Stability Analysis of a Single-Machine Dynamic-Load System with Inclusion of an SVC Device

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1 Assignment

Study the effect of load active power variations on the small-signal stability as well as the transient behavior of the system shown in Fig. 1 for the following scenarios.[1]

1. *The load is not compensated and has a voltage dynamic driven by the following differential equation:*

$$\dot{v} = \frac{1}{\tau}(-q_d - q_L) \tag{1}$$

   Use the device `Vdyn` *defined in Dome.*

2. *The load is compensated through an SVC with a simple low-pass regulator. Use the device* `Svc1` *defined in Dome.*

Use the classical synchronous machine model for the generator connected at bus 1 and a lossless transmission line. Properly describe system and controller models. Assume reasonable physical and control parameters for every system device. Discuss simulation results and draw relevant conclusions.

2 Report

2.1 Objectives

This report illustrates the effects of a static var compensator (SVC) device on the dynamic response of a simple "single-machine dynamic-load" system. The stability of the test system without SVC is investigated first. Then, static and dynamic analyses are repeated for the system with an SVC device connected at the load bus.

[1] The case study considered in this document is based on [1].

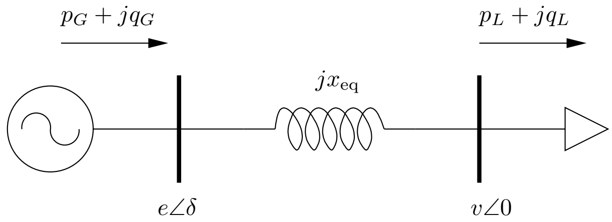

Figure 1: One-line diagram of the single machine dynamic load (SMDL) system.

2.2 Contents

The report is organized as follows. Section 2.3 illustrates the single-machine dynamic-load example considered in this report. Static and dynamic simulations are also provided in this section to illustrate the voltage collapse phenomenon. Section 2.4 discusses the use of a SVC controller as remedial action to avoid the voltage collapse. Static and dynamic simulations are also provided in this section to illustrate the effects of the SVC on the voltage stability and the birth of undamped oscillations. Finally, in Section 2.5, conclusions are duly drawn.

2.3 Single-Machine Dynamic-Load System

Figure 1 shows a simple model of single-machine dynamic-load (SMDL) system. This didactic system was originally presented in [2] for voltage stability analysis. The following set of DAE is used to model the system:

$$
\begin{aligned}
\dot{\delta} &= \Omega_b(\omega - 1) \\
\dot{\omega} &= \frac{1}{M}[p_m - p_G(v,\delta) - D(\omega - 1)] \\
\dot{v} &= \frac{1}{\tau}[q_L(v,\delta) - q_d]
\end{aligned}
\tag{2}
$$

and

$$
\begin{aligned}
p_G(v,\delta) &= \frac{ev}{x_{\mathrm{eq}}}\sin(\delta) \\
q_L(v,\delta) &= -\frac{v^2}{x_{\mathrm{eq}}} + \frac{ev}{x_{\mathrm{eq}}}\cos(\delta)\ ,
\end{aligned}
\tag{3}
$$

where δ is the generator rotor angle; ω is the generator angular speed; M is the generator inertia constant; p_m is the mechanical power of prime mover; x_{eq} is the equivalent generator and line reactance; e is the generator voltage; D is the generator damping; τ is the voltage time constant of the dynamic load; and v is the bus voltage of the dynamic load.

