Small-signal Stability

POWER SYSTEM STABILITY ANALYSIS (EEEN40340)

Prof. Federico Milano

Email: federico.milano@ucd.ie

Tel.: 01 716 1844

Room 157a – Engineering & Materials Science Centre

School of Electrical & Electronic Engineering

University College Dublin

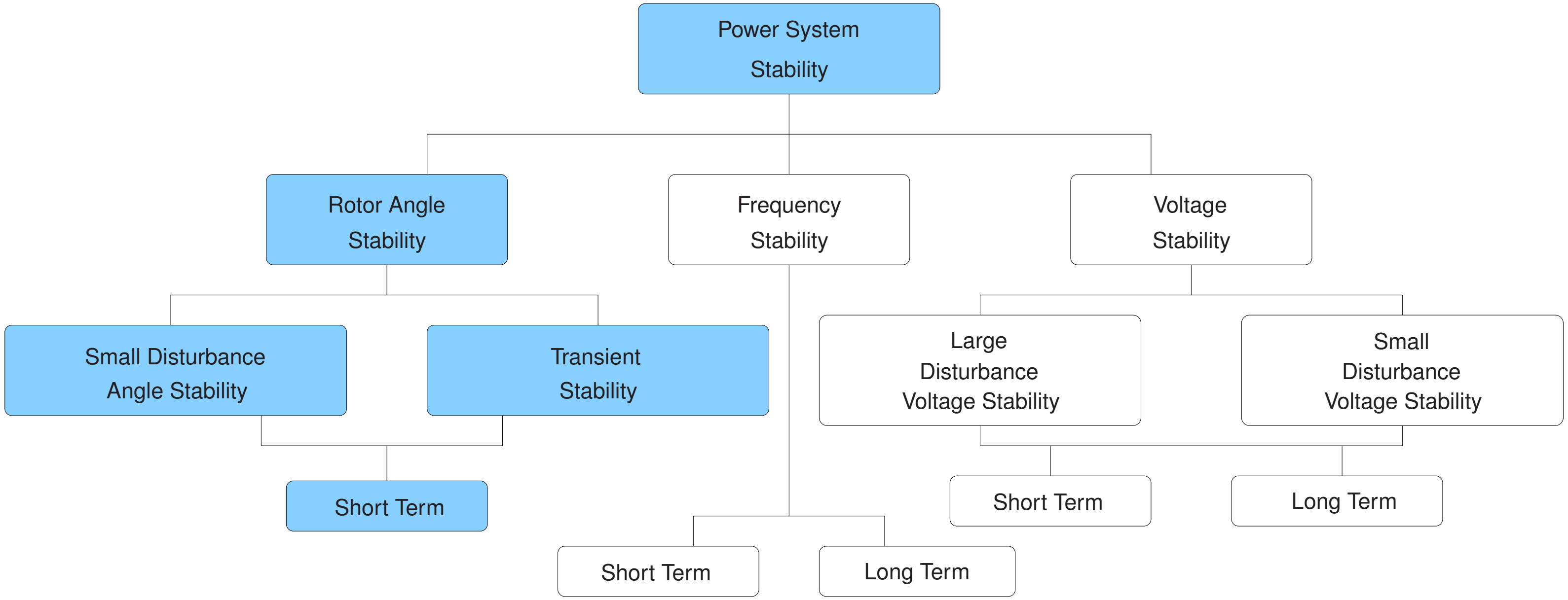
Dublin, Ireland

Angle Stability Outlines

- Definitions.
- Small-signal Stability:
  - Hopf Bifurcations.
  - Control and mitigation.
  - Practical example.
- Transient Stability
  - Time Domain.
  - Direct Methods.
    - Lyapounov function.
    - Equal Area Criterion.
    - Energy functions.
  - Practical applications.

Angle Stability Definitions

- IEEE-CIGRE classification (IEEE/CIGRE Joint Task Force on Stability) Terms and Definitions, “Definitions and Classification of Power System Stability”, *IEEE Trans. Power Systems and CIGRE Technical Brochure 231*, 2003:



Angle Stability Definitions

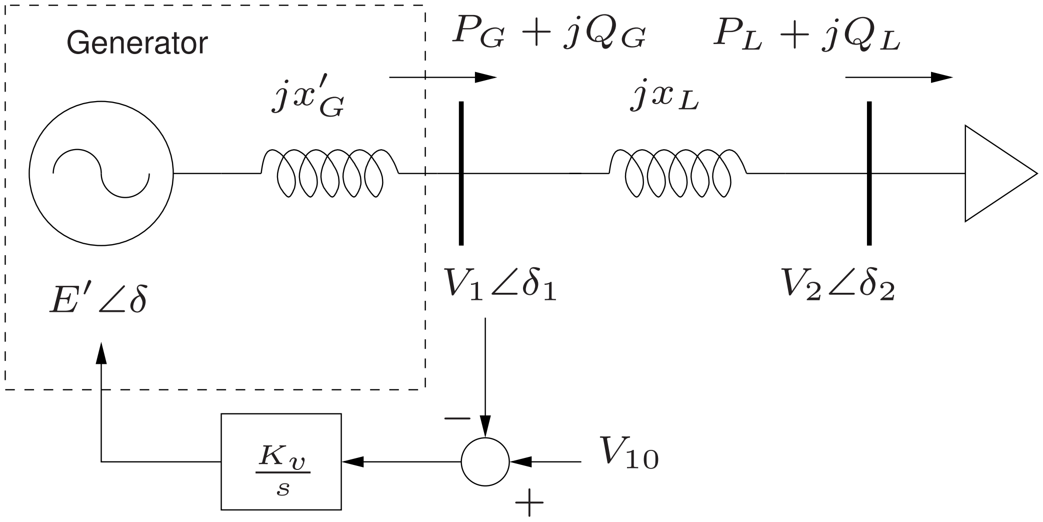
- "*Rotor angle stability* refers to the ability of synchronous machines of an interconnected power system to remain in synchronism after being subjected to a disturbance. It depends on the ability to maintain/restore equilibrium between electromagnetic torque and mechanical torque of each synchronous machine in the system."
- In this case, the problem becomes apparent through angular/frequency swings in some generators which may lead to their loss of synchronism with other generators.

Small Disturbance

- "*Small disturbance* (or *small signal*) *rotor angle stability* is concerned with the ability of the power system to maintain synchronism under small disturbances. The disturbances are considered to be sufficiently small that linearization of system equations is permissible for purposes of analysis."
- This problem is usually associated with the appearance of undamped oscillations in the system due to a lack of sufficient damping torque.
- Theoretically, this phenomenon may be associated with a s.e.p. becoming unstable through a Hopf bifurcation point, typically due to contingencies in the system (e.g. August 1996 West Coast Blackout).

