THINKING NONLINEARLY: EXPERIENCE IN TEACHING POWER SYSTEM STABILITY ANALYSIS IN ENGINEERING PROGRAMMES

Federico Milano

University College Dublin (IRELAND)

Abstract

The paper discusses the didactic challenges to teach nonlinear systems to master and undergraduate students of electrical engineering programmes. This discussion is particularly focused to high-voltage transmission systems, but main concepts can be applied to any engineering area involving sets of nonlinear differential equations. A variety of examples that illustrate peculiar and unexpected behaviours of nonlinear systems are provided. These examples are also presented during the lectures, hence, typical answers provided by the students and the rationales why such answers are typically incorrect are duly discussed. The paper also outlines relevant aspects of the modules “Power System Modelling and Control” and “Power System Stability Analysis” offered by the author at UCD as part of the ME Programme in Electric Energy Systems, as well as a discussion of difficulties encountered by the students to master the matter of the aforementioned modules.

Keywords: nonlinear systems, power system dynamics, power system stability analysis, electrical energy systems.

1 INTRODUCTION

It is well-known that nonlinear dynamic systems are way more complex and intrinsically more difficult to understand than linear ones. These are some relevant facts that makes nonlinear system conceptually more challenging than linear ones: (i) the stability of linear systems can be defined univocally and is a global property, whereas, in the best case, only local stability properties can be determined for nonlinear systems; (ii) most control techniques can guarantee to work properly only for linear systems; (iii) bifurcations, e.g., structural changes of the dynamic response of a dynamic systems, only occur in nonlinear systems.

As a result, the dynamic behaviour of nonlinear systems is, to a large extent, unpredictable. This can be overcome only with experience, which in the case of power systems, implies dedicating several hours to solve computer simulations. The lack of experience can be only partially covered by lectures and tutorials. In fact, paraphrasing a famous sentence by Leo Tolstoy, *all linear systems are alike; each nonlinear system is nonlinear in its own way*.

The author has more than one decade experience in teaching, in different institutions and countries, power systems modules, including power system modelling, control and stability analysis and in developing open-source software tools for research and educational purposes [1-9]. The inherent complexity of power systems is due to two key factors: (i) power systems are composed of several different devices; and (ii) the equations that describes power system dynamics are nonlinear. This basically means that the behaviour of an interconnected power system is not just the combination of the behaviours of each device alone.

The learning approach designed by the author in his modules is as follows: (i) to assign to the students several lab activities that require solving a large number of simulations and studying the dynamic response of several test systems; and (ii) to provide, during the lectures, a variety of examples of the idiosyncrasies of the behaviour of nonlinear systems and ask the students questions on how they would expect the system to respond following given disturbances.

The contributions of the paper are as follows.

- A discussion on the didactic challenges to teach nonlinear systems to engineering students. This discussion is particularly focused to electric power systems, but main concepts can be applied to any engineering area involving sets of nonlinear differential equations.
- A variety of examples that illustrate peculiar and unexpected behaviours of nonlinear systems. These examples are also presented during the lectures, hence, typical answers provided by the students and the rationales why such answers are typically incorrect are duly discussed.

- An example of the lab activities that are proposed for the module “Power System Stability Analysis” offered by the author at UCD as part of the ME Programme in Electric Energy Systems.

The remainder of the paper is organized as follows. Section 2 briefly describes the modules Power System Modelling and Control and Power System Stability Analysis taught to 4^{th}-stage BE and ME students of the Electrical Energy Systems programme at UCD and presents the challenges to teach nonlinear system dynamics to undergraduate students. Section 3 discusses a set of examples taken from the lectures of the modules Power System Modelling and Control and Power System Stability Analysis that illustrates how the behaviour of nonlinear dynamic systems can be counterintuitive and far away from common sense. This section also discusses the approach used by the author to stimulate the students to think “nonlinearly”. Finally, Section 4 duly draws conclusions and future work to improve the learning curve of students with respect to the understanding of nonlinearity.

2 OUTLINES OF THE MODULES

The examples discussed in this paper are based on the modules Power System Modelling and Control and Power System Stability Analysis taught at during the academic years from 2013 to 2016 at the University College Dublin. These are 4^{th}-stage modules of the ME in Electrical Energy Systems programme that was implemented in 2013. This programme has received the accreditation from Engineering Ireland in 2014. Reference books of the modules described in this section are [10-14]. A brief description of the programme of each module is given below. An example of lab assignment of the module Power system Stability Analysis is given in the Appendix.

2.1 Power System Modelling and Control

The module introduces the main control requirements of most important devices that compose a HV transmission system. The focus of the module is on the dynamic behaviour of power systems. This includes frequency control, voltage control and auxiliary controllers aimed to improve the stability of the network. All topics are explained both theoretically and with simulation-based examples.