The load power demand is $p_d + jq_d$. For the sake of simplicity but without loss of generality, the resistance of the transmission line is neglected, i.e. $p_m =$

p_d. Furthermore, it is assumed that the load, in steady-state conditions, has a constant power factor, i.e. $q_d = kp_d$, where k is a given constant. Thus (2) can be simplified as follows:

$$\begin{aligned}
\dot{\delta} &= \Omega_b(\omega - 1) \\
\dot{\omega} &= \frac{1}{M}[p_d - \frac{ev}{x_{\mathrm{eq}}}\sin(\delta) - D(\omega - 1)] \\
\dot{v} &= \frac{1}{\tau}[-kp_d - \frac{v^2}{x_{\mathrm{eq}}} + \frac{ev}{x_{\mathrm{eq}}}\cos(\delta)] \ ,
\end{aligned} \tag{4}$$

Observe that algebraic equations (3) have been eliminated from (4), thus leading to a set of ODE. Variables and parameters are defined in Appendix A. Observe also that eliminating explicitly the algebraic variables is possible only in simple systems such as the one discussed in this paper.

The active power demand p_d of the dynamic load is the parameter that can be varied. All other parameters are given in Appendix B.

2.3.1 Small Signal Stability Analysis

Figure 2 depicts the changes of relevant eigenvalues of the state matrix A as the active power demand p_d of the dynamic load increases. One eigenvalue crosses the imaginary axis for $p_d \approx 0.61804$ p.u. The equilibrium point for $p_d = 0.61804$ p.u. is:

$$(\delta_0, \omega_0, v_0) = (0.5533, 1, 0.5881) \ .$$

To define the rotor angle δ above, the load bus voltage phase angle has been assumed as the reference. This point cannot be obtained with a Newton-Raphson method as the Jacobian matrix of the system is singular (or very badly conditioned) for $0.61803 < p_d \leq 0.61804$. Moreover, for $p_d > 0.61804$, the system has no solution. However, in this case, the solution of (4) can be obtained explicitly.

2.3.2 Time Domain Simulation

Figure 3 shows the time domain simulation for a step change of p_d from 0.60 p.u. to 0.61 p.u. at $t = 1$ s. As expected fromt he previous small-signal stability analysis, the system is stable and gets to a new steady-state operating point. Observe that the voltage magnitude at the load bus is too low and is not acceptable from a practical viewpont. Thus, the SVC device is required to provide a reactive power support.

Figure 4 shows the time domain simulation for a step change of p_d from 0.61 p.u. to 0.619 p.u. at $t = 1$ s. For $t \approx 1.05$ s, the system undergoes a voltage collapse driven by the dynamic of the load voltage.

2.4 Single-Machine Dynamic-Load System with SVC

SVC devices are commonly used in power systems to control bus voltages and improve stability [3]. In this paper an SVC is used to control the voltage at the load bus of the SMDL system, as illustrated in Fig. 5.

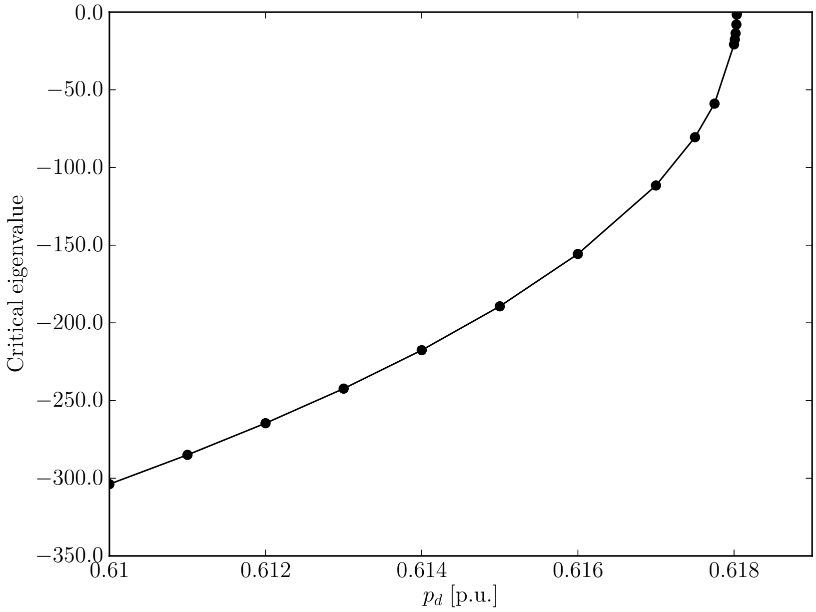

Figure 2: SMDL system without SVC: Eigenvalue loci.