Hopf Bifurcation I

- For the generator-load example, with AVR but no Q_G limits:



Hopf Bifurcation I

- The DAE model is:

$$
\begin{aligned}
\dot{\omega} &= \frac{1}{M}\left(P_m \frac{E'V_2}{X}\sin\delta - D_G\omega\right) \\
\dot{\delta} &= \omega - \frac{1}{D_L}\left(\frac{E'V_2}{X}\sin\delta - P_d\right) \\
\dot{E}' &= K_v(V_{10} - V_1) \\
\dot{V}_2 &= \frac{1}{\tau}\left(-\frac{V_2^2}{X} + \frac{E'V_2}{X}\cos\delta - kP_d\right) \\
0 &= \frac{V_1V_2}{X_L}\sin\delta' - \frac{E'V_2}{X}\sin\delta \\
0 &= V_2^2\left(\frac{1}{X_L} - \frac{1}{X}\right) + \frac{E'V_2}{X}\cos\delta - \frac{V_1V_2}{X_L}\cos\delta'
\end{aligned}
$$

Hopf Bifurcation I

- Observe that the algebraic constraint can be eliminated, since:

$$\begin{aligned} V_{1r} &= V_1 \cos\delta' \\ V_{1i} &= V_1 \sin\delta' \end{aligned}$$

Thus:

$$0 = \frac{V_{1i}V_2}{X_L} - \frac{E'V_2}{X}\sin\delta \Rightarrow V_{1i} = \frac{E'X_L}{X}\sin\delta$$

and

$$\begin{aligned} 0 &= V_2^2\left(\frac{1}{X_L} - \frac{1}{X}\right) + \frac{E'V_2}{X}\cos\delta - \frac{V_{1r}V_2}{X_L} \\ \Rightarrow \quad V_{1r} &= V_2\left(1 - \frac{1}{X_L}\right) + \frac{E'X_L}{X}\cos\delta \end{aligned}$$

Hopf Bifurcation I

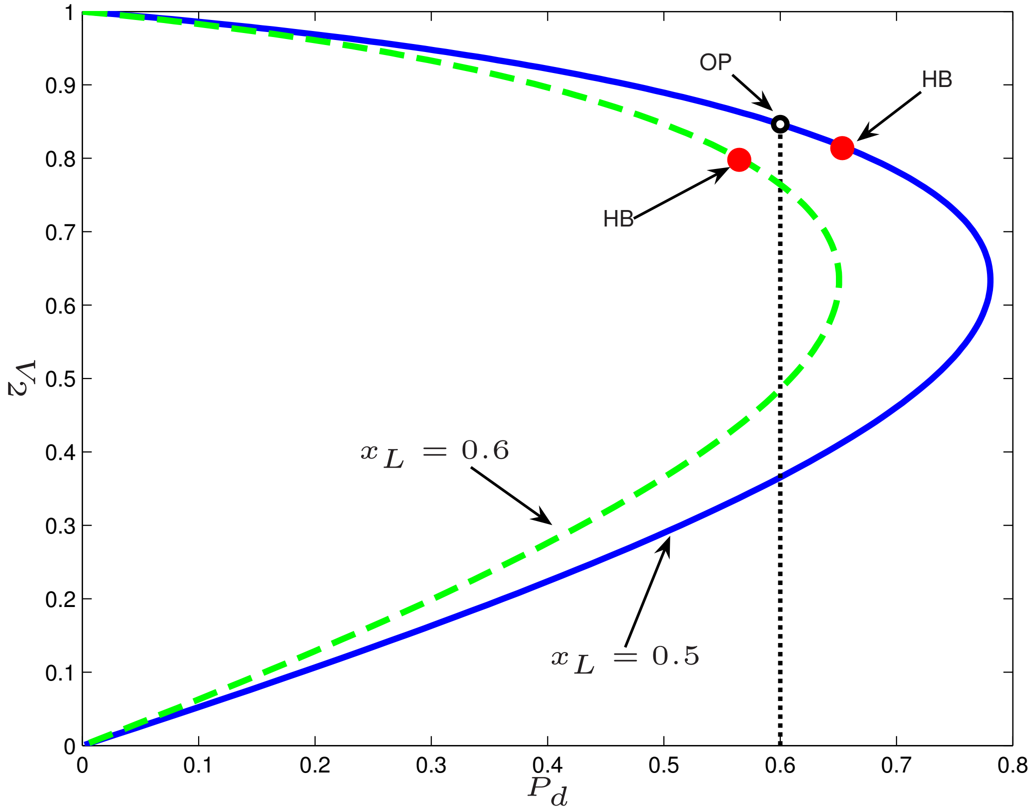
- This yields the following equations, which are better for numerical time domain simulations:

$$
\begin{aligned}
\dot{\omega} &= \frac{1}{M}\left(P_m \frac{E' V_2}{X}\sin\delta - D_G\omega\right)\\
\dot{\delta} &= \omega - \frac{1}{D_L}\left(\frac{E' V_2}{X}\sin\delta - P_d\right)\\
\dot{E}' &= K_v\left(V_{10} - \sqrt{V_{1r}^2 + V_{1i}^2}\right)\\
\dot{V}_2 &= \frac{1}{\tau}\left(-\frac{V_2^2}{X} + \frac{E' V_2}{X}\cos\delta - k P_d\right)
\end{aligned}
$$

- Observe that in this case, $P_m = P_d$, i.e. generation and load are assumed to be balanced.

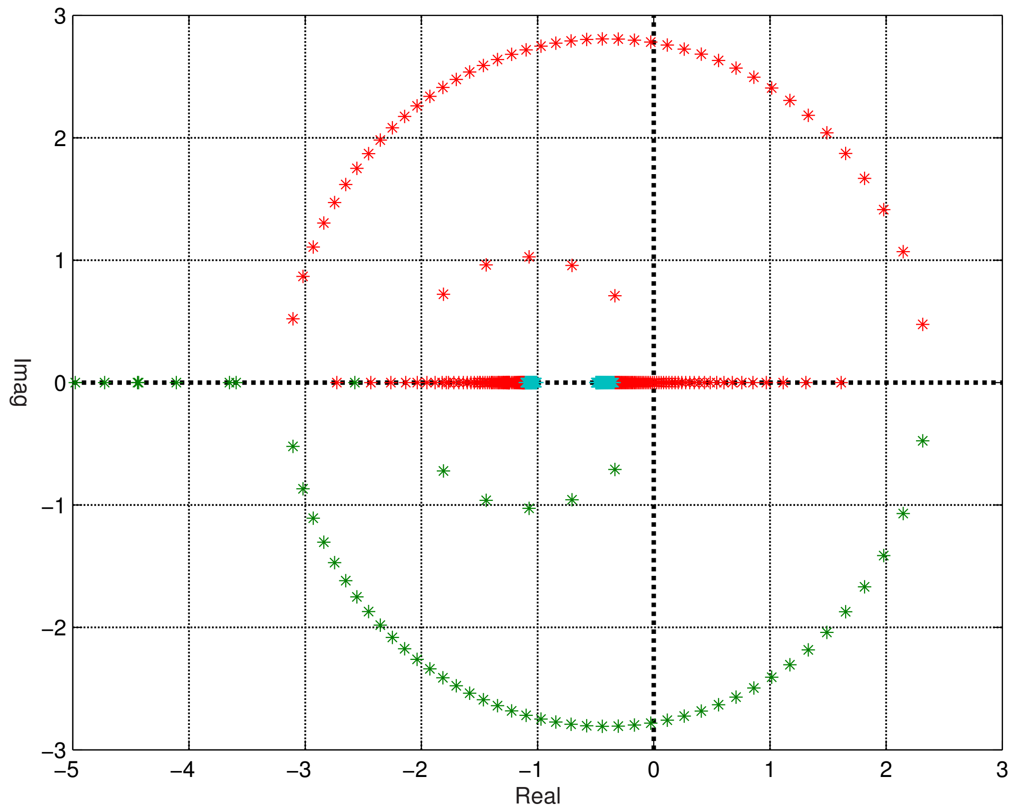
Hopf Bifurcation I

- The PV curves for $M = 0.1$, $D_G = 0.01$, $D_L = 0.1$, $\tau = 0.01$, $K_v = 10$, $X'_G = 0.5$, $V_{10} = 1$, $k = 0.25$ are:



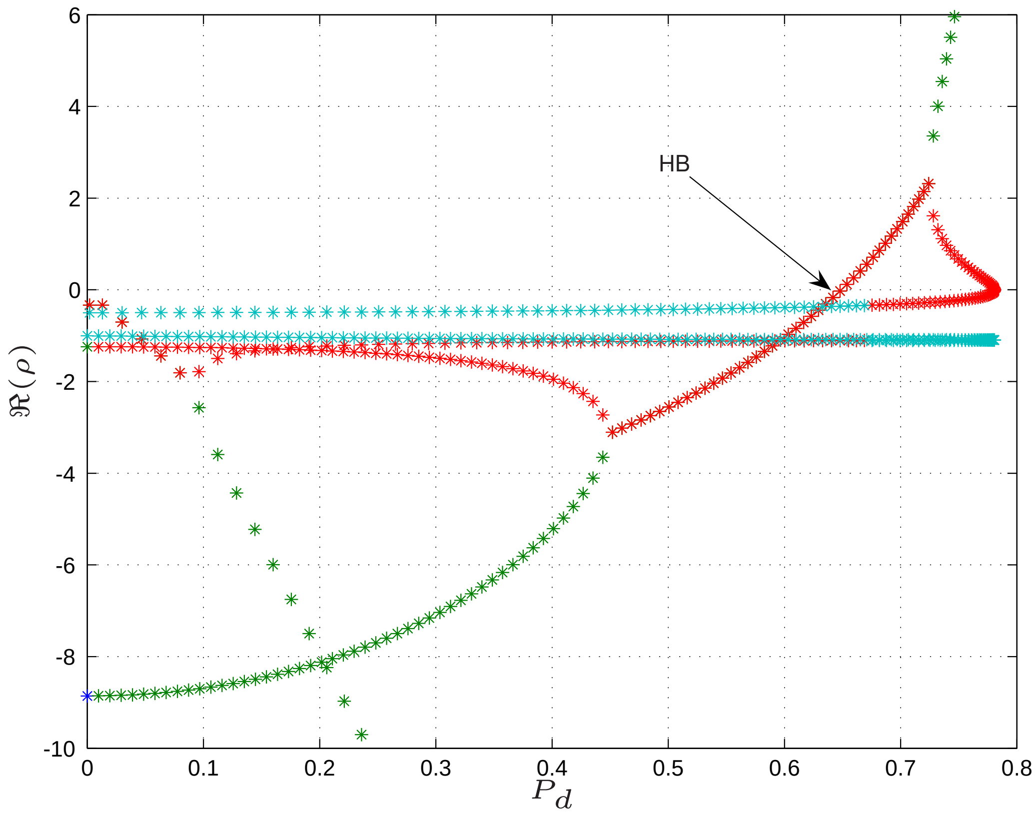
Hopf Bifurcation I

- The eigenvalues for the system with respect to changes in P_d for $x_L = 0.5$:



Hopf Bifurcation I

- There is a Hopf bifurcation for $P_d = 0.65$, $x_L = 0.5$:



Hopf Bifurcation I

- A Hopf bifurcation with eigenvalues $\rho = \pm j\beta$ yields a periodic oscillation of period:

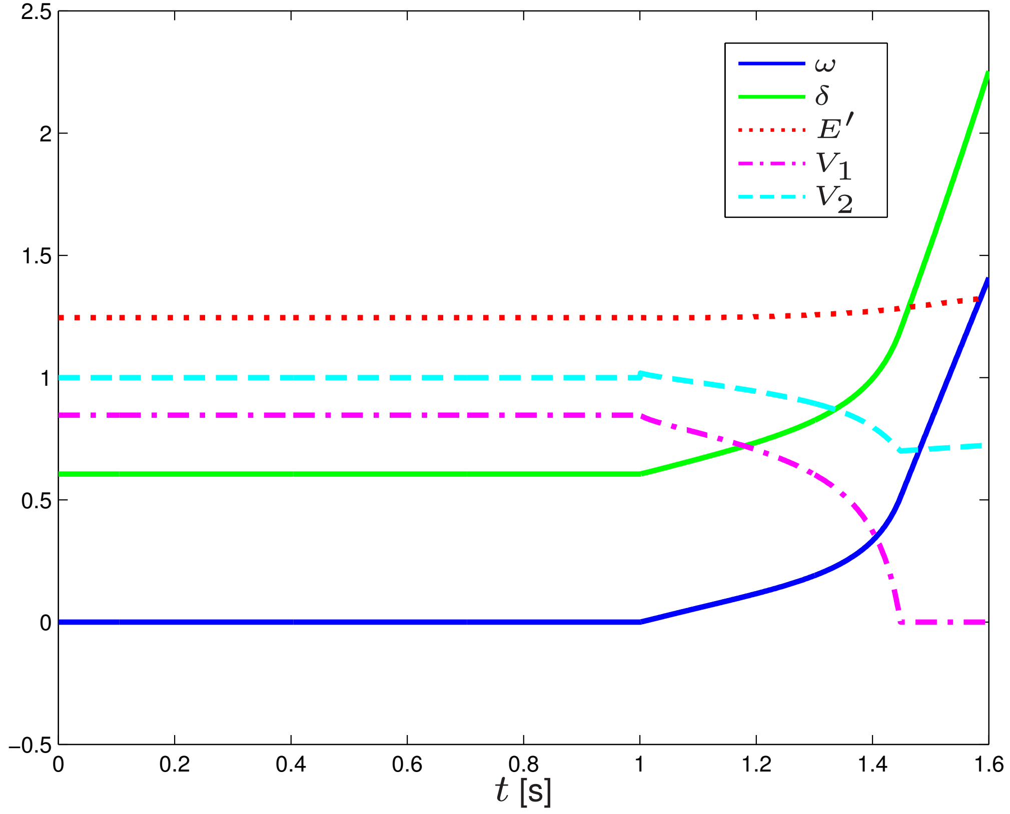
$$T = \frac{2\pi}{\beta}$$

- Hence, for the example:

$$\begin{aligned} \rho &\approx \pm j3 \\ \Rightarrow \quad T &\approx 2 \text{ s} \end{aligned}$$

Hopf Bifurcation I

- The contingency $x_L = 0.5 \rightarrow 0.6$ yields:

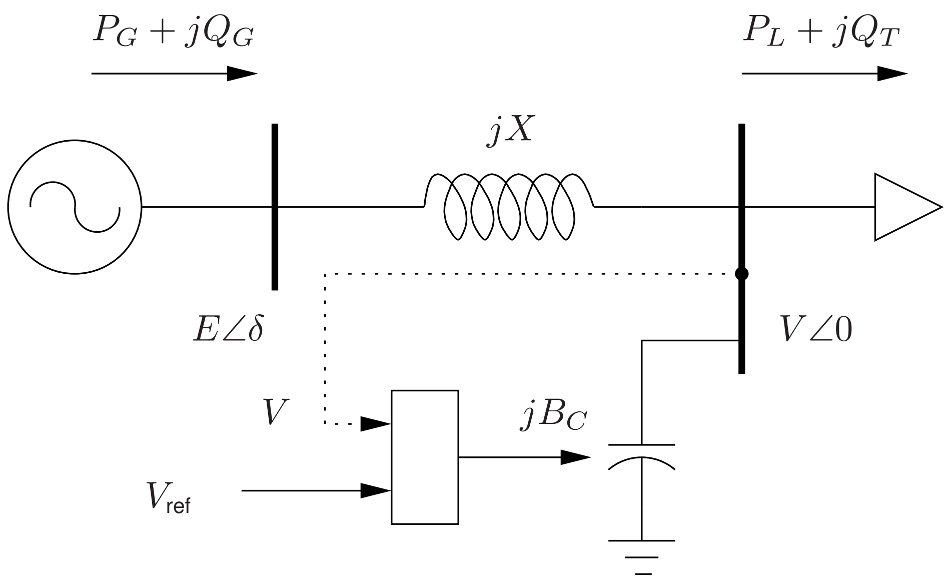


Hopf Bifurcation I

- Notice that in this case no oscillations are observed, which is a “trademark” of Hopf bifurcations and small-disturbance angle instabilities.
- The reason for this is that the oscillation period is 2 s (typical in practice where these kinds of oscillations are in the 0.1-1 Hz range), but the bus voltage collapses well before the oscillations appear, which is atypical and is probably due to the chosen impedances and time constants.
- This example stresses the point that angle instabilities do lead to voltage collapse, and vice versa, voltage instabilities lead to angle/frequency oscillations, even though the reason behind each stability problem are fairly different.