The module is divided into three parts.

- Part I: Park-Concordia model of the synchronous machine. Control of the synchronous machine, as follows. Primary and secondary controls including automatic voltage regulators, turbine and turbine governors, under and over-excitation limiters, and power system stabilizers. Synchronous machine secondary controls including automatic generator controllers and secondary voltage regulators. Tertiary frequency control.
- Part II: Transformer and FACTS device controllers, including under-load tap changers and phase shifters. VSC model and controls. Control of shunt and series FACTS devices. Basic models of VSC-based HVDC connections.
- Part III: Distributed energy sources control with particular emphasis on models and controllers of wind turbines and photovoltaic panels (MPPT, voltage control, frequency control, etc.). Basic control schemes of VSC-based distributed energy resources, e.g., PV power plants and battery energy storage devices.

The learning outcomes of the module are: basic concepts of power system frequency and voltage control; knowledge of control systems of all principal devices for high voltage transmission systems; and practical examples based on numerical simulations.

2.2 Power System Stability Analysis

The module explains the mathematical background of the phenomena that lead to power system instability, studying numerical methods to tackle such phenomena. The module is divided into four parts.

- Part I: long term voltage stability. Bifurcation theory (saddle-node, limit-induced and singularity-induced bifurcations) and the voltage collapse phenomenon. Continuation power flow analysis. Direct methods. Voltage stability constrained OPF. Voltage stability indexes. Cascade line tripping phenomenon.

- Part II: large perturbation angle stability (transient stability). Lyapunov theory. Direct methods. Time domain analysis methods. Hybrid methods (e.g. SIME [15]). Transient stability constrained OPF. Multi-swing phenomenon.
- Part III: small-signal angle stability analysis. Hopf bifurcations and limit cycles. Monodromy matrix. Routes to chaos, Poincaré maps and Lyapunov exponents. Small-signal stability constrained OPF. Effect of delays and analytical methods to assess the stability of delayed DAEs.
- Part IV: frequency stability. Load shedding problem. Frequency stability with renewable energy sources. Effect of thermostatically controlled loads.

Each part is completed by real-world examples (large scale blackouts), practical remedial actions and several computer-based simulation examples to support theoretical aspects.

The learning outcomes of the module are: basic principles of stability analysis of nonlinear differential-algebraic equation systems; definitions, causes and concepts of voltage, transient, angle and frequency stability; applied numerical methods to assess power system stability of large power systems; and practical control and protection strategies to avoid power system instabilities and blackouts.

2.3 Challenges in teaching nonlinear systems

The two modules above deal with the full set of nonlinear dynamic equations that describe the behaviour of power systems. Main sources of nonlinearity are the power flow equations, which are basically quadratic expressions, synchronous machine mechanical equations, control limits and saturations, discrete variables (e.g., tap positions of under-load tap changer transformers) and discrete events (e.g., line outages). In the module Power System Stability Analysis, delays and chaotic motions are also considered.

There are several challenges when teaching nonlinear systems to undergraduate students. In the experience of the author, the following are the main ones.

- Nonlinearity makes each system unique. The response of the system to a given disturbance can vary consistently. For example, a three-phase fault cleared after 200 ms can be harmless if it occurs at a certain bus and can lead to collapse at another one. This issue is also known as “transient instability” and is so relevant that a vast literature has been dedicated to its study and understanding (see for example the literature review in [15]). However, despite several techniques have been proposed to study such a problem, including direct methods based on Lyapunov and/or transient energy functions [16-18], the only reliable tool to define the transient stability of a power system is still numerical time domain integration.
- Perturbations around an operating point can drastically change the response of a system to the same disturbance. These phenomena are typically tackled by means of the bifurcation theory which provides a taxonomy of “special” points, namely bifurcation points, at which the structure and thus the behaviour of the system show substantial qualitative changes. A well-known example is saddle-node bifurcation that is used to explain voltage collapse [19].
- The dynamic coupling of continuous and discrete variables is particularly hard to understand. This can be due to the effects of saturations and control limits. If a control limit is binding, in fact, the control is effectively turned off. Recent studies on grazing bifurcations have attempted to study the role of discontinuities and control limits [20].
- Large perturbations can trigger unexpected changes in the global dynamic behaviour of the system. This can be due, for example, to the birth of strange attractors that leads to chaotic motions [21]. Another relevant example is the multi-swing instability which is similar to the transient instability described above but occurs unexpectedly and cannot be detected through direct methods [22]. It has also been conjectured that multi-swing instability is a special case of short-terms chaotic motion [23].

Since it is impossible, a priori, to know which phenomenon among the ones discussed above will actually occur in the system under study, it is not hard to understand how difficult can be for an undergraduate students to master the topic.