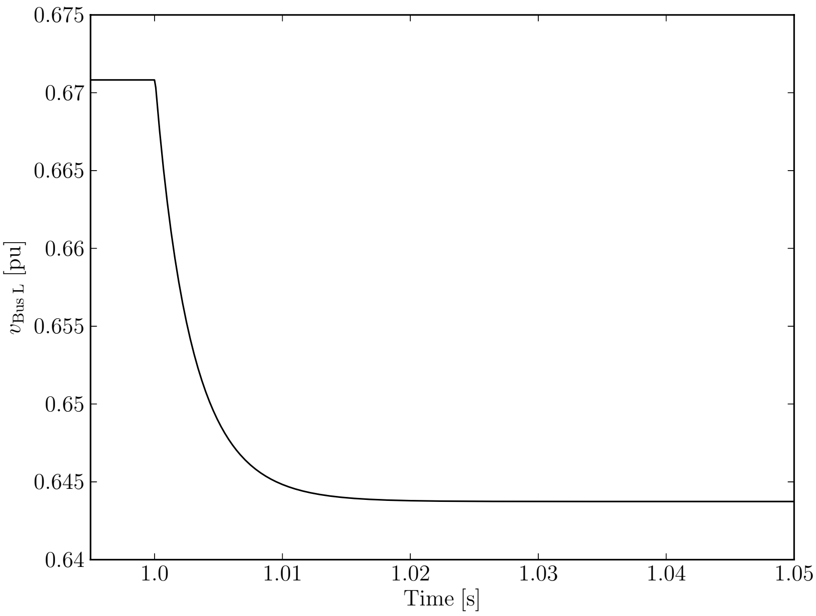

Figure 3: SMDL system without SVC: voltage trajectory induced by a sudden load increase (from 0.60 to 0.61 p.u.).

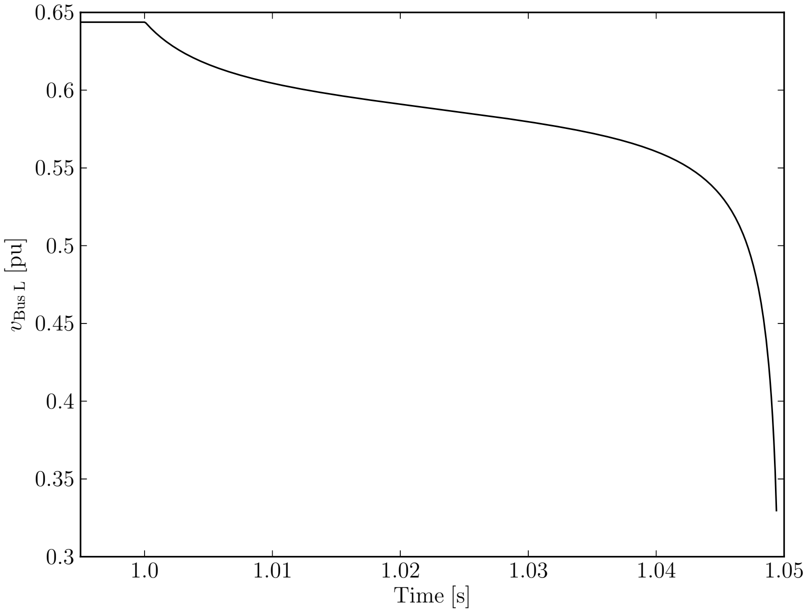

Figure 4: SMDL system without SVC: voltage collapse induced by a sudden load increase (from 0.61 to 0.619 p.u.).