Hopf Bifurcation II

- Single-machine dynamic-load system with SVC:

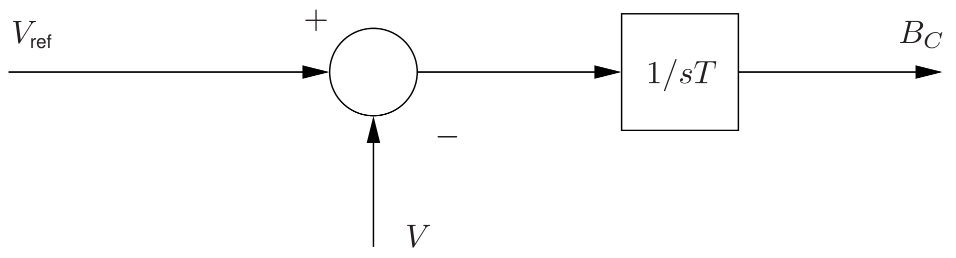


Hopf Bifurcation II

- The total reactive power absorbed by the load and the SVC is as follows:

$$Q_T(V, \delta) = -\frac{V^2}{X} + \frac{EV}{X} \cos(\delta) + V^2 B_C$$

- The SVC controller is modeled as a first order pure integrator.



Hopf Bifurcation II

- The resulting differential equations of the SMDL system with SVC are as follows:

$$
\begin{aligned}
\dot{\delta} &= \omega \\
\dot{\omega} &= \frac{1}{M}[P_d - \frac{EV}{X}\sin(\delta) - D\omega] \\
\dot{V} &= \frac{1}{\tau}[-kP_d + V^2(B_C - \frac{1}{X}) + \frac{EV}{X}\cos(\delta)] \\
\dot{B}_C &= \frac{1}{T}(V_{\text{ref}} - V)
\end{aligned}
$$

Hopf Bifurcation II

- B_C is the equivalent susceptance of the SVC; T and V_{ref} are the SVC time constant and reference voltage, respectively.
- In the following, it is assumed that $T = 0.01$ s and $V_{\text{ref}} = 1.0$ p.u.
- Observe that also in this case it is possible to deduce the set of ODE, i.e. the algebraic variables can be explicitly expressed as a function of the state variables and the parameters.

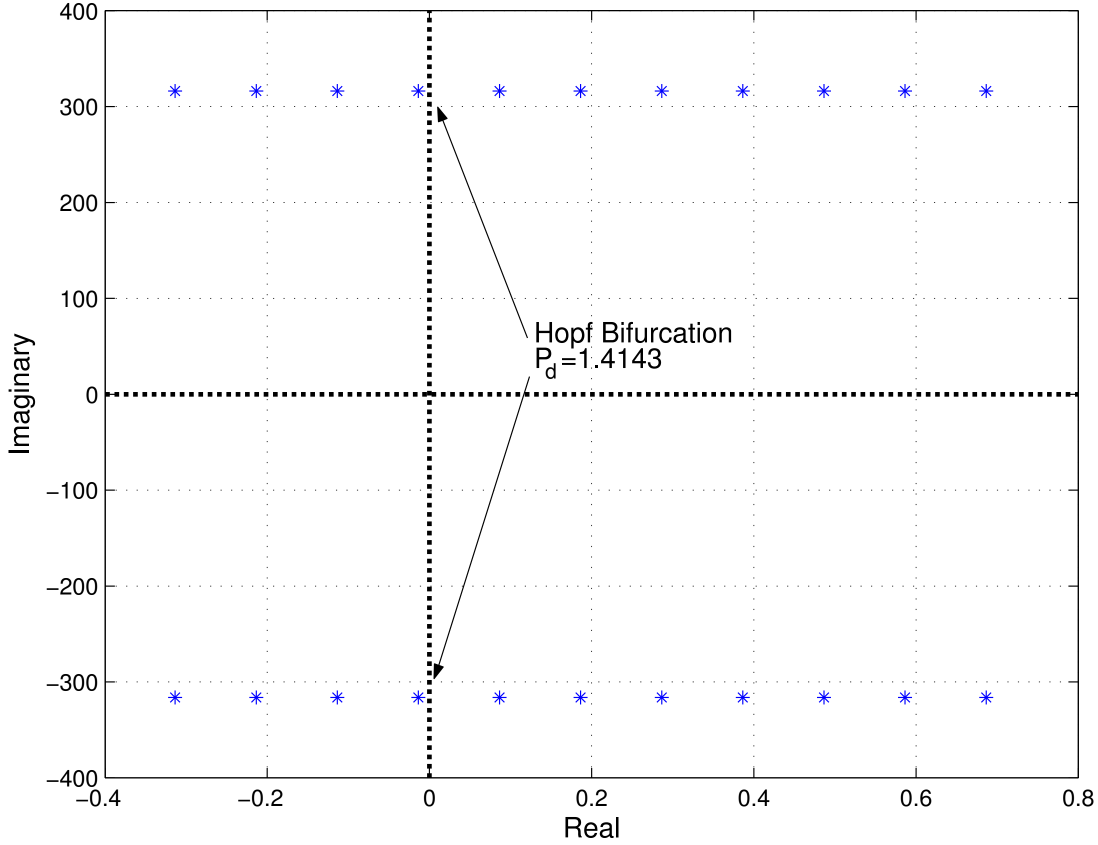
Hopf Bifurcation II

- The state matrix of the system is as follows:

$$A = \left[\begin{array}{c|c|c|c} 0 & 1 & 0 & 0 \\ \hline -\frac{EV}{MX}\cos(\delta) & -\frac{D}{M} & -\frac{E}{MX}\sin(\delta) & 0 \\ \hline -\frac{EV}{\tau X}\sin(\delta) & 0 & \frac{1}{\tau X}[E\cos(\delta) - 2V + 2VB_C X] & \frac{V^2}{\tau} \\ \hline 0 & 0 & -\frac{1}{T} & 0 \end{array}\right]$$

Hopf Bifurcation II

- Eigenvalue loci:



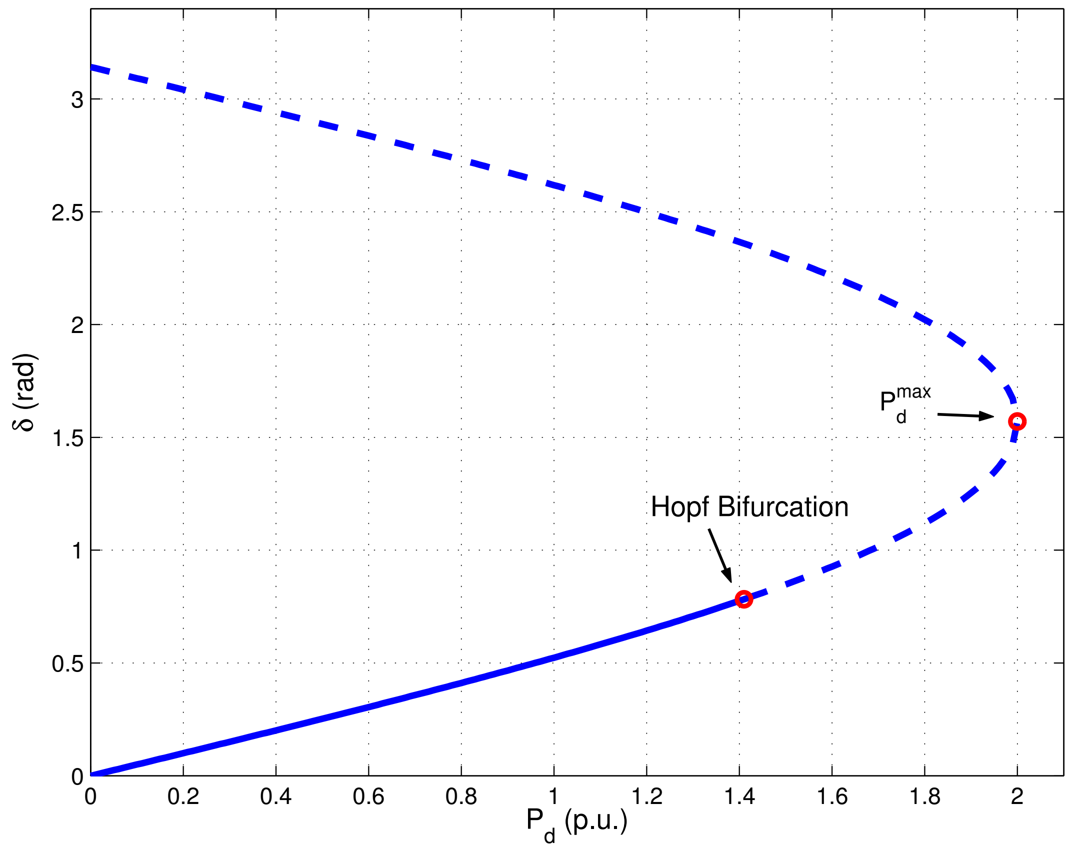
Hopf Bifurcation II

- A complex conjugate pair of eigenvalues crosses the imaginary axis for $P_d = 1.4143$, thus leading to a Hopf bifurcation.
- The HB point is:

$$(\delta_0, \omega_0, V_0, B_{C0}, P_{d0}) = (0.7855, 0, 1, 1.2930, 1.4143)$$

Hopf Bifurcation II

- Bifurcation diagram P_d-δ:

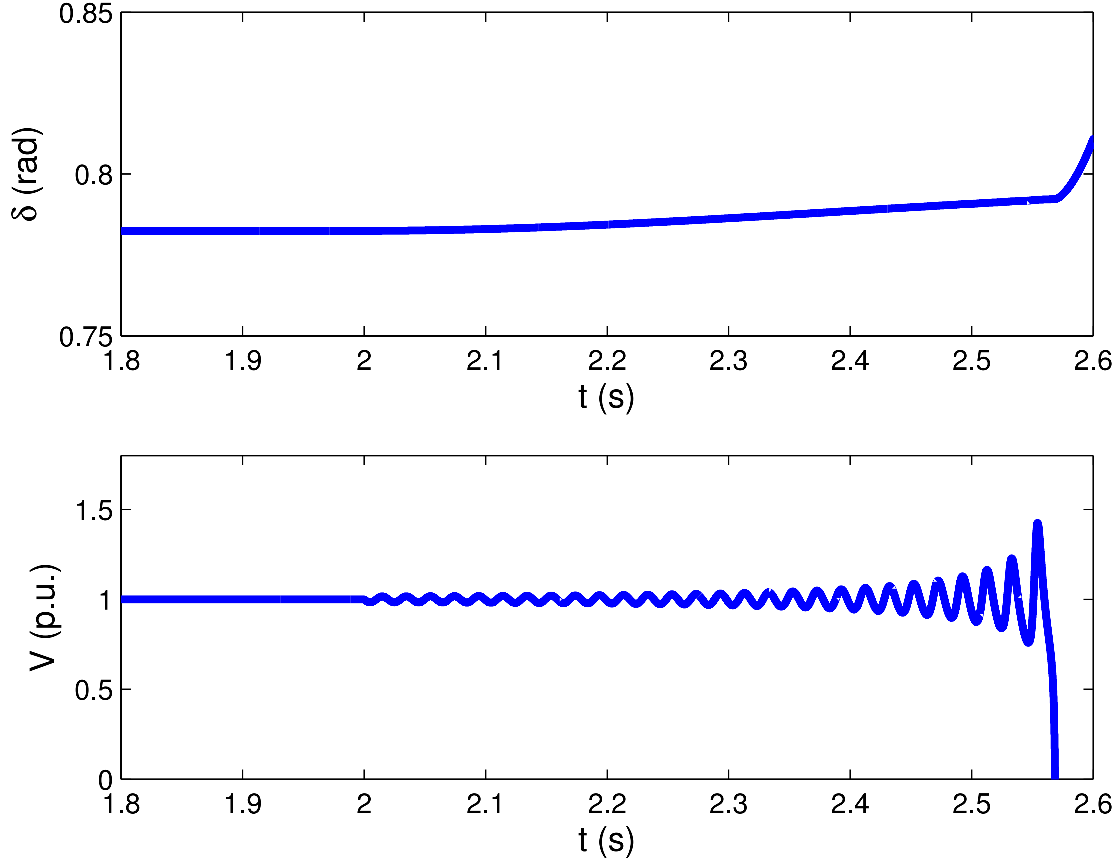


Hopf Bifurcation II

- We simulate a step change in P_d from 1.41 p.u. to 1.42 p.u. for $t = 2$ s.
- For $t > 2$ s the system does not present a stable equilibrium point and shows undamped oscillations (likely an unstable limit cycle), as expected from the P-δ curve.
- For $t = 2.57$ s, the load voltage collapses.
- Note that, in this case, the generator angle shows an unstable trajectory only after the occurrence of the voltage collapse at the load bus.

Hopf Bifurcation II

- Time domain simulation results:

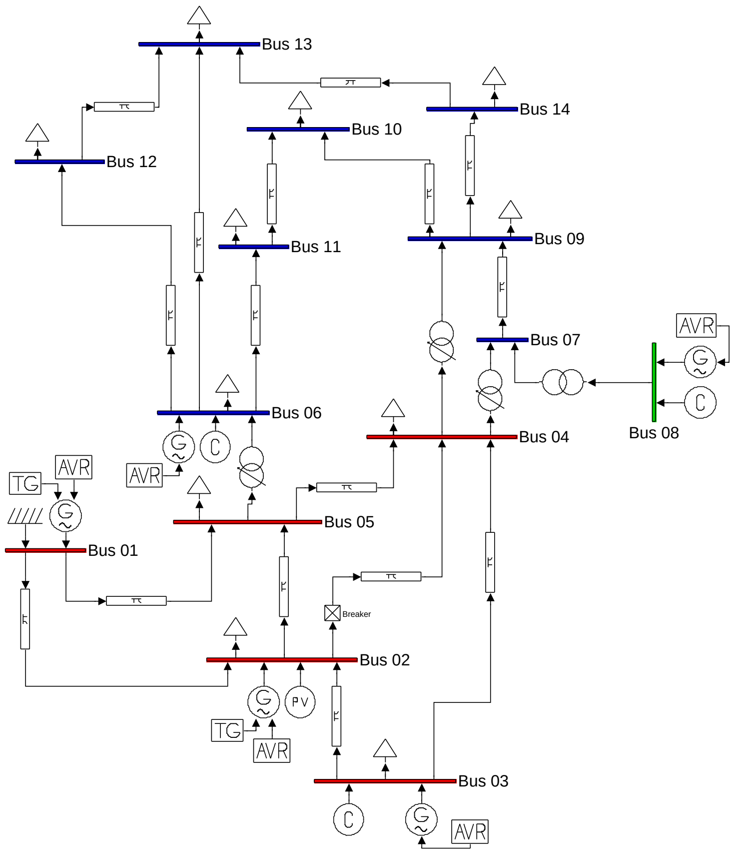


Hopf Bifurcation II

- The use of the SVC device gives a birth to a new bifurcation, namely a Hopf bifurcation.
- This Hopf bifurcation cannot be removed by simply adjusting system parameters.
- However SVC and load dynamics can be coordinated so that the loadability of the system can be increased.

