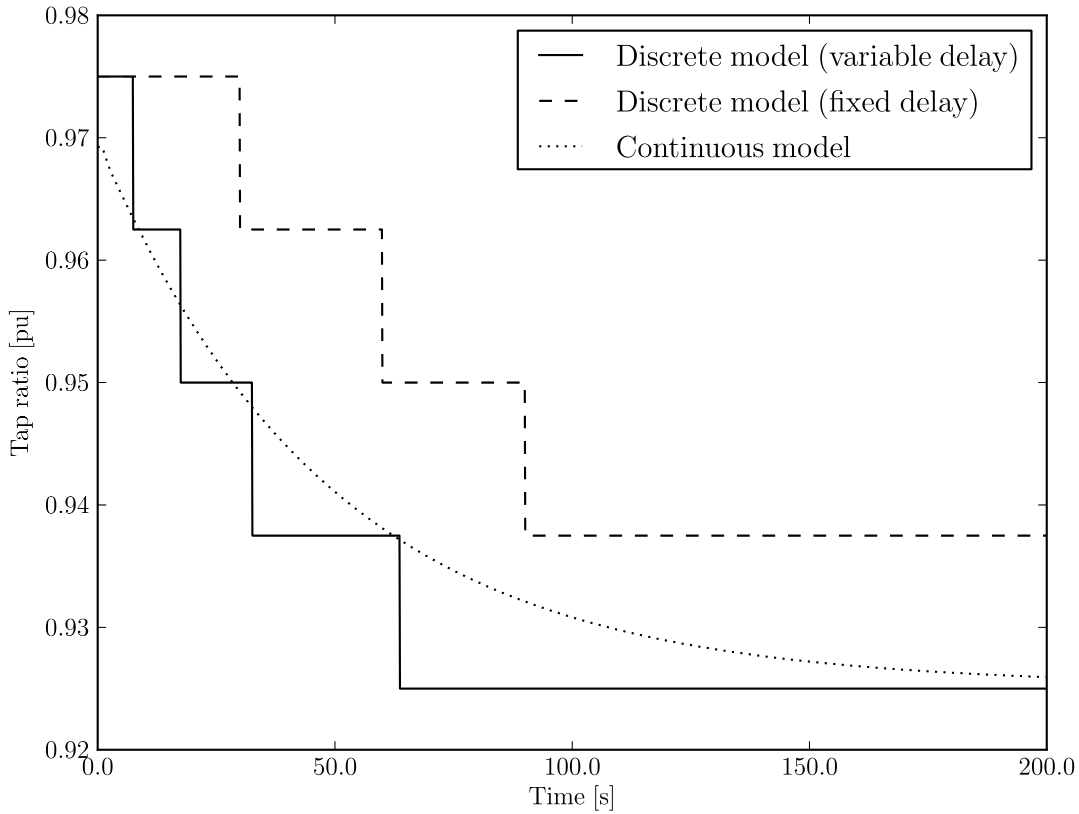
Remarks on the Hybrid Control Model

- The discrete model is closer to the real behavior of the ULTC control, but introduces a time delay (difficult to implement) and a discrete variable. Observe that if we have discrete variables, defining eigenvalues is not possible.

- The continuous models is easier to handle and allows defining the stability of the ULTC controller through the eigenvalue analysis. However, its behaviour is not that of the real controller, which is discrete.

- The hybrid model is slightly more complex but takes the best parts of the discrete and continuous models.