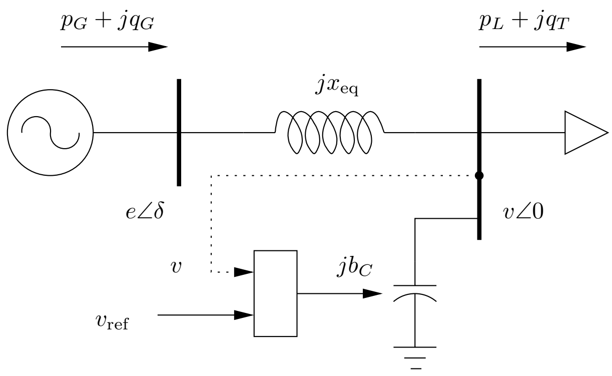

Figure 5: One-line diagram of the SMDL system with SVC.

The total reactive power absorbed by the load and the SVC is as follows:

$$q_T(v,\delta) = -\frac{v^2}{x_{\mathrm{eq}}} + \frac{ev}{x_{\mathrm{eq}}}\cos(\delta) + v^2 b_C \tag{5}$$

The SVC controller is modeled as a first order pure integrator, as depicted in Fig. 6.

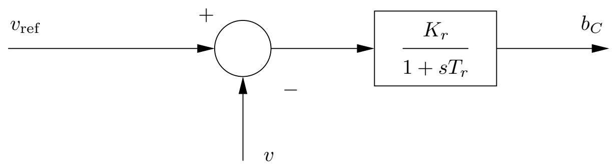

Figure 6: SVC control scheme.

The resulting differential equations of the SMDL system with SVC are as follows:

$$\begin{aligned}
\dot{\delta} &= \Omega_b(\omega - 1) \\
\dot{\omega} &= \frac{1}{M}[p_d - \frac{ev}{x_{\mathrm{eq}}}\sin(\delta) - D(\omega - 1)] \\
\dot{v} &= \frac{1}{\tau}[-kp_d + v^2(b_C - \frac{1}{x_{\mathrm{eq}}}) + \frac{ev}{x_{\mathrm{eq}}}\cos(\delta)] \\
\dot{b}_C &= \frac{1}{T_r}(K_r(v_{\mathrm{ref}} - v) - b_c)\ ,
\end{aligned} \tag{6}$$

where b_C is the equivalent susceptance of the SVC; and T_r, K_r v_{ref} are the SVC controller time constant, gain and reference voltage, respectively. In the following, it is assumed that $T = 0.01$ s and v_{ref} is the value required to impose in steady-state $v = 1$ p.u. at the load bus (for example, $v_{\mathrm{ref}} = 1.0768$ p.u. for $p_d = 1$ p.u.). Observe that also in this case it is possible to deduce the set of ODE, i.e., the algebraic variables can be explicitly expressed as a function of the state variables and the parameters.

2.4.1 Small Signal Stability Analysis

The inclusion of an SVC in the system leads to the birth of a pair of complex conjugate eigenvalues. Figure 7 shows the real part of such eigenvalues as a function of the load active power demand p_d. The real part of the pair of complex conjugate eigenvalues crosses the imaginary axis for $p_d \approx 1.08325$, thus leading to an unstable equilibrium point characterized by undamped oscillations. Observe that the inclusion of the SVC, apart from maintaining the load voltage at 1 p.u., allows increasing the load consumption well beyond 0.62 p.u. Hence, the SVC improve both the static and dynamic behvior of the system.

In this case, the system solution can be always obtained using a standard Newton-Raphson method as the Jacobian matrix of the system is well-conditioned close to the bifurcation point. The equilibrium point for $p_d = 1.08325$ is:

$$(\delta_0, \omega_0, v_0, b_{C0}) = (0.5724, 1, 1, 0.8604) \ .$$

To define the rotor angle δ above, the load bus voltage phase angle has been assumed as the reference.

2.4.2 Time Domain Simulation

Figure 8 shows the time domain simulation for a step change in p_d from 1.0 p.u. to 1.05 p.u. at $t = 1$ s. As expected fromt he small-signal stability analysis above, the system trajectory is stable. The new equilibirum point is reached aftaer about 15 s.