To provide a large variety of examples clearly helps, but the examples are not very effective if the students do not learn to "think" in a new way. A big challenge is that the students dealing with nonlinear system dynamics have to learn to "expect the unexpected". Another relevant challenge for the student is to learn how to determine the inner causes of the dynamic response of a system. This is not a trivial task as the number of variables is generally high and because the original cause of instability can trigger others. Strictly speaking, to determine the cause of an unstable behaviour can also be impossible. With this aim, the interested reader can find in [24] an interesting monograph on the concept of "causality".

3 ON THE COUNTERINTUTIVE BEHAVIOUR OF NONLINEAR SYSTEMS

In his 15-year long experience in teaching nonlinear system dynamics, the author has found that challenging the students with tricky problems and questions on the behaviour of nonlinear systems, the interest of the students is kept high during the lectures and their motivation is increased. This section presents a set of such problems, which ultimately are aimed to teach the student how to think "nonlinearly".

3.1 Unstable equilibrium points and stable limit cycles

One of the most widely used techniques to define the stability of an equilibrium point of a set of differential equations is the Lyapunov first criterion [12]. In lay language, this criterion states that the eigenvalues of the state matrix of the differential equations computed at the equilibrium point allow inferring its *local* stability. In particular, if all eigenvalues have negative real part, then the system is stable; whereas if at least one eigenvalue has positive real part the system is unstable. The criterion is inconclusive if at least one eigenvalue shows a null real part. Often such a case is a bifurcation point, further study is necessary. Leaving out these special cases, students are typically used to deal with linear systems, for which the Lyapunov first criterion is a *global* property, i.e., it can determine the stability of the system everywhere, not just at the equilibrium point. However, for smooth nonlinear systems, the Lyapunov first criterion is only true in a *neighbourhood* of the operating point. Now, the main issue that is difficult for the students to understand and, more importantly, to apply in practice, is that one cannot know *a priori* how big is such a neighbourhood. Not just that, nonlinear system can have many possible behaviours and, hence, once the trajectories of the system go outside the neighbourhood, any behaviour is possible and the information given by the state matrix at the equilibrium point is useless.

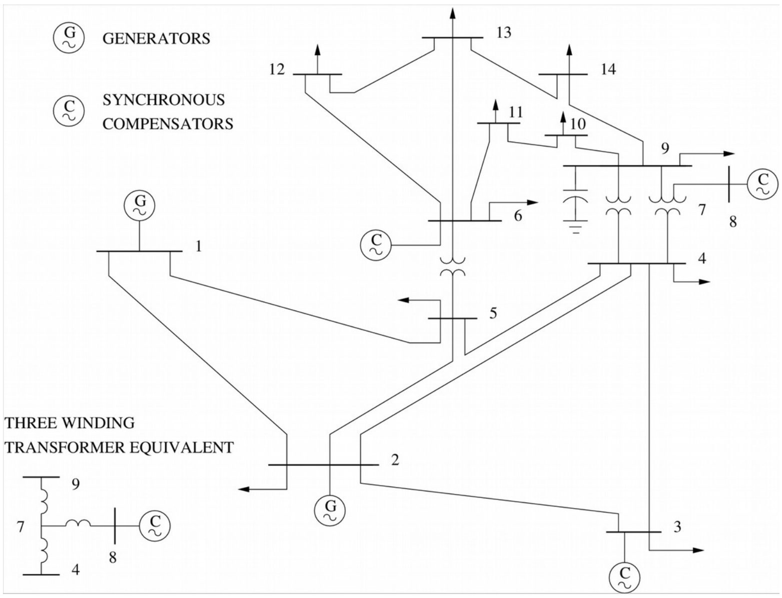

Figure 1: Single-line diagram of the IEEE 14-bus system.

The following example, based on the well-known IEEE 14-bus system (see Figure 1), is particularly appropriate to illustrate the issue above. Such a system, if the loading level is properly increased, shows a Hopf bifurcation, namely a pair of complex eigenvalues crosses the imaginary axis [13]. When the state matrix shows a pair of complex conjugate eigenvalues with positive real part, the oscillations of the system are not damped. In a linear system, this would lead to oscillations whose amplitude increases as the simulation time increases and, eventually, the system would collapse. This is the behaviour expected by students as they do not think of the pair of unstable eigenvalue as a local property, but rather as a global one. Hence, when they are shown the time domain simulation following a line outage of the system, they tend to be surprise to see that the trajectories are actually bounded and that the system does not collapse. This is due to the fact that the Hopf bifurcation originates a stable limit cycle which attracts the trajectories of the system departing from the unstable equilibrium point. The eigenvalue analysis and the time domain simulation of the IEEE 14-bus system with a 20% overloading with respect to the base-case and line 2-4 outage are shown in Figure 2.

