Power System Stability Analysis

Example of Exam Paper I

Problem 1

Consider the Volterra-Lotka equations, also known as the *predator-prey equations*:

$$\begin{aligned} \dot{x} &= x(\alpha - \beta y) \\ \dot{y} &= -y(\gamma - \delta x) \end{aligned} \tag{1}$$

where x is the number of preys (e.g., rabbits), and y is the number of predators (e.g., foxes). Determine:

a. The equilibrium points of (1). Considering the physical meaning of x and y, assign a physical meaning to the equilibrium points. 20%

b. The stability of the equilibrium points assuming that all parameters α, β, γ and δ are positive. 20%

c. Using the phase-plane, sketch some system trajectories around the equilibrium points and indicate the direction of the flows. 20%

d. Assume that a delay τ is introduced in (1) as follows: 20%

$$\begin{aligned} \dot{x} &= x(\alpha - \beta y(t - \tau)) \\ \dot{y} &= -y(\gamma - \delta x(t - \tau)) \end{aligned}$$

and write the characteristic equation of the delay ODE.

e. Decide whether the following function:

$$V = \frac{\delta}{2}x^2 + \frac{\beta}{2}y^2 - \alpha\delta x - \beta\gamma y + \beta\delta xy$$

is a Lyapunov function for (1) and $\alpha = \beta = \gamma = \delta = 1$. Note that: 20%

$$\dot{V} = \frac{\partial V}{\partial x}\dot{x} + \frac{\partial V}{\partial y}\dot{y}$$

Problem 2

The following equations describe the dynamics of a simple power system composed of a synchronous generator and a load (see Fig. 2).

Neglecting for simplicity losses, electromagnetic dynamics, and the transient impedance in the d-axis transient model, the generator can be simulated with:

$$\begin{aligned} \dot{\delta}_1 &= \omega_1 = \omega_r - \omega_0 \\ \dot{\omega}_1 &= \frac{1}{M}(P_m - P_G - D_G\omega_1) \end{aligned} \tag{2}$$

The load can be simulated using the *mixed* models. For P, neglecting voltage dynamics ($T_{pv} = 0$) and voltage dependence ($\alpha = 0$):

$$\begin{aligned} P_L &= K_{pf} f_2 + K_{pv}[V_2^\alpha + T_{pv}\dot{V}_2] \\ \Rightarrow \quad P_L &= P_d + D_L\omega_2 \\ \dot{\delta}_2 &= \omega_2 = \frac{1}{D_L}(P_L - P_d) \end{aligned} \tag{3}$$

For Q, neglecting frequency dependence ($K_{qf} = 0$) and voltage dependence ($\beta = 0$)

$$\begin{aligned} Q_L &= K_{qf} f_2 + K_{qv}[V_2^\beta + T_{qv}\dot{V}_2] \\ \Rightarrow \quad Q_L &= Q_d + \tau\dot{V}_2 \\ \dot{V}_2 &= \frac{1}{\tau}(Q_L - Q_d) \end{aligned} \tag{4}$$

Assume also $P_m = P_d$, $Q_d = kP_d$ and a lossless transmission line.

a. Write the full set of system equations assuming $Q_{G_{\min}} \leq Q_G \leq Q_{G_{\max}}$. Is it a DAE or an ODE? 25%

b. Assuming a lossless transmission line, write the full set of system equations assuming $Q_G = Q_{G_{\max}}$. Is it a DAE or an ODE? 25%

c. Determine the system Jacobian matrices of the system for the case $Q_G = Q_{G_{\max}}$ and indicate the expression to determine the state matrix at the equilibrium point. 25%

d. How many equilibrium points are expected for the system? What kind of bifurcations can be expected? 25%

Hint. It may be useful to define:

$$\begin{aligned} \delta &= \delta_1 - \delta_2 \\ \omega &= \omega_1 \\ \Rightarrow \quad \dot{\delta} &= \omega - \omega_2 \end{aligned}$$

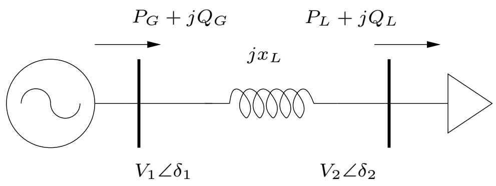

Figure 1

Questions

Answer to the following questions.

a. Provide the definitions of voltage stability and voltage collapse provided by the IEEE/CIGRE Joint Task Force on Stability. In your opinion, what limitations have the definitions above? 20%

b. Provide an example of ODE system that shows a saddle-node bifurcations and show that the equilirium point satisfies the transversality conditions of the saddle-node bifurcation. 20%

c. Sketch the bifurcation diagram of a system showing a transcritical bifurcation and indicate in which way the digram can *degenerate*, thus leading the transcritical bifurcation to disappear. 20%

d. Describe the most common remedial actions to prevent the occurrence of Hopf bifurcations on power systems. 20%

e. Describe, qualitatively, the principle on which Lyapunov exponents are based. 20%