Figure 9 shows the time domain simulation for a step change in p_d from 1.05 p.u. to 1.09 p.u. at $t = 1$ s. For $t > 1$ s, the system does not have a stable equilibrium point and shows undamped oscillations. For $t \approx 1.06$ s, the system collapses.

2.5 Conclusions

This report has presented a case study on the effects of a SVC controller on the stability of single-machine dynamic-load test system. The stability is defined on the well-known eigenvalue analysis and time domain simulations. The main results obtained in this report are summarized as follows:

1. The SMDL without SVC presents a monotonic instability that limits the loadability of the system. If the load active power is increased, the system can show a voltage collapse.

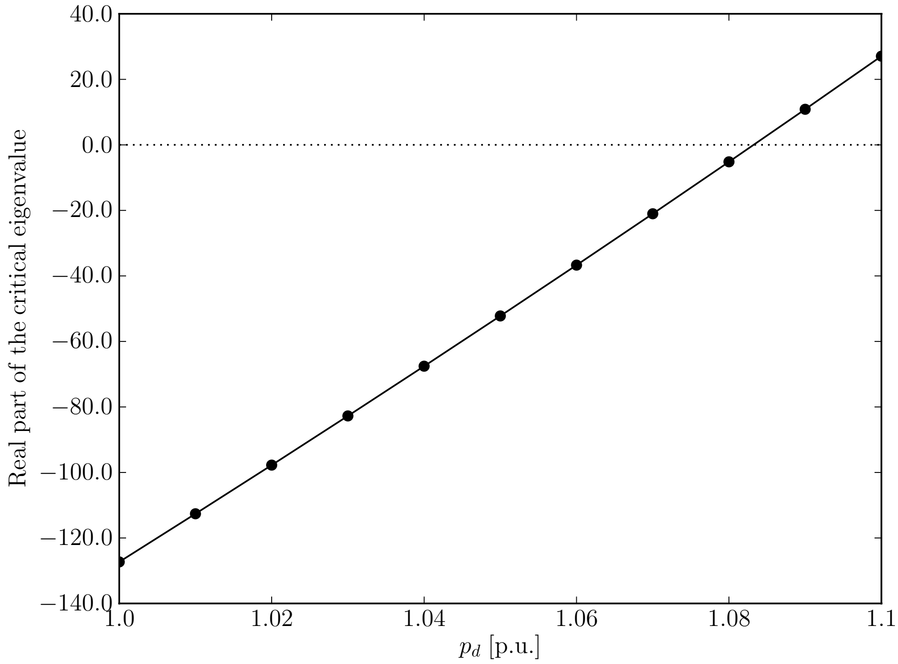

Figure 7: SMDL system with SVC: real part of the critical eigenvalue as a function of the load active power demand p_d.