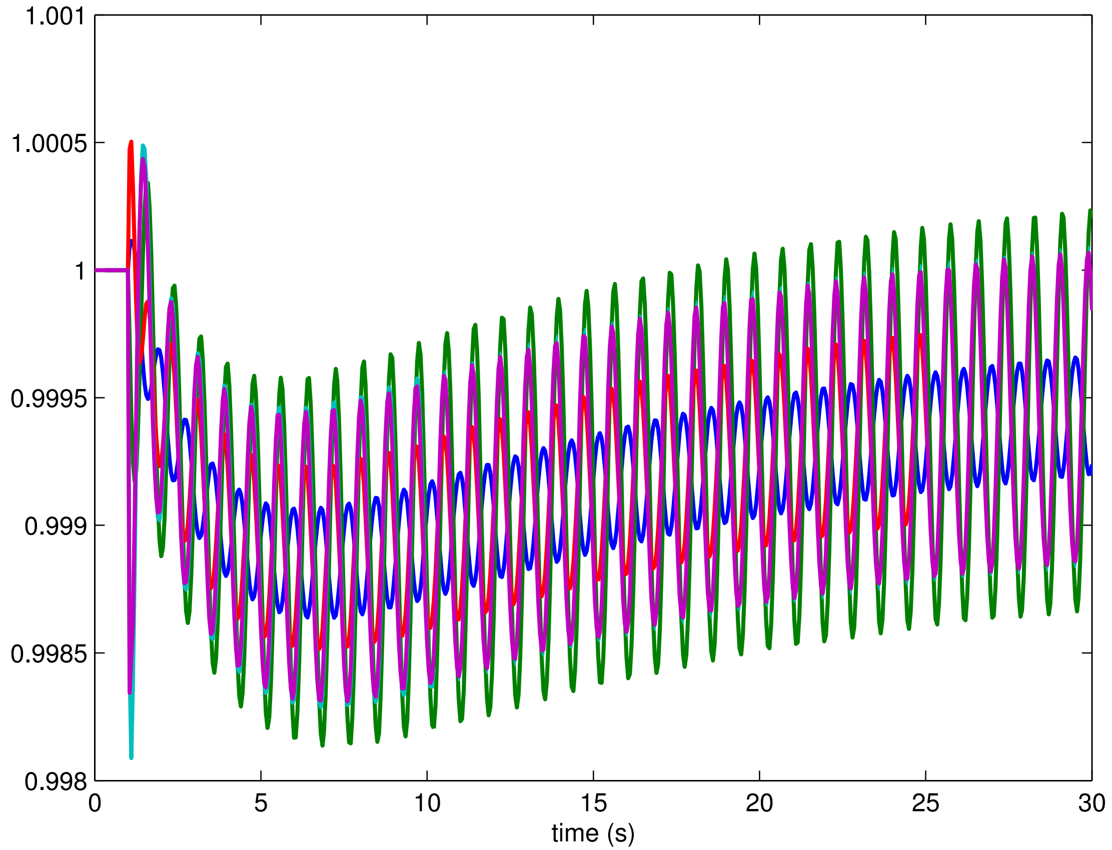
Control and Mitigation

- For the IEEE 14-bus test system:



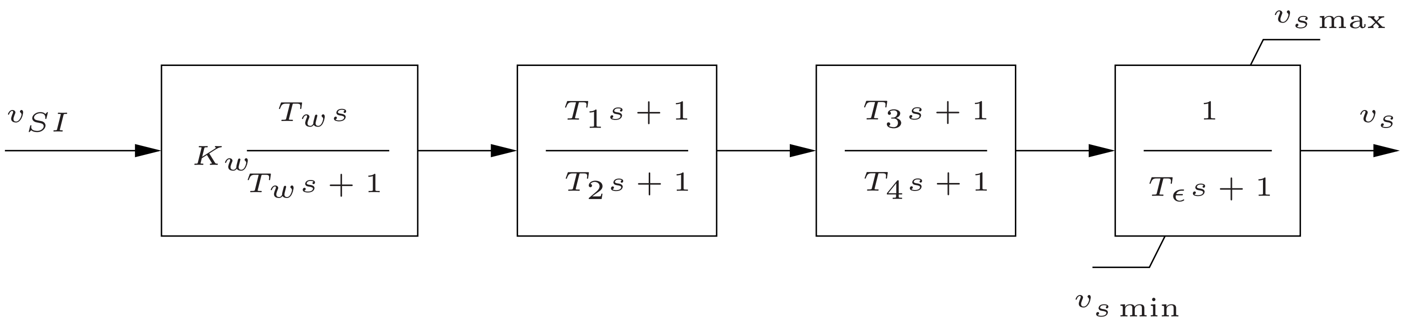
Control and Mitigation

- Generator speeds for the line 2-4 outage and 40% overloading:



Control and Mitigation

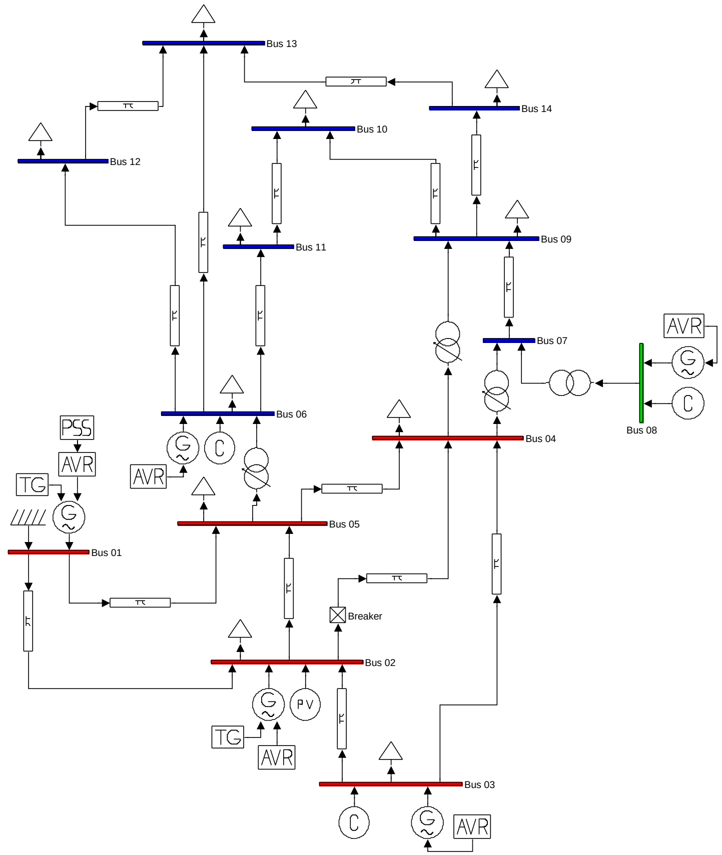
- This has been typically solved by adding Power System Stabilizers (PSS) to the voltage controllers in “certain” generators, so that equilibriun point is made stable, i.e. the Hopf is removed.



- FACTS can also be used to address this problem.