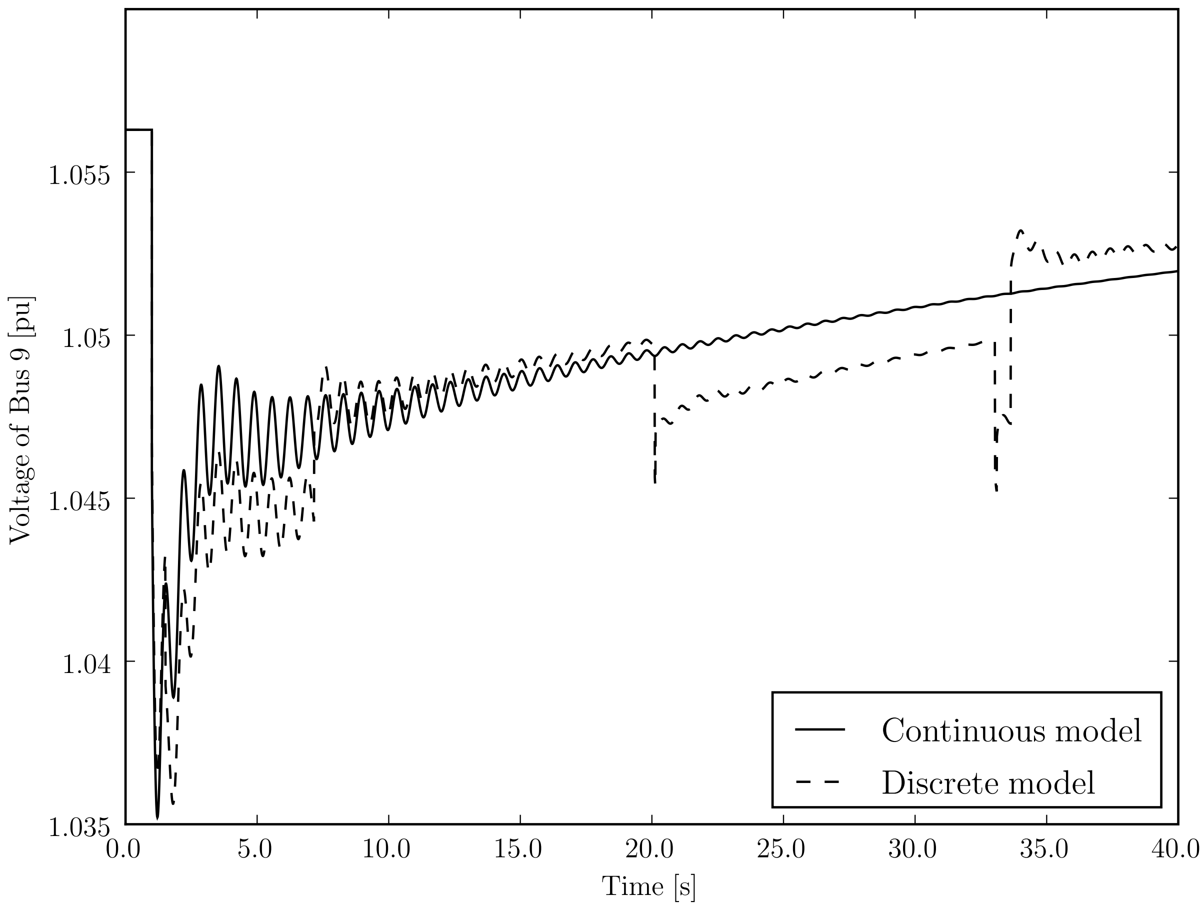
Comparison of the Discrete and Continuous Models



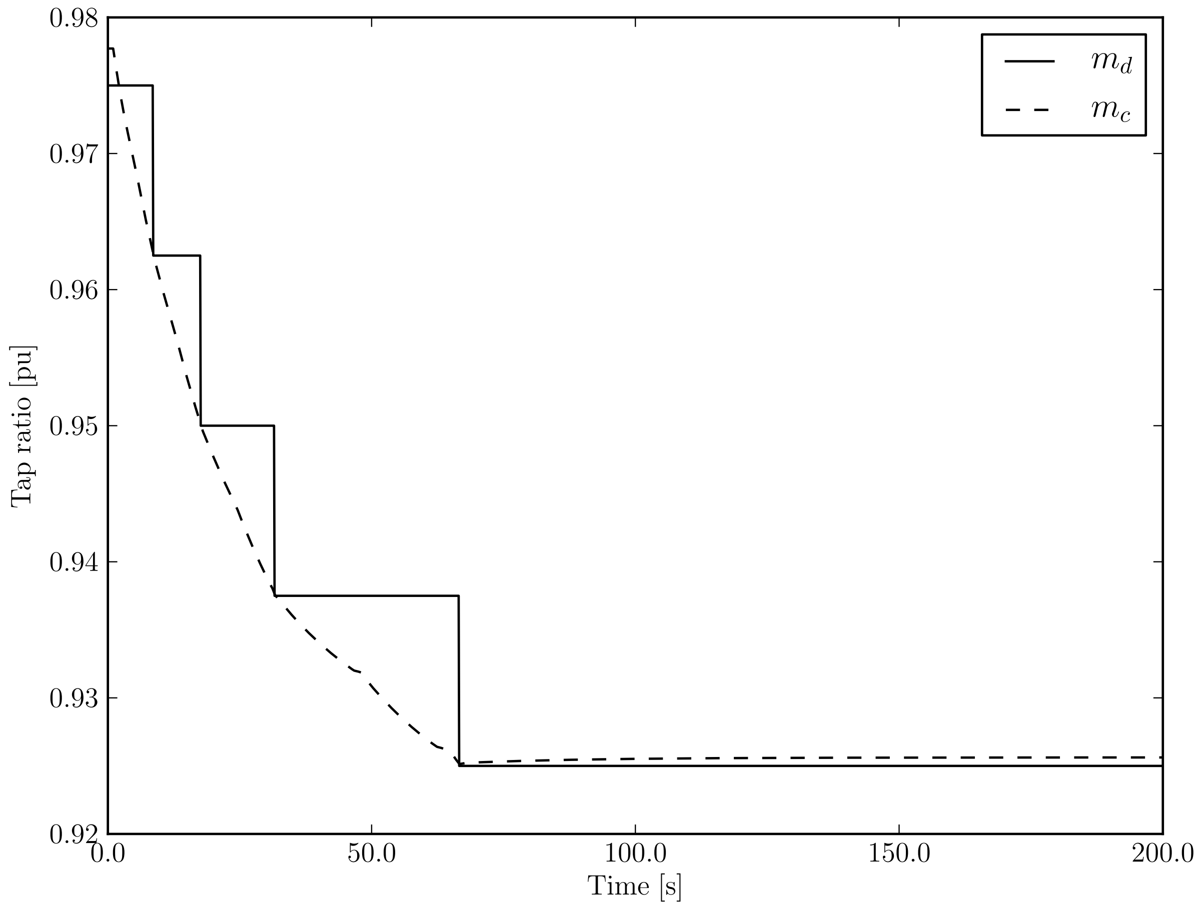
The transformer with ULTC is that connecting buses 4 and 9