Power System Stability Analysis

Example of Exam Paper II

Problem 1

Consider the test function:

$$\dot{x} = f(x) = \lambda x \tag{5}$$

Determine:

a. The maps obtained by applying the forward and backward Euler integration methods to (5). 10%

b. The fixed points of the maps determined above. 10%

c. The stability of the fixed points of the maps determined above as a function of the product $h\lambda$ ($h\lambda \in \mathbb{R}$). 20%

d. The stability of the two maps above as a function of the product $h\lambda$, assuming $\lambda \in \mathbb{C}$. With this aim, for both maps, sketch the stability region of the fixed point in the complex plain ($\Re\{h\lambda\}, \Im\{h\lambda\}$). 20%

e. The stability of (5) and of both maps for $\lambda = 0$. Indicate also whether $\lambda = 0$ is a limit point for (5). 20%

f. Whether the integration schemes are stable or unstable with respect to the original continuous system (5). With this aim, consider only the case $\lambda \in \mathbb{R}$, $\lambda \neq 0$, and $h > 0$. 20%

Note: The generic step of the forward Euler integration scheme is:

$$x^{n+1} = x^n + hf(x^n) \tag{6}$$

whereas the generic step of the backward Euler integration scheme is:

$$x^{n+1} = x^n + hf(x^{n+1}) \tag{7}$$

where $h \in \mathbb{R}^+$ is the time step.

Problem 2

The following equations describe, using a standard notation, a simple power system composed of a synchronous generator with AVR and no reactive power limits, a transmission line and a dynamic load (see Fig. 2).

$$
\begin{aligned}
\dot{\omega} &= \frac{1}{M}\left(P_m - \frac{E'V_2}{X}\sin\delta - D_G\omega\right) \\
\dot{\delta} &= \omega - \frac{1}{D_L}\left(\frac{E'V_2}{X}\sin\delta - P_d\right) \\
\dot{E}' &= K_v(V_{10} - V_1) \\
\dot{V}_2 &= \frac{1}{\tau}\left(-\frac{V_2^2}{X} + \frac{E'V_2}{X}\cos\delta - kP_d\right) \\
0 &= \frac{V_1V_2}{X_L}\sin\delta' - \frac{E'V_2}{X}\sin\delta \\
0 &= V_2^2\left(\frac{1}{X_L} - \frac{1}{X}\right) + \frac{E'V_2}{X}\cos\delta - \frac{V_1V_2}{X_L}\cos\delta'
\end{aligned}
\tag{8}
$$

where X_L is the reactance of the lossless transmission line that connects the generator to the load and $X = X_L + X'_G$, where X'_G is the internal transient reactance of the synchronous machine.

a. Is the set of equations (8) an ODE or a DAE? Indicate the state variables $\boldsymbol{x}$, algebraic variables $\boldsymbol{y}$ and parameters $\boldsymbol{p}$. 20%

b. Determine an analytic expression of the state matrix of the system assuming that $\delta = \delta' \approx 0$ and $V_1 \approx V_2 \approx 1$ pu. Discuss whether the suggested approximations lead to reasonable results. 40%

c. Assume that the exogenous parameter is the load demand $\mu = P_d$. Assume also that $P_m = P_d$. Determine $\boldsymbol{f}_\mu$. 10%

d. What bifurcation points are to be expected by increasing P_d? 30%

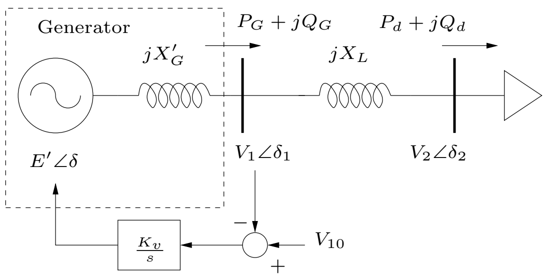

Figure 2

Questions

Answer to the following questions.

a. Assume that v_i is the voltage magnitude at a certain bus of a power system. Indicate whether the trajectories $v_i(t)$ shown in Fig. 3 are stable or unstable according to the definitions provided by the IEEE/CIGRE Joint Task Force on Stability. Duly justify your answers. 25%

b. Describe the Hopf bifurcations and its consequences on the transient behaviour of set of differential algebraic equations. Indicate which kind of power system devices and/or interaction among devices are expected to lead to Hopf bifurcations. 25%

c. Describe advantages and disadvantages of most common indices used to define the voltage stability margin of a power system. 25%

d. Describe the meaning of the fundamental solution matrix $\mathbf{\Phi}(t)$ and the properties of its eigenvalues for periodic orbits. 25%

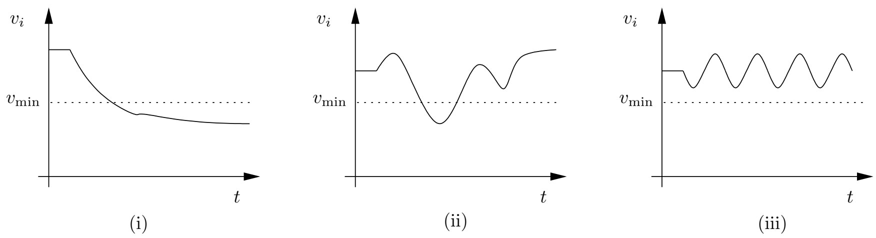

Figure 3