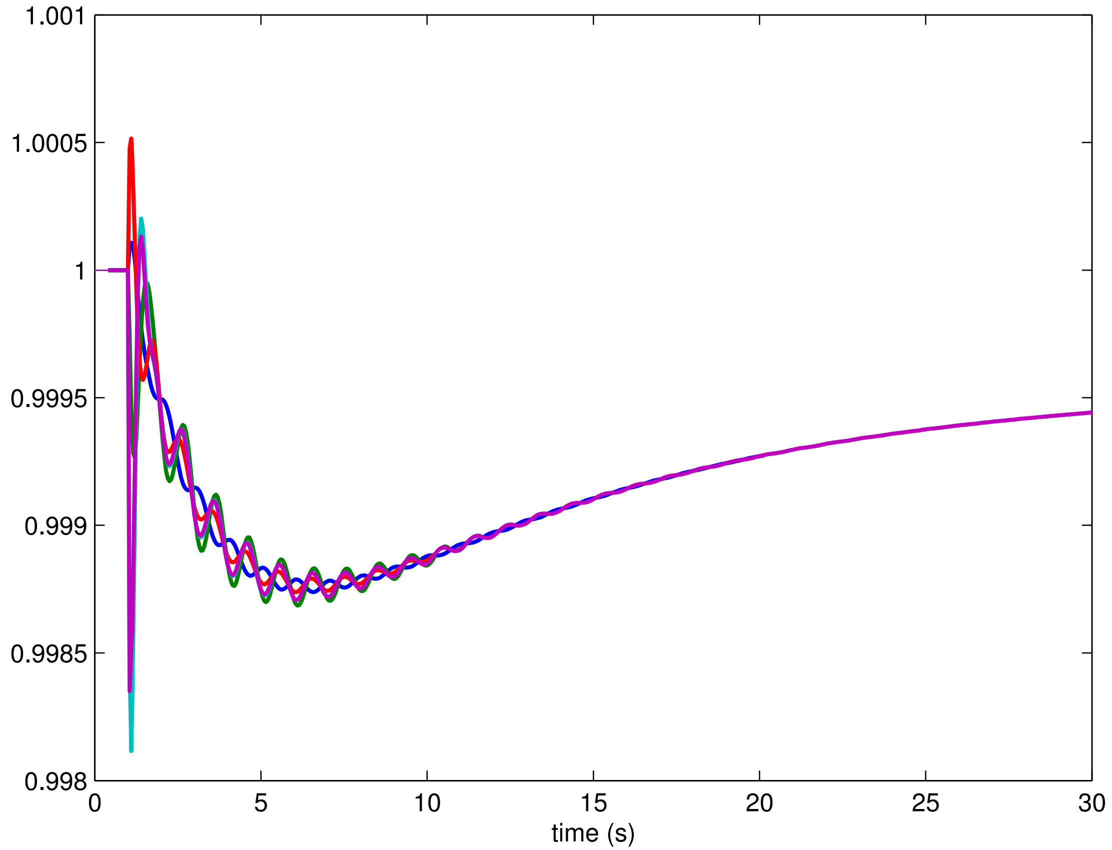
Control and Mitigation

- For the IEEE 14-bus test system with PSS at bus 1:



Control and Mitigation

- Generator speeds for the line 2-4 outage and 40% overloading:



Control and Mitigation

- Data regarding this system are available at:

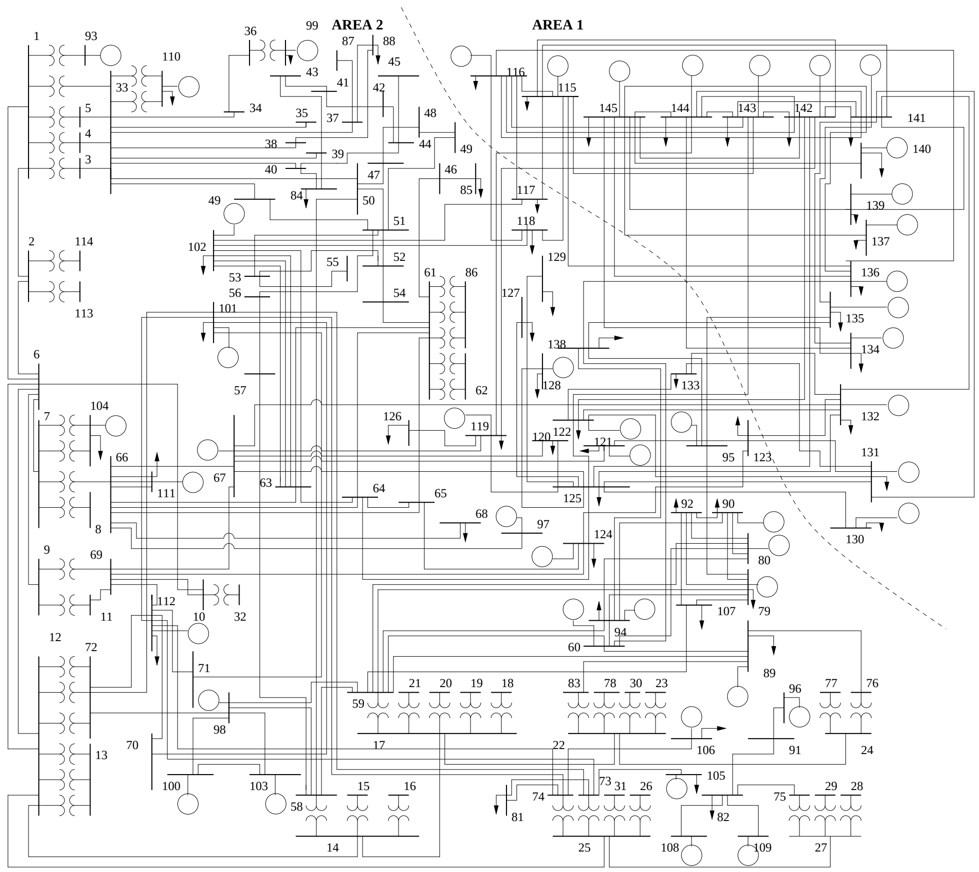
  http://thunderbox.uwaterloo.ca/∼claudio /papers/IEEEBenchmarkTFreport.pdf

- More details regarding this example can be found in:

  F. Milano, “An Open Source Power System Analysis Toolbox”, accpeted for publication on *IEEE Trans. On Power Systems*, March 2004.