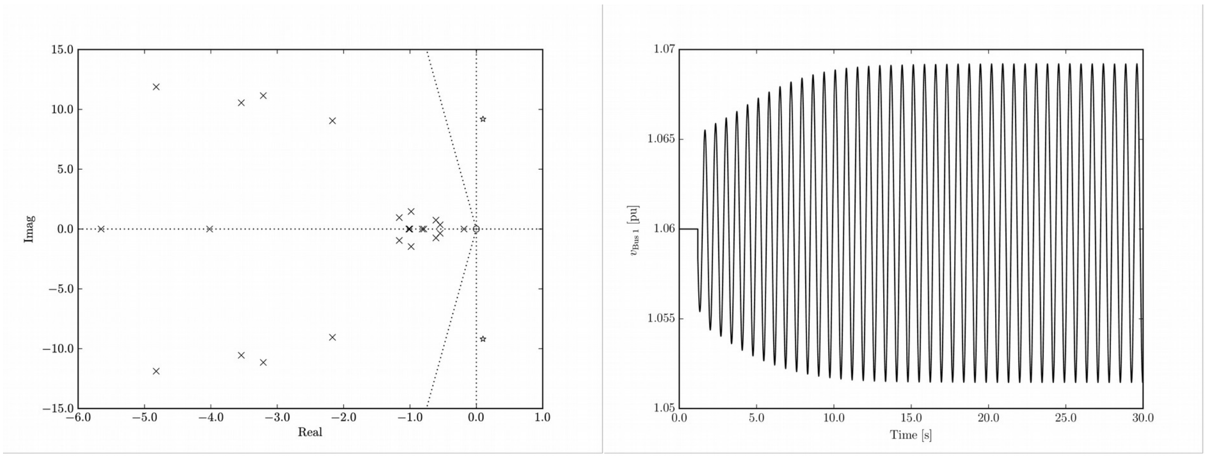

Figure 2: IEEE 14-bus system with 20% overloading with respect to the base case condition. Left panel: Eigevalue analysis considering line 2-4 outage. Right panel: Time domain simulation following line 2-4 outage at t = 1 s.

3.2 Finding "all" solutions of a set of nonlinear equations

One of the most popular techniques to study voltage stability and define the maximum loading condition of a power system is the continuation power flow analysis [19]. The main advantage of this approach is that it retains the full set of nonlinear equations and is able to determine the distance of the current operating point to the closest bifurcation point, which is typically the point at which the system becomes unstable. The most notable application of this technique is the determination of saddle-node and limit-induced bifurcations. From the mathematical point of view, such a technique is an application of the homotopy theory [25], which has the property that, if a solution of the set of nonlinear equations is known, then it is able to find all solutions along a path defined by a given parameter (typically the loading level of the system). Varying the parameter, it is thus possible to obtain a "path", which has also the property to be always closed. An example of such a closed path is shown in Figure 3, which has been obtained based on the IEEE 14-bus system.

Once the concepts of continuation power flow and the properties of the homotopy theory are clear to the students, the following question is asked to them. Since the path is closed, and assuming that we know how to compute a point on this path, is it possible to know all solutions of the power flow equations of the system? The question is particularly relevant because the power flow problem, i.e., the problem to determine the operating point of a power system given load and generation powers, is known to have multiple solutions, only few of which however are acceptable and feasible from the practical point of view. The ability to find all solutions, thus, would provide the ability to select the one that is acceptable (e.g., stable and feasible) and to know how close to such an operating condition are other solutions (in particular, unstable or unfeasible).

In the experience of the author, the students always answer "yes" to the question above. In other words, they implicitly assume that the "path" identified by the known solution of the power flow equations is unique. Actually, there is no reason why such a path should be unique and, indeed, it can be shown that it is unique only for a 2-bus system. In [26], the authors show a simple counterexample for which there are multiple paths. Hence, one should know at least one solution per every existing path to be able, through a homotopy technique, to find all solutions of the power flow equations. Unfortunately, there is no systematic technique, nor any mathematical tool that allows finding at least one solution per path. Actually, it is not even possible to know a priori how many independent paths there are. So, the problem of finding all solutions of the power flow equations remains unsolvable.

