Basic Stability Concepts

Proposed Exercises

EEEN40340 - Power System Stability

Exercise 1

Find the sationary solutions (equilibria) of the system:

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -x_1 - \mu x_2$$

for $\mu = -1$, $\mu = 0$ and $\mu = 1$. When the equilibrium is stable, asymptotically stable, unstable?

Exercise 2

Discuss the stability of the equilibria of:

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -x_1 - \mu x_2$$

for all values of μ.

Exercise 3

Find the stationary points of the following second order differential equation, which is a special case of the Duffing equation:

$$\ddot{x} + \dot{x} - x + x^3 = 0$$

Determine the properties of the stationary points.

Exercise 4

Consider the stationary point $x_0 = 0$ of the Van der Pol equation:

$$\ddot{x} - \mu(1 - x^2)\dot{x} + x = 0$$

Determine the properties of the stationary point $x_0 = 0$ as a function of the parameter μ.

Exercise 5

A simple mathematical model of the heartbeat is given by:

$$\begin{aligned}\dot{x}_1 &= -a(x_1^3 - bx_1 + x_2)\\ \dot{x}_2 &= x_1 - c\end{aligned}$$

where x_1 is the length of a muscle fiber and x_2 electrochemical control. Discuss the equilibrium (diastole) for $a = 100$, $b = 1$, $c = 1.1$.

Exercise 6

Sketch the bifurcation diagram in the plane (μ, x_1) for the following system:

$$\begin{aligned}\dot{x}_1 &= \mu - x_1^2\\ \dot{x}_2 &= x_1 + cx_2\end{aligned}$$

Consider two cases $c > 0$ and $c < 0$ and define stable and unstable branches. Finally, prove that the equilibrium point for $\mu = 0$ is a saddle-node bifurcation.

Exercise 7

Determine the equilibrium point of the synchronous machine classical model:

$$\begin{aligned}\dot{\delta} &= \omega\\ \dot{\omega} &= \frac{1}{M}(p_m - \frac{e'v}{x}\sin\delta - D\omega)\end{aligned}$$

for $p_m = e' = v = 1$ pu, $x = 0.1$ pu, $D = 0$, and $\delta \in [0, \pi/2]$. Discuss whether the equilibrium point is a Hopf bifurcation and discuss how its properties vary for $D \neq 0$.

Exercise 8

Consider the mechanical equation of the induction motor:

$$\begin{aligned}\dot{\omega} &= \frac{1}{M}(\tau_e(\sigma) - \tau_m)\\ \sigma &= 1 - \omega\\ \tau_e &= \frac{v_1^2}{(r_1 + r_2'/\sigma)^2 + x'^2_{cc}}\frac{r_2'}{\sigma}\end{aligned}$$

where all quantities have usual meaning and are in per units. Show that the equilibrium point for $\tau_m = \tau_e^{\max}$ is a saddle-node bifurcation.

Exercise 9

Consider the following map:

$$x_1^{n+1} = x_1^n + \mu - b x_1^n x_2^n + \beta x_2^n - d_1 x_1^n$$
$$x_2^{n+1} = x_2^n + b x_1^n x_2^n - \beta x_2^n - d_2 x_2^n$$

This map can be used to model student migration within an engineering graduate school, x_2^{n+1} denotes the number of students choosing in year $n+1$ to major in a specific field (electrical engineering, say), and x_1^{n+1} is the number of students who prefer any other field. μ is the constant input rate of fresh students per year, b describes the migration, dominated by communication, β is the backflow, and d_1, d_2 are the successful examination rates.

Assuming $d_1 = 0.1$, $d_2 = 0.3$, and $\beta = 0.1$, establish the fixed point for $x_2 = 0$ and establish its stability.

Exercise 10

Study the stability of the fixed points of the map obtained by applying the Euler integration method to the following equation (logistic differential equation):

$$\dot{x} = x(x-1)$$