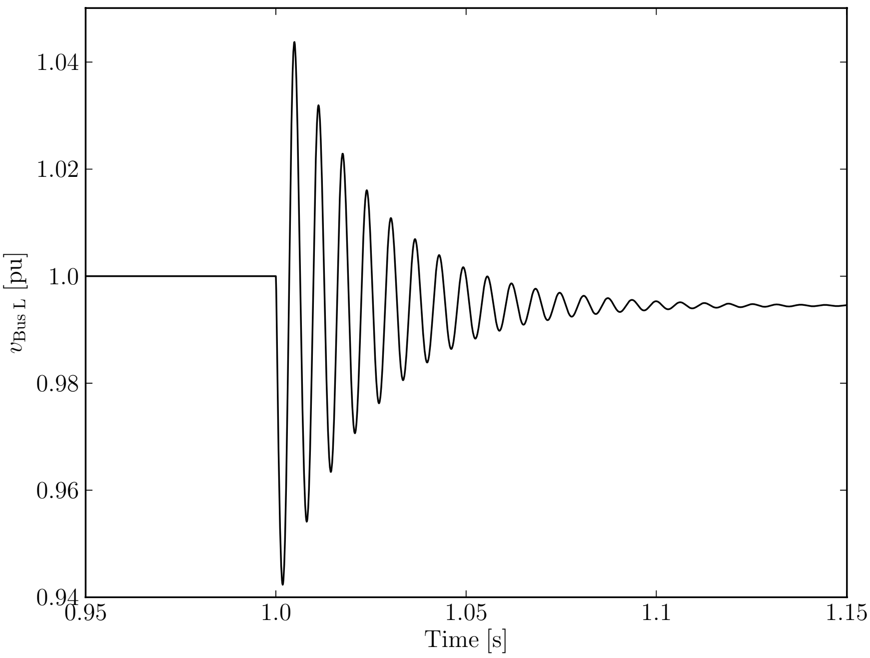

Figure 8: SMDL system with SVC: voltage trajectory following an sudden load increase (from 1.0 to 1.05 p.u.).

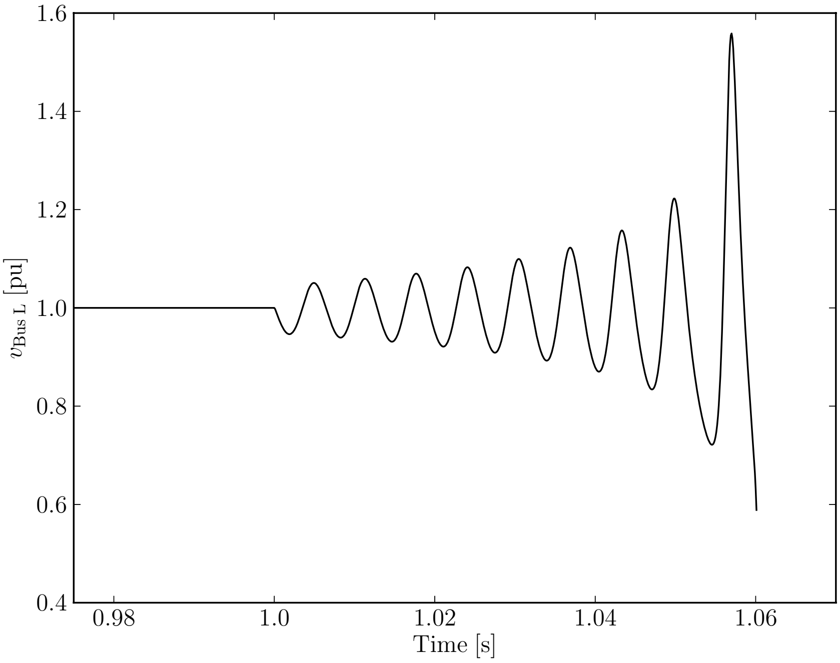

Figure 9: SMDL system with SVC: voltage collapse induced by a sudden load increase (from 1.05 to 1.09 p.u.).

2. If an SVC device is placed at the load bus, the loading level of the system can be increased. However, the inclusion of the SVC controller can lead to undamped oscillations.

A Notation

This Appendix provides the notation of the equations, variables and parameters used in this report. Abbreviations are also provided in this section.

State Variables

b_C	Susceptance of the SVC.
v	Load bus voltage.
δ	Generator rotor angle.
ω	Generator rotor speed.

Algebraic Variables

p_G	Generator active power.
q_L	Load active power.
q_d	Load reactive power demand.
q_L	Load reactive power.
q_T	Load and SVC reactive power.

Parameters

D	Generator damping.
e	Generator regulated bus voltage.
k	Constant used for simulating a constant power factor load ($k = \tan\phi$).
K_r	SVC controller gain.
M	Generator rotor inertia.
p_d	Load active power demand.
p_m	Generator mechanical power.
T_r	SVC controller time constant.
v_{ref}	Voltage reference of the SVC controller.
x_{eq}	Equivalent generator and transmission line reactance.
τ	Load time constant.
Ω_b	Synchronous frequency base in rad/s.