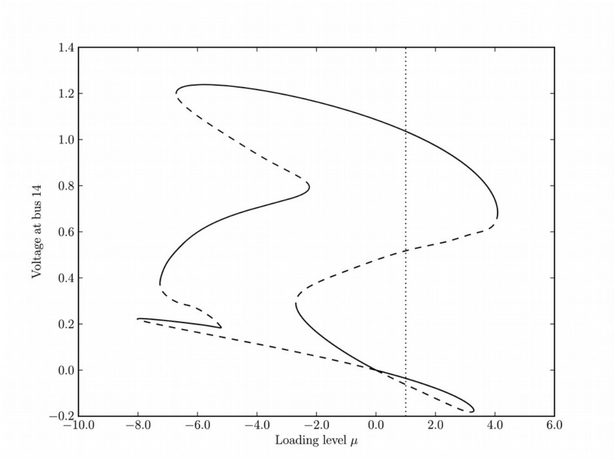

Figure 3: Full path obtained using a continuation power flow technique for the IEEE 14-bus system. After each saddle-node bifurcation, the line is changed from continuous to dashed and *vice versa* [13].

The fact that students never answer correctly is not surprising. It would be surprising, actually, if there were some student able to imagine the existence of multiple independent paths, as such an intuition would require either an exceptionally brilliant mind or a long experience with nonlinear systems. However, the purpose of posing the question is that the students learn through their error and understand that “prejudice” do not generally work with nonlinear systems. The very lesson is thus that one has to be very careful to draw final conclusions whenever a nonlinear system is involved.

3.3 Impact of delays on power system stability

An advanced topic of the module “Power System Stability Analysis” is the impact of time delays on power system stability. In the vast majority of cases, delays reduce the stability and, possibly, drive power systems to instability. This is easy to understand if one thinks of the limit case of an infinite delay which, in turns, is equivalent to an open-loop and thus effectively deactivates the control affected by such a delay. This point is discussed in detail in [27].

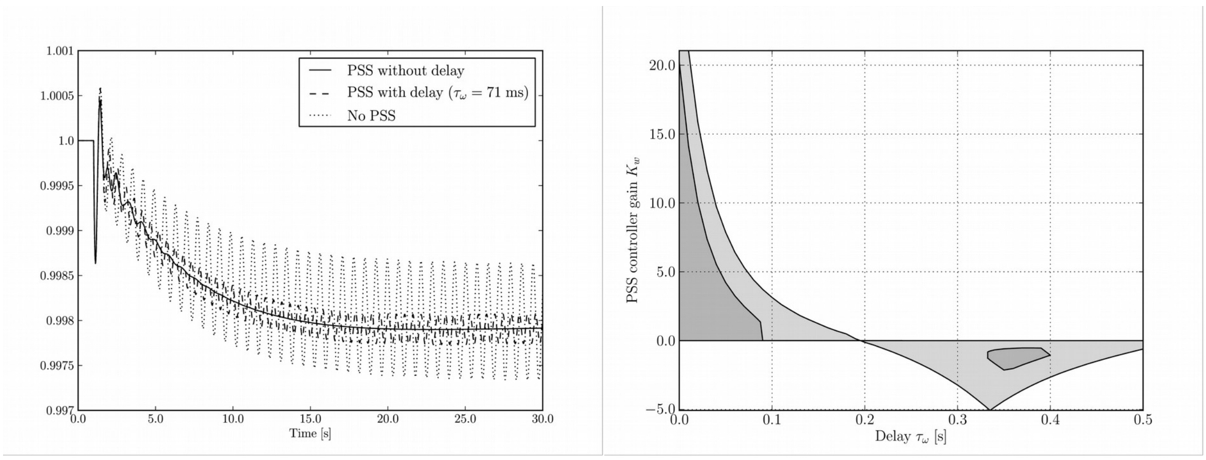

Figure 4: IEEE 14-bus sytem with a PSS connected to generator 1. Left panel: Time domain simulation following line 2-4 outage; the case without PSS is equivalent to the case with a PSS with a infinite delayed input signal [28]. Right panel: stability map for different values of the delay and the gain of the PSS; grey regions are stable while white regions are not [29].

The left panel of Figure 4 shows the expected effect of a time in the measured signal of a power system stabilizer (PSS) for the IEEE 14-bus system. The interested reader can find a detailed discussion of this case study in [28]. It is possible, however, that delays have an unexpected effect on the system. For example, in case of oscillatory behaviours, a time delay equal to the period of the oscillation would not harm the system. Reference [29] shows, for example, that increasing the delay affecting the PSS in the IEEE 14-bus system does not necessarily make the system unstable, provided that the PSS gain is properly tuned (see right panel of Figure 4). While this conclusion becomes obvious once the rationale based on the oscillation period is pointed out, students do not generally answer correctly to the question “do delays always reduce the stability of power systems?”. The tricky part of the question is the word “always” which, while generally appropriate for linear systems, it is often not suitable for nonlinear ones. There are in fact very few known general propositions and theorems on nonlinear systems and this is, in turn, the reason why they are so difficult (likely impossible) to master.