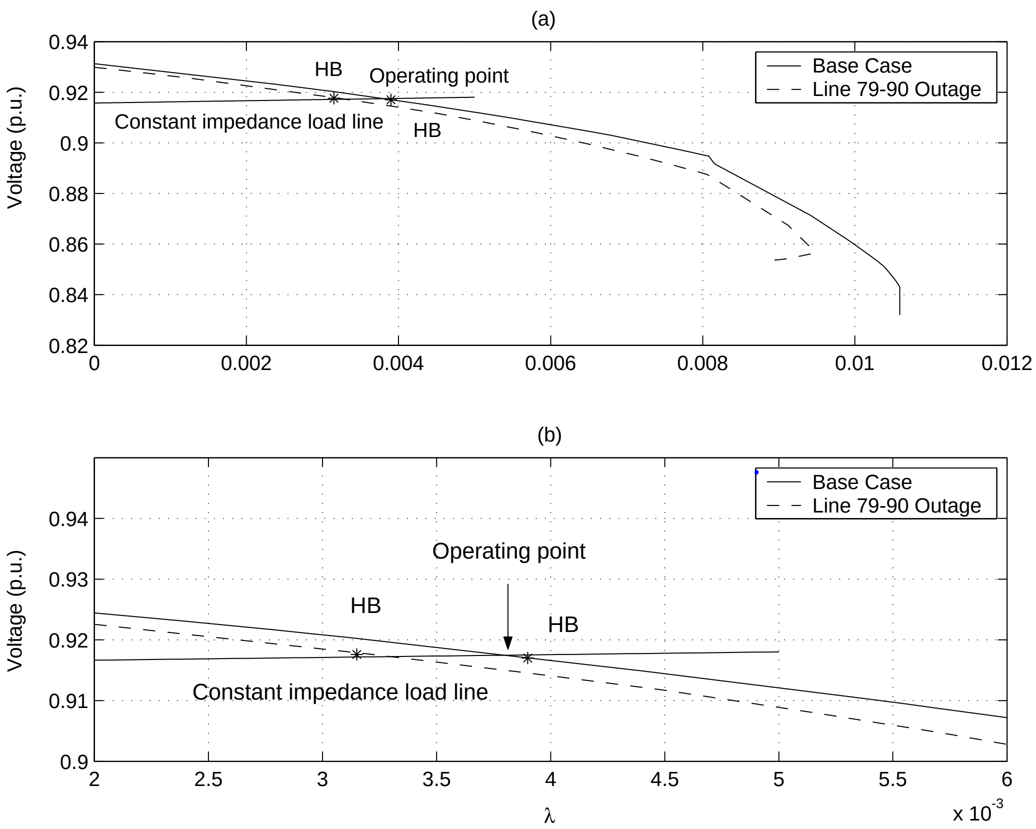
Control and Mitigation

- For the IEEE 145-bus, 50-machine test system:



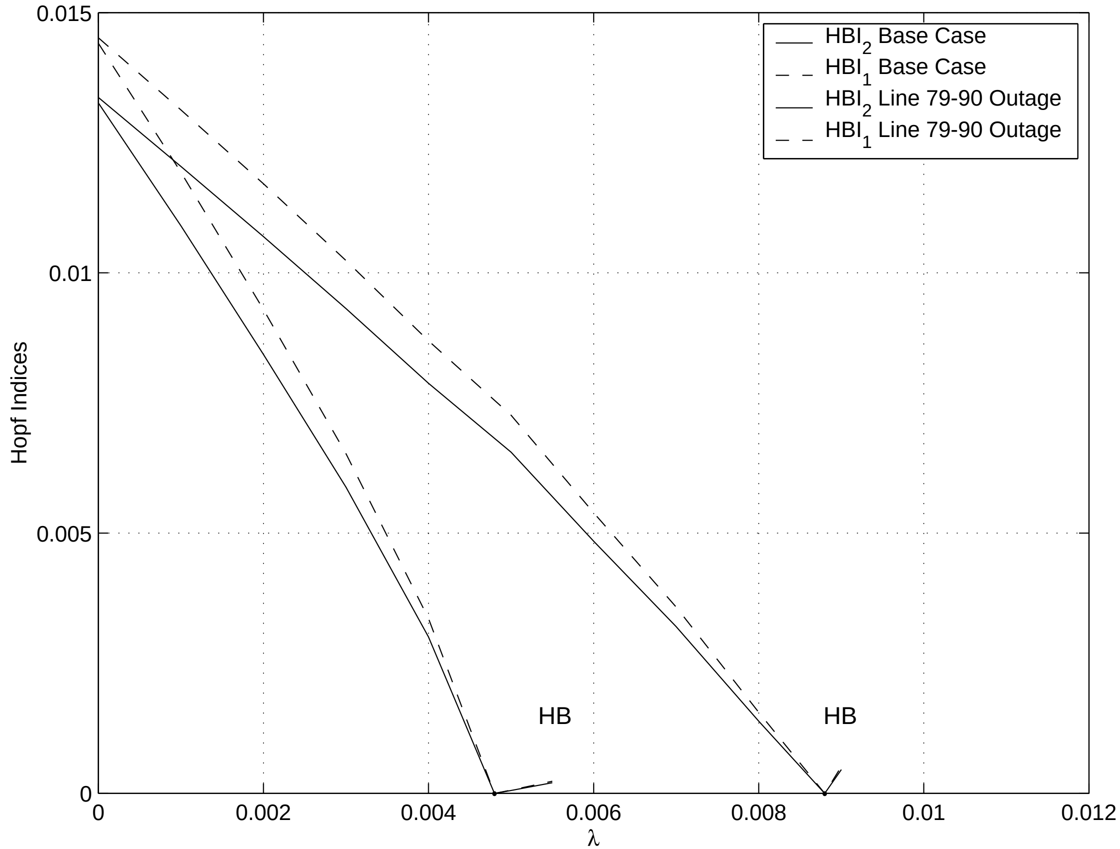
Control and Mitigation

- For an impedance load model, the PV curves yield:



Control and Mitigation

- Indices based on the singular values have been proposed to predict Hopf bifucations:



Control and Mitigation

- More details regarding this example can be found in:

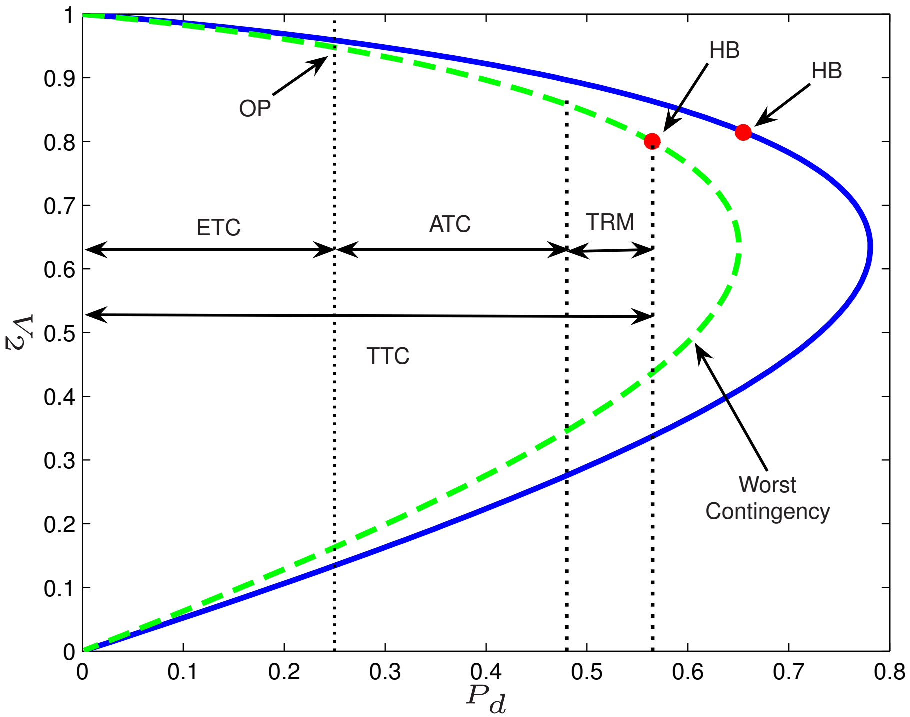
  C. A. Cañizares, N. Mithulananthan, F. Milano, and J. Reeve, “Linear Performance Indices to Predict Oscillatory Stability Problems in Power Systems”, *IEEE Trans. On Power Systems*, Vol. 19, No. 2, May 2004, pp. 1104-1114.

Small Disturbance Applications

- In practice, some contingencies trigger plant or inter-area frequency oscillations in a "heavily" loaded system, which may be directly associated with Hopf bifurcations.
- This is a "classical" problem in power systems and there are many examples of this phenomenon in practice, such as the August 10, 1996 blackout of the WSCC (now WECC) system.

Small Disturbance Applications

- Observe that the maximum loadability of the system is reduced by the presence of the Hopf bifurcation.
- This leads to the definition of a "dynamic" ATC value.



Impact of Wind Generation on Small-Signal Stability

- If the penetration of wind power is high and the voltage of the grid connection point is regulated, then it is possible that the interaction among wind generators and conventional synchronous machines produces undamped oscillations (Hopf bifurcation).
- The solution is using power sytem stabilizers, SVCs or special power oscillation dampers.