Abbreviations

DAE	Differential algebraic equations.
ODE	Ordinary differential equations.
SMDL	Single-machine dynamic-load.
SVC	Static var compensator.

B System Data

Constant parameters of the SMDL system are as follows: $M = 0.1$ s, $D = 0.1$, $x_{\mathrm{eq}} = 0.5$ p.u., $e = 1$ p.u., $k = 0.5$, $\tau = 0.001$ s, $K_r = 10$, $T_r = 0.01$ s. Values in p.u. are referred to the following bases: $S_b = 100$ MVA, $V_b = 220$ kV, and $f_b = 50$ Hz ($\Omega_b = 314.16$ rad/s).

The data file that describes the SMDL without SVC in Dome format is the following:

```
# DOME format version 1.0

Bus, Vn = 220.0, idx = 1, name = "Bus G"
Bus, Vn = 220.0, idx = 2, name = "Bus L"

Line, Vn = 220.0, Vn2 = 220.0, b = 0.0, bus1 = 1, bus2 = 2,
      fn = 50.0, idx = 1, name = "Line 1", r = 0.0, x = 0.5

Slack, Vn = 220.0, bus = 1, idx = 1, name = "Slack", v0 = 1.00

Synem, Sn = 100, Vn = 220, M = 0.1, D = 0.1, bus = 1, gen = 1,
       idx = 1, name = "Gen", xd1 = 0.00001

Vdyn, bus = 2, pd = 0.60, k = 0.5, tau = 0.001

Perturbation, function = "load", module = "pert"

Tuning, setting = "TDS", fixt = True, t0 = 0, tstep = 0.0001,
        tf = 4
```

Observe that `xd1` is set to a small value to impose $e \approx 1$ at the generator bus.

The data file that describes the SMDL with SVC in Dome format is the following:

```
# DOME format version 1.0

Bus, Vn = 220.0, idx = 1, name = "Bus G"
Bus, Vn = 220.0, idx = 2, name = "Bus L"

Line, Vn = 220.0, Vn2 = 220.0, b = 0.0, bus1 = 1, bus2 = 2,
      fn = 50.0, idx = 1, name = "Line 1", r = 0.0, x = 0.5

Slack, Vn = 220.0, bus = 1, idx = 1, name = "Slack", v0 = 1.00

Synem, Sn = 100, Vn = 220, M = 0.1, D = 0.1, bus = 1, gen = 1,
       idx = 1, name = "Gen", xd1 = 0.00001
```

```
Vdyn, bus = 2, pd = 1.0, k = 0.5, tau = 0.001

Svc1, bus = 2, Kr = 10, Tr = 0.01, v0 = 1

Perturbation, function = "load", module = "pert"

Tuning, setting = "TDS", fixt = True, t0 = 0, tstep = 0.0001,
        tf = 4
```

The Python script the defines the load increase is reported below:

```
# modules that defines the load step variation

def load(system, t):

    """define a load step at t = 1 s"""

    p1 = 1.05
    p2 = 1.09
    k = 0.5

    if t <= 1:
        system.Vdyn.upd[0] = p1
        system.Vdyn.uqd[0] = k * p1
    else:
        system.Vdyn.upd = p2
        system.Vdyn.uqd = k * p2
```

References

[1] W. Gu, F. Milano, P. Jang, G. Tang, Bifurcations Induced by SVC Controllers: A Didactic Example, Electric Power Systems Research 77 (2007) 234–240.

[2] C. A. Cañizares, On Bifurcation Voltage Collapse and Load Modeling, IEEE Trans. Power Systems 10 (1) (1995) 512–522.

[3] N. Mithulananthan, C. A. Cañizares, J. Reeve, G. J. Rogers, Comparison of PSS, SVC and STATCOM Controllers for Damping Power System Oscillations, IEEE Trans. Power Systems 18 (2) (2003) 786–792.