3.4 Linear indices to predict voltage collapse

An interesting application of the continuation power flow analysis that is described in the previous section is that, given the quadratic nature of power flow equations, it is possible to define a linear index to predict the voltage collapse [30]. This rationale of this idea is as follows. The curves obtained using the continuation power flow are highly nonlinear. Such a curve is, in fact, often called “nose curve” for its peculiar shape. Thus, one has to compute the whole curve to actually know the maximum loading condition and, hence, for which loading level the voltage collapse occurs. It would be clearly more efficient to be able to predict the point at which the voltage collapse occurs if the curve were linear. Now, while the nose curve itself cannot be linearized, a particular index based on the minimum singular value or minimum eigenvalue computed at each point of the curve actually shows an almost linear behaviour. This is due to the quadratic structure of power flow equations. Moreover, another relevant property is that the voltage collapse occurs when such an index is null. The interested reader can find more details on how such an index is defined in [19] and [30]. In this context, it suffices to say that, with a linear index, one would need to compute only two points for two different loading levels. An operating point is always known, i.e., it is the current base loading condition. So one needs to compute only another point for a different loading level and then extrapolate the intersection of the linear index with the abscissa.

The reasoning above typically convince the students. The fact that a complex phenomenon such as the occurrence of the voltage collapse can be reduced to a linear problem is certainly reassuring. Unfortunately, this is not true in general. The conclusions above are correct only if no limit (e.g., generator reactive power limit) becomes active before the occurrence of the voltage collapse. If, on the other hand, some limit is binding, the linear index changes its slope and, as a result, the voltage collapse happens for a smaller value than that predicted without taking into account the limits. This behaviour is illustrated in Figure 5. Since it is impossible to know, especially for a large system, how many limits will become binding before the occurrence of the voltage collapse, then one has to compute the complete nose curve and cannot rely on the information extrapolated by means of the linear index. Even more important, the collapse may also occur for non-null values of the index. In fact, the condition that the voltage collapse happens for a null value of the index is only sufficient, not necessary.

The reactions of the students to the discussion above is often a slight disappointment due to the understanding that, when dealing with nonlinear systems, there are no shortcuts. It is an illusion, in fact, to hope that one can get rid of the issues of nonlinearity with some smart index which is easy to compute. To understand that easy shortcuts rarely are the solutions to our problems is actually very important and, in the opinion of the author, not limited to the study of power systems.

3.5 Multi-swing phenomenon in transient instability

As discussed in the previous section, the multi-swing phenomenon is a particular, relatively uncommon, type of transient instability driven by the dynamic of three or more synchronous machines following a large disturbance, e.g., a three-phase short circuit. The conventional transient instability consists of the loss of synchronism of one or more machines in the few initial hundreds of milliseconds following the disturbance. Transient instability is also often called first-swing instability as it happens in the first oscillation of the rotor angles of the synchronous machines. All direct methods (e.g., transient energy function and extend area criterion [16-18]) assume that the instability occurs during the first swing and assume that a system where no machine loses synchronism after the first oscillation is stable.

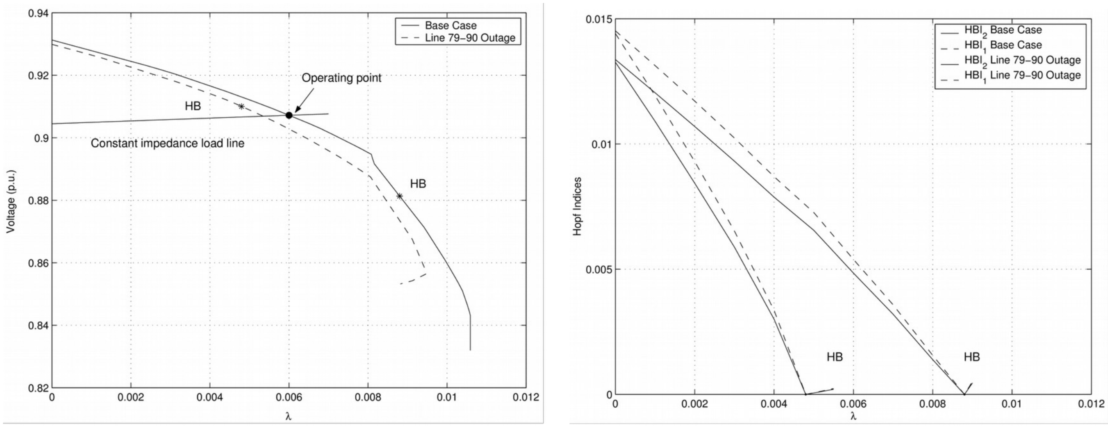

Figure 5: IEEE 145-bus system: Effect of generator reactive power limits on nose curves (left panel) and linear voltage stability indexes (right panel). HBI1 and HBI2 are the indices based on the minimum eigenvalue and minimum singular value of the system state matrix, respectively [30].