The simulation illustrates line 2-4 outage of the IEEE 14-bus system.

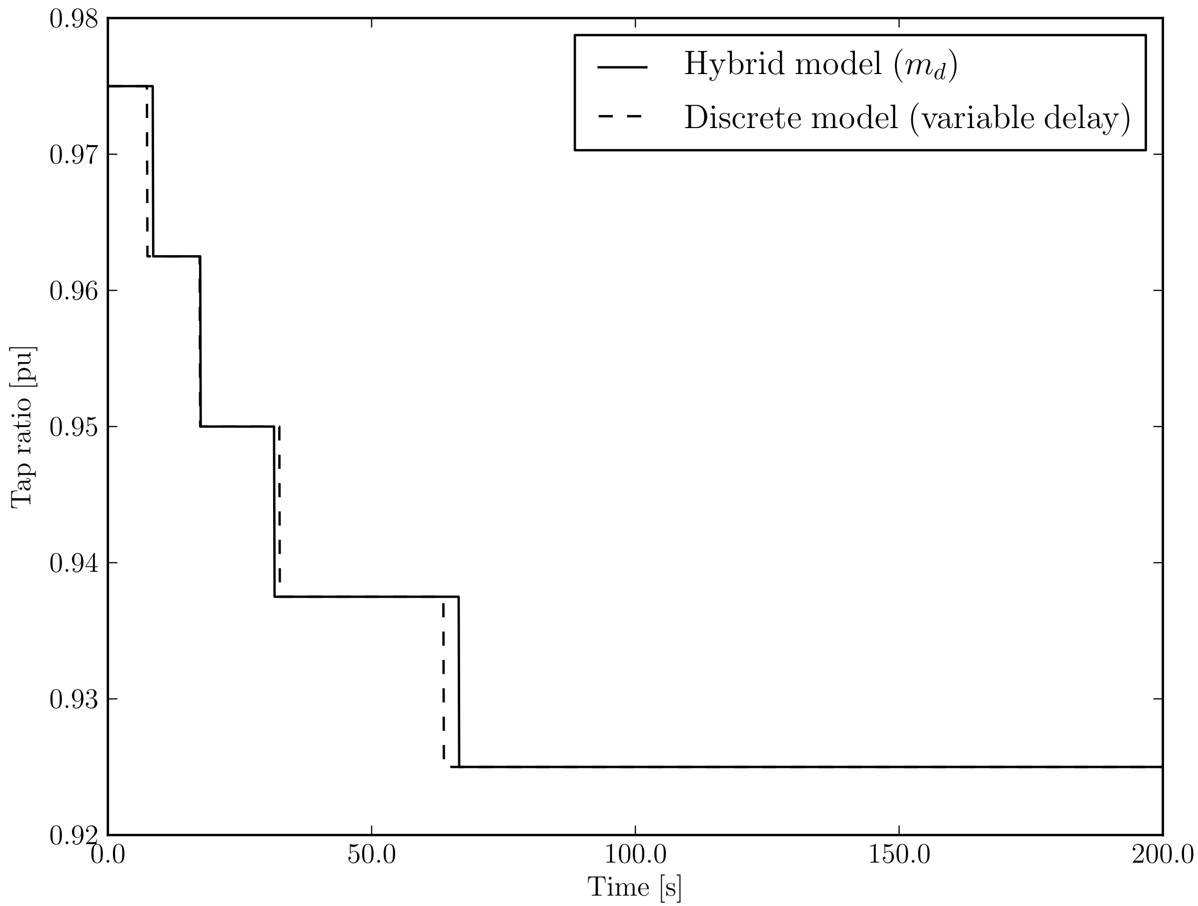
Comparison of the Discrete and Continuous Models