Multi-swing instability, while ultimately causing the loss of synchronism, is a substantially different phenomenon with respect to the first swing and that explains why it cannot be tackled using direct methods. As introduced in the previous section, it has been conjectured that multi-swing instability is originated by the birth of a strange attractor that drives the trajectories of the machines to fall onto a chaotic motion [23]. This is actually hard to prove because chaotic motions are, by definition, bounded, whereas the loss of synchronism is not. Moreover, to prove that a chaotic motion is such, one have to solve a time domain integration for a sufficiently long time (e.g., a few minutes). However, a system that faces a multi-swing instability can only survive a few seconds after the contingency. On the other hand, similarly to chaotic motions, the multi-swing instability shows a high sensitivity to initial conditions and, hence, share the same "unpredictable" behaviour typical of chaos. Multi-swing instability, in fact, can appear or disappear by varying of a very little amount the initial conditions of any system parameter.

Teaching chaos (or chaos-like) phenomena allows to explain to the student that nonlinear systems not only can show different dynamic responses depending on the operating point and the contingency, but also can be genuinely unpredictable or, in other words, the system can respond differently even if starting at (almost) the same operating point and for the same contingency. This fact helps the students understand that simulating the system is always necessary and that it is important not to assume a given response even if one think to know well the system under study.

4 CONCLUDING REMARKS

The paper has presented a variety of examples and related questions that are proposed to the students during the lectures of the modules Power System Modelling and Control and Power System Stability Analysis that have been taught by the author at the University College Dublin since 2013. The examples focus on the idiosyncrasies of dynamic nonlinear systems in general and power systems in particular. Some concluding remarks and common guidelines can be drawn, as follows.

Nonlinear dynamical systems are unpredictable, challenge the common sense, and, often, show behaviours that only experience or a brilliant mind can anticipate. The underline message is thus that when approaching the study of nonlinear systems, one has to be as humble as possible and always open to new ideas and possibilities. Words such as "always", "never" or "all" should be used with caution and a certain behaviour should never be accepted *a priori* without carrying out appropriate tests. The caveats and issues above come, however, with a reward: the intellectual pleasure of the discovery of unexpected phenomena and the ability to capture through mathematical equations the complexity of dynamic systems (power systems in this case). This is generally appreciated by the students, who often provide positive feedback for the modules considered in this paper – in particular, Power System Stability Analysis.

The author is currently working on expanding the set of examples and case studies on power systems to present to the students and continuously looking for new approaches capture students' attention on the study of dynamic nonlinear systems.

ACKNOWLEDGMENTS

Federico Milano is funded under Programme for Research in Third Level Institutions and co-funded under the European Regional Development Fund (ERDF). Federico Milano is also a beneficiary of financial support from the EC Marie Skłodowska-Curie Career Integration Grant No. PCIG14-GA-2013-630811.

APPENDIX

This appendix reports an example of lab activity of the module "Power System Stability Analysis". The assignment focuses on small-signal stability analysis and Hopf bifurcations and is based on the IEEE 14-bus sytem discussed in Section 3.

Exercises

1. Solve the continuation power flow analysis of the IEEE 14-bus system with inclusion of dynamic synchronous machines and AVRs and determine the maximum loading condition at which the system can be operated.
2. Solve the continuation power flow analysis of the IEEE 14-bus system with inclusion of dynamic synchronous machines, AVR and one PSS connected at the machine 1 and determine the maximum loading condition at which the system can be operated. Discuss the effect of the PSS comparing the results obtained for Exercise 1.
3. Determine which of the following contingencies is the most critical for the system: line 2-4 outage, line 2-5 outage and line 4-5 outage.
4. Check the solution of one contingency analysis of Exercise 3 through a time domain simulation. Discuss whether the Hopf bifurcation is super-critical of sub-critical.

REFERENCES

[1] F. Milano, Power System Software Tools, available at: http://faraday1.ucd.ie/software.html

[2] F. Milano, An Open Source Power System Analysis Toolbox, IEEE Trans. on Power Systems, Vol. 20, No. 3, pp. 1199-1206, August 2005.

[3] L. Vanfretti, and F. Milano, Application of the PSAT, an Open Source Software, for Educational and Research Purposes, IEEE PES General Meeting, Tampa, USA, 24-28 June 2007.

[4] F. Milano, L. Vanfretti, J.C. Morataya, An Open Source Power System Virtual Laboratory: The PSAT Case and Experience, IEEE Transactions on Education, Vol. 51, No. 1, pp. 17-23, February 2008.

[5] L. Vanfretti, F. Milano, The Experience of PSAT as a Free and Open Source Software for Power System Education and Research, International Journal of Electrical Engineering Education, Vol. 47, No. 1, pp. 47-62, January 2010.

[6] F. Milano, R. Zárate-Miñano, Using Python for the development if Electrical Engineering Projects. ICERI, Madrid, Spain, November 2011.

[7] F. Milano, A Python-based Software Tool for Power System Analysis. PES General Meeting, Vancouver, Canada, July 2013.

[8] F. Milano, Experience of Unix Terminal-based Labs for Undergrounduate Modules on Power System Analysis, EDULEARN14, Barcelona, Spain, 7-9 July 2014.

[9] F. Milano, L. Vanfretti, Role of Non-commercial Software in Undergraduate Electric Energy Systems Programmes, EDULEARN15, Barcelona, Spain, 6-8 July 2015.

[10] P. Kundur, *Power System Stability and Control*. McGraw- Hill, 1994.

[11] P. M. Anderson, A. A. Fouad, *Power System Control and Stability*, 2nd Edition, IEEE Press, John Wiley Interscience, 2003.

[12] P. W. Pai, M. A. Sauer, *Power System Dynamics and Stability.* Prentice Hall, 1998.

[13] F. Milano, *Power System Modelling and Scripting*. Springer-Verlag, 2010.

[14] A. Gómez Expósito, A. J. Conejo, C. Cañizares, *Electric Energy Systems: Analysis and Operation*, CRC, Boca Raton, 2008.

[15] M. Pavella, D. Ernst, D. Ruiz-Vega, *Transient stability of power systems: a unified approach to assessment and control*, Kluwer Academic Publishers, Dordrecht, 2000.

[16] M. A. Pai, *Energy Function Analysis for Power System Stability*. Kluwer Academic, 1989.

[17] A. A. Fouad and V. Vital, *Power system transient stability analysis using the Transient energy function method*. Prentice-Hall, 1992.

[18] H. D. Chiang, *Direct Methods for Stability Analysis of Electric Power Systems*. Wiley, 2011.

[19] Voltage Stability Assessment: Concepts, Practices and Tools, IEEE/PES Power Syst. Stability Subcommittee, Tech. Rep. SP101PSS, Aug. 2002.

[20] V. Donde and I. A. Hiskens, Dynamic Performance Assessment: Grazing and Related Phenomena, IEEE Transactions on Power Systems, vol. 20, no. 4, pp. 1967-1975, Nov. 2005.

[21] B. Lee and V. Ajjarapu, Period-doubling Route to Chaos in an Electrical Power System, IEE Proceedings C - Generation, Transmission and Distribution, vol. 140, no. 6, pp. 490-496, Nov. 1993.

[22] M. Yin, C. Y. Chung, K. P. Wong, Y. Xue and Y. Zou, An Improved Iterative Method for Assessment of Multi-Swing Transient Stability Limit, IEEE Transactions on Power Systems, vol. 26, no. 4, pp. 2023-2030, Nov. 2011.

[23] Chih-Wen Liu, J. S. Thorp, Jin Lu, R. J. Thomas and Hsiao-Dong Chiang, Detection of Transiently Chaotic Swings in Power Systems using Real-time Phasor Measurements, IEEE Transactions on Power Systems, vol. 9, no. 3, pp. 1285-1292, Aug 1994.

[24] J. Pearl, *Causality – Models, Reasoning, and Inference*, 2nd ed., Cambridge University Press, 2009.

[25] M. Ilic, J. Zaborski, *Dynamics and Control of Large Electric Power System*s. Wiley & Sons, 2000.

[26] D. K. Molzahn, B. C. Lesieutre and H. Chen, Counterexample to a Continuation-Based Algorithm for Finding All Power Flow Solutions, in IEEE Transactions on Power Systems, vol. 28, no. 1, pp. 564-565, Feb. 2013.

[27] F. Milano, Problem Solving through Limit-case Analysis: Experience in Teaching Electric Engineering Programmes, EDULEARN16, Barcelona, Spain, 4-6 July 2016.

[28] F. Milano and M. Anghel, Impact of Time Delays on Power System Stability, IEEE Transactions on Circuits and Systems - I: Regular Papers, vol. 59, no. 4, pp. 889–900, Apr. 2012.

[29] V. Bokharaie, R. Sipahi, and F. Milano, Small-Signal Stability Analysis of Delayed Power System Stabilizers, PSCC 2014, Wrocław, Poland, Aug. 2014.

[30] C. A. Cañizares, N. Mithulananthan, F. Milano and J. Reeve, Linear performance indices to predict oscillatory stability problems in power systems, in IEEE Transactions on Power Systems, vol. 19, no. 2, pp. 1104-1114, May 2004.