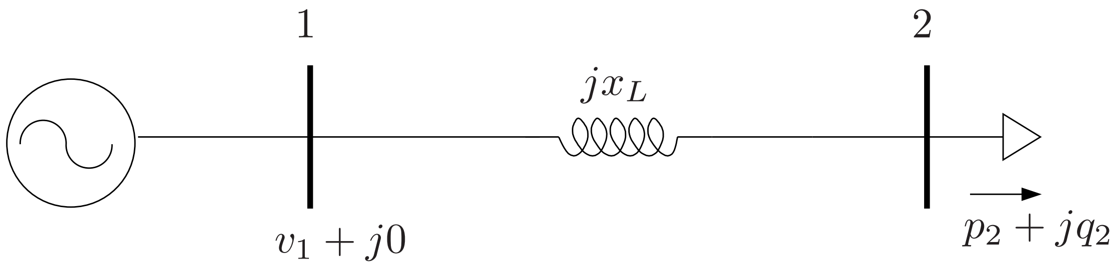
Behaviour of the Hybrid Model



Comparison of the Discrete and Hybrid Models



Qualitative Definition of PV Curves



- The power flow equations are:

$$-p_2 = \frac{v_2 v_1^{\text{ref}}}{x_L} \sin \theta_2$$

$$-q_2 = \frac{v_2^2}{x_L} - \frac{v_2 v_1^{\text{ref}}}{x_L} \cos \theta_2$$

Qualitative Definition of PV Curves

- Let's manipulate the power flow equations to obtain $v_2(p_2)$:

$$
\begin{aligned}
p_2^2 &= \frac{v_2^2 (v_1^{\text{ref}})^2}{x_L^2} \sin^2\theta_2 \\
q_2^2 \frac{v_2^4}{x_L^2} + 2 q_2 \frac{v_2^2}{x_L} &= \frac{v_2^2 (v_1^{\text{ref}})^2}{x_L^2} \cos^2\theta_2 \\
\Rightarrow \quad 0 &= p_2^2 + q_2^2 \frac{v_2^4}{x_L^2} + 2 q_2 \frac{v_2^2}{x_L} - \frac{v_2^2 (v_1^{\text{ref}})^2}{x_L^2}
\end{aligned}
$$

- That can be rewritten as:

$$
v_2 = \sqrt{-\left(q_2 x_L - \frac{(v_1^{\text{ref}})^2}{2}\right) \pm \sqrt{\left(q_2 x_L - \frac{v_1^{\text{ref}})^2}{2}\right)^2 - x_L^2 (p_2^2 + q_2^2)}}
$$

Qualitative Definition of PV Curves

- Let's assume, for simplicity, that $q_2 = p_2 \tan(\phi_2)$ (constant power factor):

$$v_2 = \sqrt{-a \pm \sqrt{a^2 - x_L^2 p_2^2 (1 + \tan^2(\phi_2)}}$$

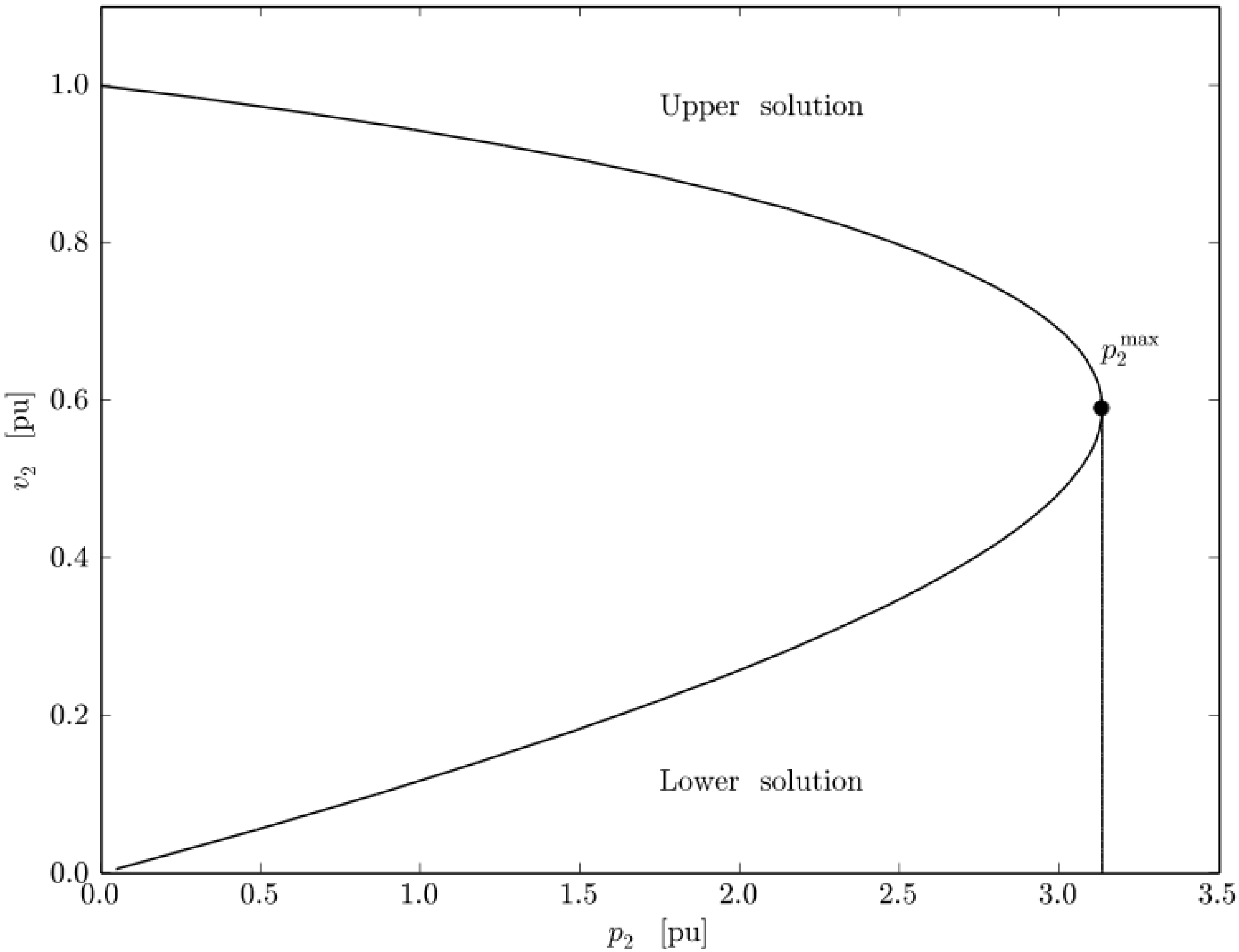
where:

$$a = p_2 \tan(\phi_2) x_L - \frac{(v_1^{\text{ref}})^2}{2}$$

- The other way round, we can obtain $p_2(v_2)$:

$$p_2 = \frac{v_2^2}{x_L} \left(\frac{-\tan(\phi_2) + \sqrt{\tan^2(\phi_2) - \left(1 - \frac{(v_1^{\text{ref}})^2}{v_2^2}\right)}}{1 + \tan^2(\phi_2)} \right)$$

Qualitative Definition of PV Curves

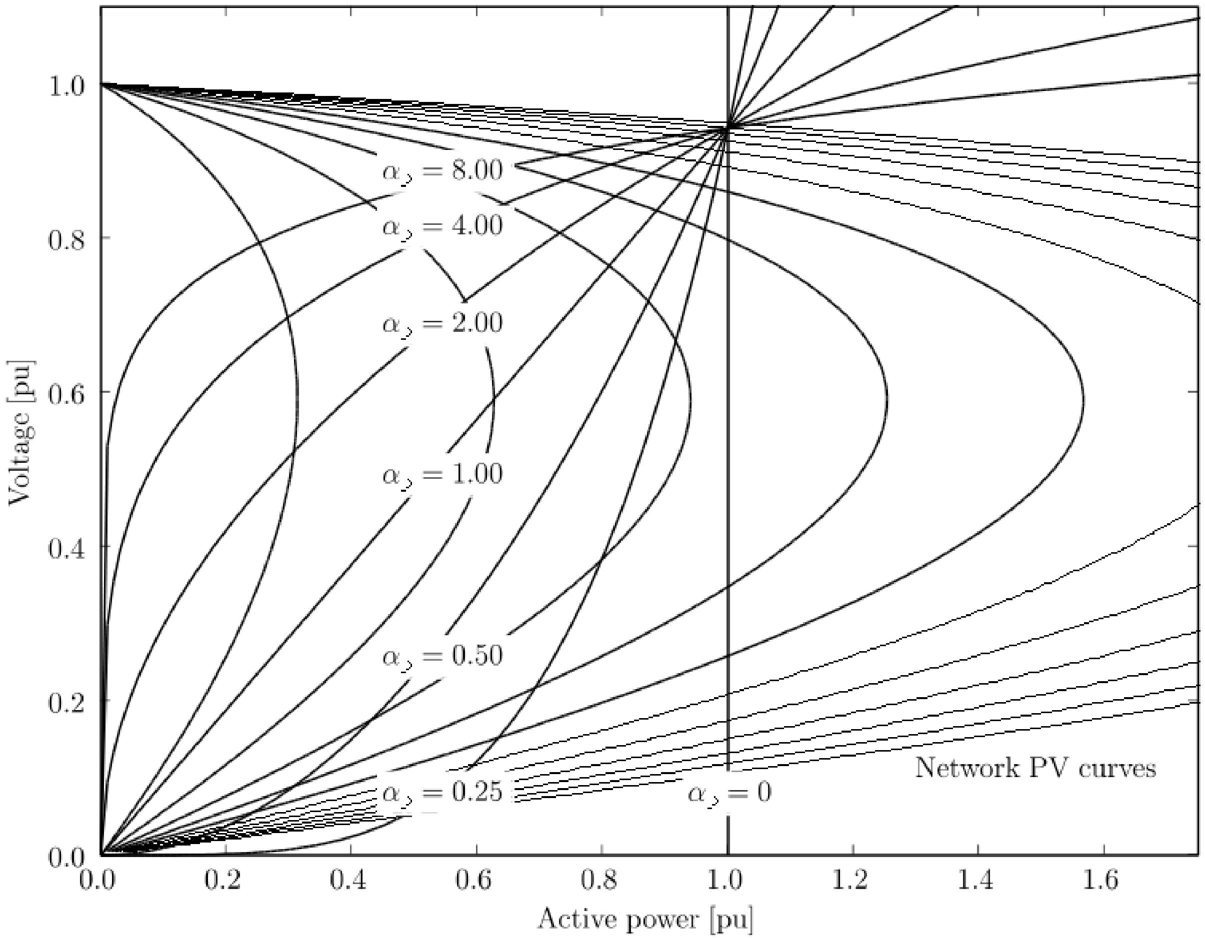


Voltage Dependent Load Model

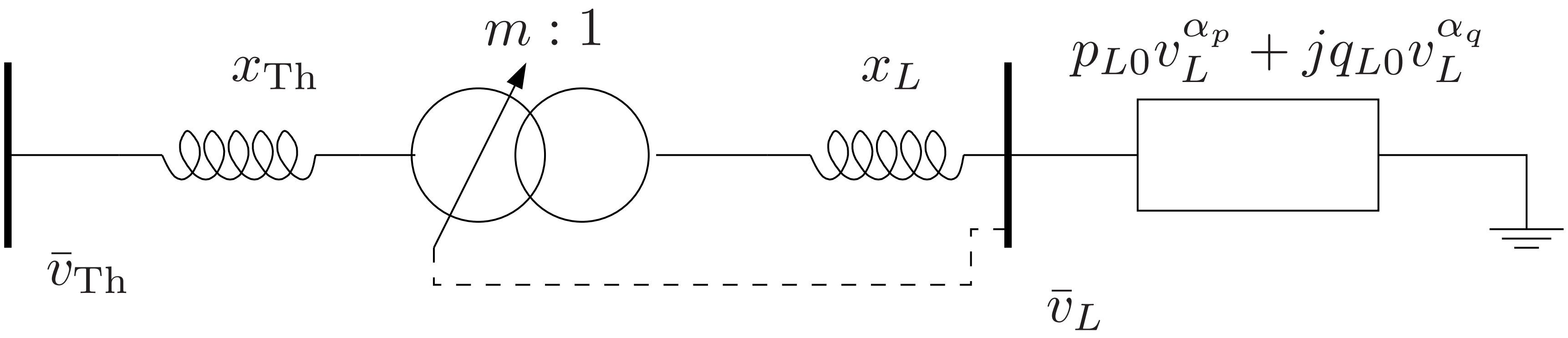
- A typical and well-accepted load model is the following:

$$
\begin{aligned}
p_h &= p_0 \left(\frac{v}{v_0} \right)^{\alpha_p} \\
q_h &= q_0 \left(\frac{v}{v_0} \right)^{\alpha_q}
\end{aligned}
$$

Effect of Load Models on the PV Curves



Example: Load Fed by a ULTC



Example: Load Fed by a ULTC

- If we assume $\alpha_p = \alpha_q = 2$ (constant impendance), then we have: $r_L + jx_L$. Hence:

$$\bar{v}_L = \bar{v}_{\rm Th} \frac{m(r_L + jx_L)}{m^2 r_L + j(x_{\rm Th} + m^2 x'_L)}$$

where:

$$x'_L = x_T + x_L$$

- The maximum voltage is obtained for:

$$m^* = \sqrt{\frac{x_{\rm Th}}{\sqrt{r_L^2 + (x'_L)^2}}}$$

Example: Load Fed by a ULTC

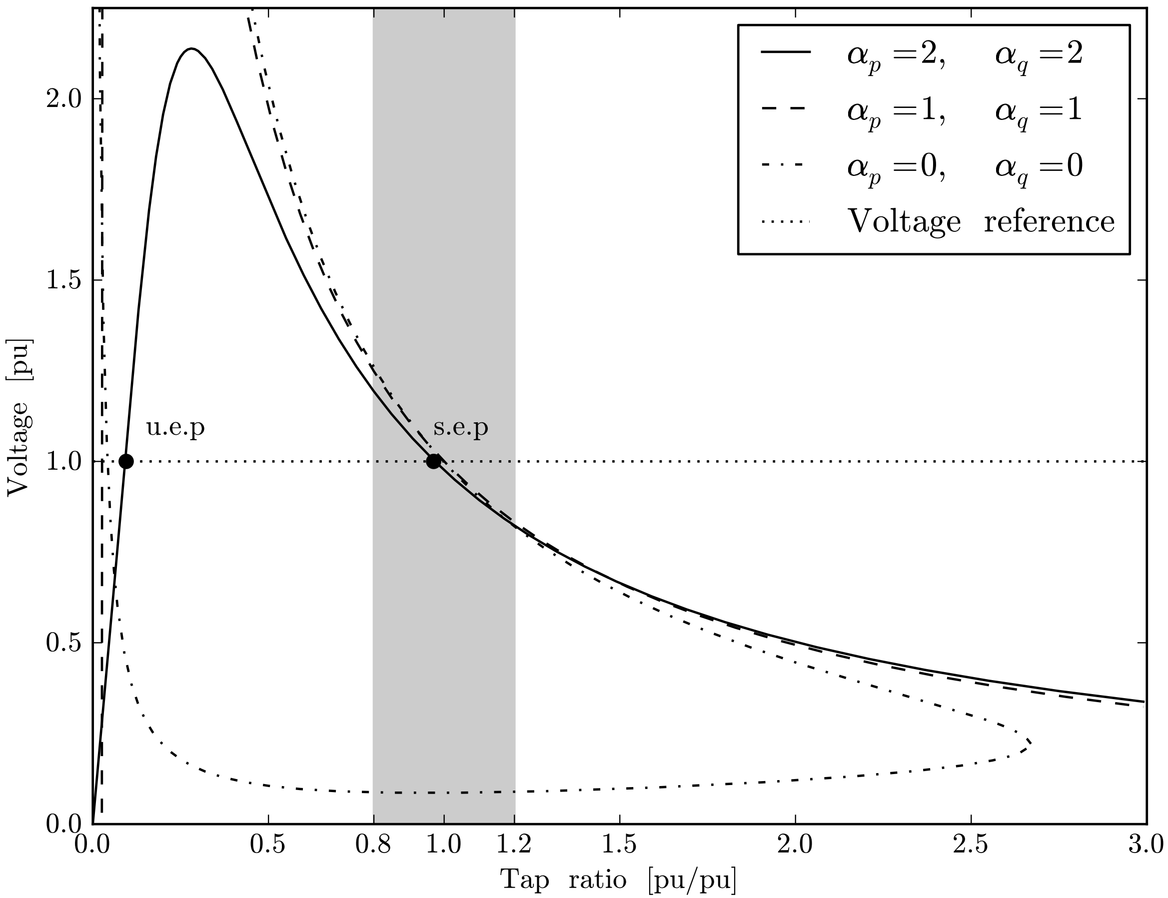
- The maximum voltage is:

$$v_L^{\max} = \sqrt{v_{\mathrm{Th}}^2 a \frac{r_L}{2x_{\mathrm{Th}}}}$$

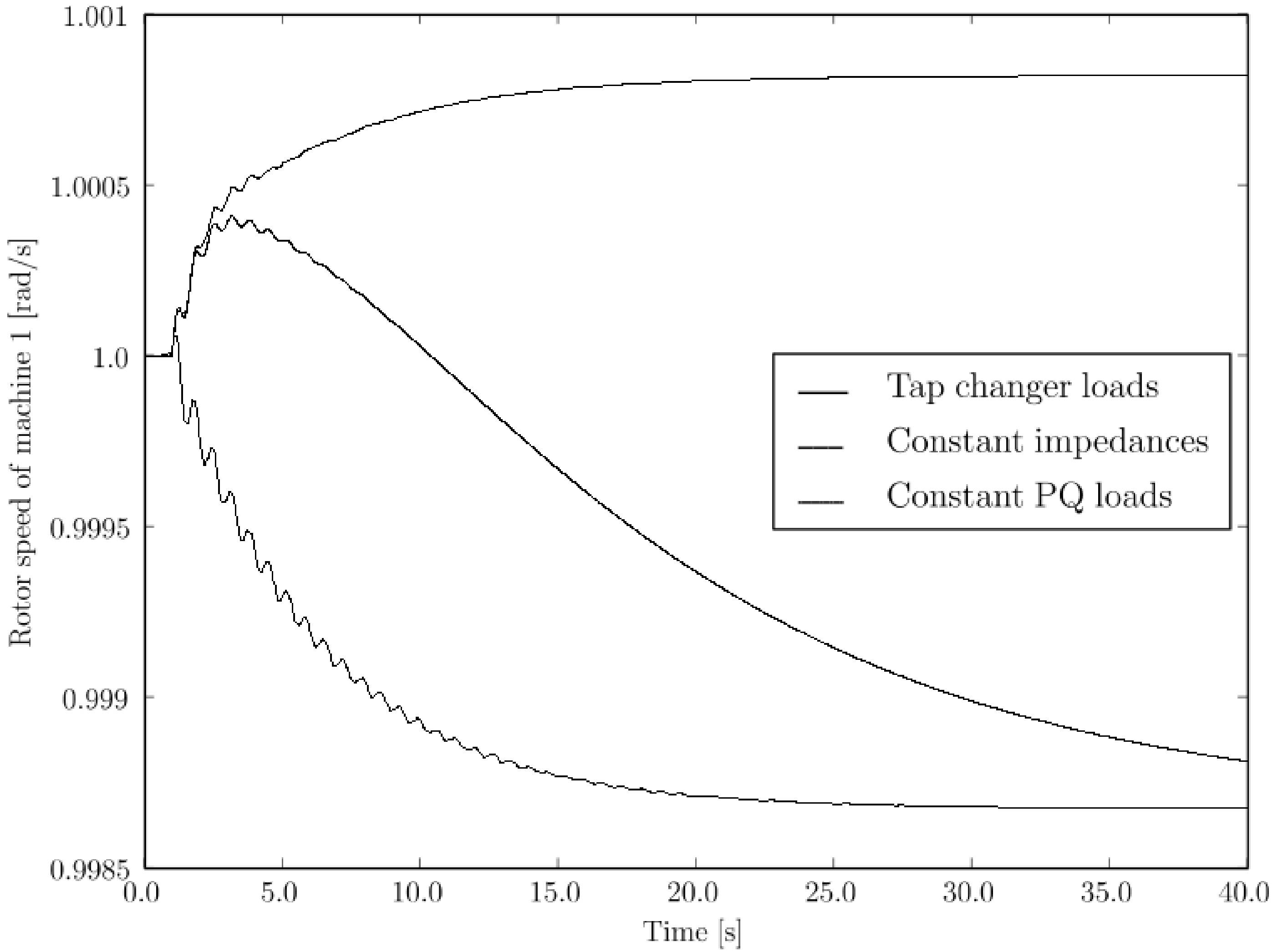
where

$$\begin{aligned} a &= 1 + b^2(\sqrt{1+c^2} - c) \\ b &= \frac{x_L}{r_L} \\ c &= \frac{x'_L}{r_L} \end{aligned}$$

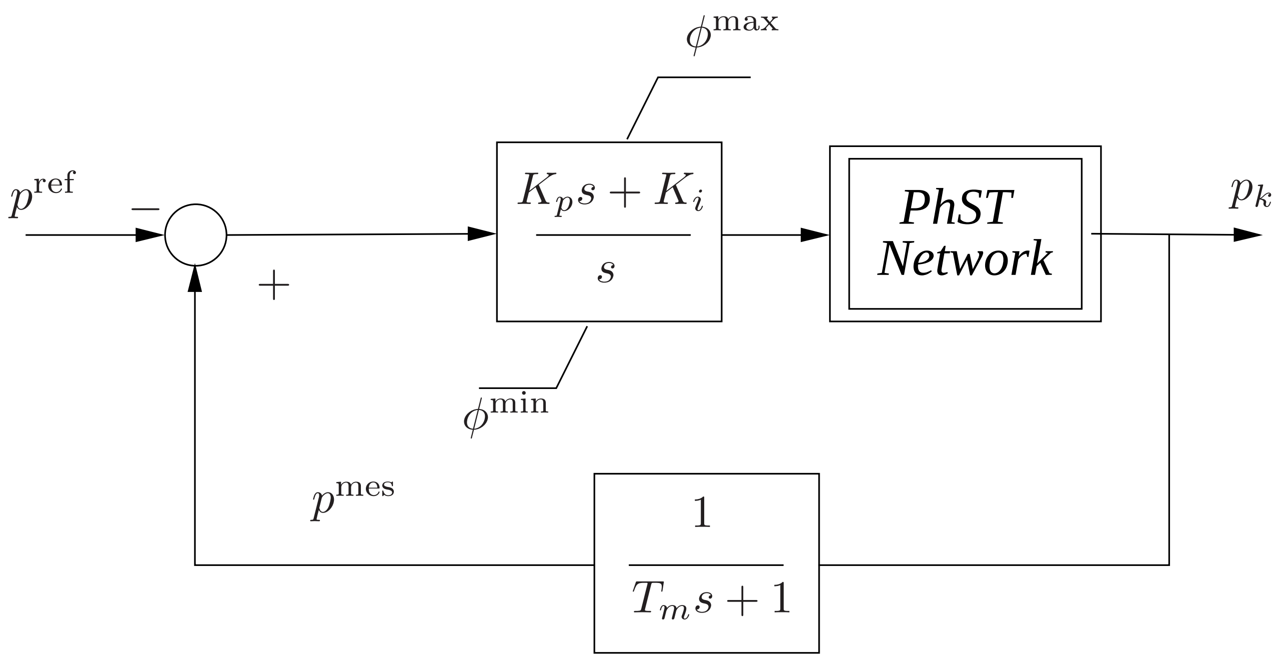
Static Characteristic of a Load Fed by a ULTC



Effect of Tap Changer Dynamics on Transient Analysis



Control Scheme of a Phase-Shifting Transformer (Continuous)



$$\dot{\phi} = K_p \frac{p_k - p^{\mathrm{mes}}}{T_m} + K_i (p^{\mathrm{mes}} - p^{\mathrm{ref}})$$

$$\dot{p}^{\mathrm{ref}} = \frac{p_k - p^{\mathrm{mes}}}{T_m}$$