Solutions

Section A

1. $$\bar{I}_{aA} = \frac{\bar{V}_a}{\bar{Z}_A} = \frac{110\angle 0^\circ}{50 + j80} = 1.1659 \angle -58^\circ \text{ A (rms)}$$

$$\bar{I}_{bB} = \frac{\bar{V}_b}{\bar{Z}_b} = \frac{110\angle -120^\circ}{j50} = 2.2 \angle 150^\circ \text{ A (rms)}$$

$$\bar{I}_{cC} = \frac{\bar{V}_c}{\bar{Z}_c} = \frac{110\angle 120^\circ}{100 + j25} = 1.07 \angle 106^\circ \text{ A (rms)}$$

2. $$\bar{I}_{nN} = \bar{I}_{aA} + \bar{I}_{bB} + \bar{I}_{cC} = 1.95 \angle 125.8^\circ \text{ A (rms)}$$

3. $$\bar{S} = \bar{S}_A + \bar{S}_B + \bar{S}_C =$$

$$= \bar{V}_A \bar{I}_{aA}^* + \bar{V}_B \bar{I}_{bB}^* + \bar{V}_C \bar{I}_{cC}^*$$

$$= (421 \angle 64.34^\circ) \text{ VA}$$

Section B

4. $$T(\theta) = \frac{1}{2}\frac{\partial L_1}{\partial\theta} i_1^2 + \frac{1}{2}\frac{\partial L_2}{\partial\theta} i_2^2 + \frac{\partial M}{\partial\theta} i_1 i_2$$

$$= -\frac{1}{2} 6 \sin 2\theta \; i_1^2 - \frac{1}{2} \sin 2\theta \; i_2^2 - 11 \sin\theta \; i_1 i_2$$

$$= (-2.75 \sin 2\theta - 6.16 \sin\theta) \text{ Nm}$$

5. $$W(\theta) = \frac{1}{2} L_1(\theta)\, i_1^2 + \frac{1}{2} L_2(\theta)\, i_2^2 + M i_1 i_2$$

$$= \frac{1}{2}(11 + 3\cos 2\theta)\, i_1^2 + \frac{1}{2}(7 + 2\cos 2\theta)\, i_2 + 11\cos\theta\, i_1 i_2$$

$$= [4.935 + 1.375\cos 2\theta + 6.16\cos\theta]\ \text{J}$$

Section C

6. the synchronous speed is:

$$\omega_1 = \frac{60 f_1}{p} = \frac{60 \cdot 50}{2} = 1500 \text{ rpm}$$

hence the slip factor is:

$$\sigma = \frac{\omega_1 - \omega}{\omega_1} = \frac{1500 - 1435}{1500} = 4.33\%$$

7. The power flowing in the air gap is:

$$P_a = P_1 - P_{Losses1} = 4760 - 265 = 4495 \text{ W}$$

hence, the losses in the ~~stator~~ rotor windings are:

$$P_{j2} = P_{Cu2} = \sigma P_a = 0.0433 \cdot 4495 = 194.78 \text{ W}$$

8. The net mechanical power is:

$$P_{mech} = P_a - P_{mech\ losses} - P_{j2} = 4495 - 300 - 194.78$$

$$= 4000.2 \text{ W}$$

9. The efficiency is:

$$\eta = \frac{P_{mech}}{P_1} = \frac{4000.2}{4760} = 84.04\ \%$$

Section D

10. Full load current:

$$I = \frac{P}{\sqrt{3}\, V \cos\varphi}$$

$$= \frac{10^6}{\sqrt{3} \cdot 6600 \cdot 1} = 87.48\ A$$

$$\Rightarrow \bar{I} = I \angle 0^\circ = 87.48 \angle 0^\circ\ A$$

→ f.e.m.

$$\bar{E}_0 = \frac{6600}{\sqrt{3}} + (0.4 + j6)\, 87.48 \angle 0^\circ =$$

$$= 3881.17 \angle 7.8^\circ\ V$$

$$\rightarrow \varepsilon_{\%} = \frac{3881.17 - 3810.51}{3810.51} = 1.85\ \% \qquad \text{where } \frac{V}{\sqrt{3}} = \frac{6600}{\sqrt{3}} = 3810.51$$

11. Full load current: $I = \dfrac{P}{\sqrt{3}\, V \cos\varphi}$

$$\Rightarrow I = \frac{10^6}{\sqrt{3} \cdot 6600 \cdot 0.866} = 101.01\ A \quad , \quad \bar{I} = 101.01 \angle -30^\circ\ A$$

f.e.m. $\Rightarrow \bar{E}_0 = \dfrac{6600}{\sqrt{3}} \angle 0^\circ + (0.4 + j0.6)\, 101.01 \angle -30^\circ = 4179 \angle 20.86^\circ\ V$

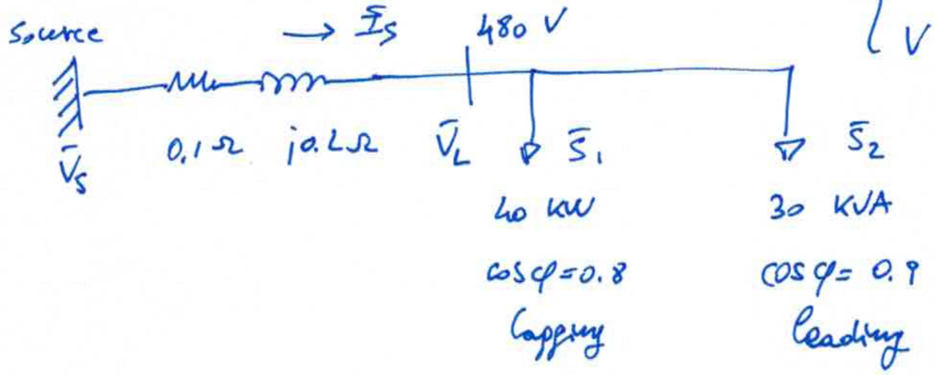
$$\varepsilon_{\%} = \frac{4179.6 - 3810.51}{3810.51} = 9.67\ \%$$

12. $\Rightarrow \quad \bar{I} = 101.01 \angle 30^\circ \text{ A}$

$$\bar{E}_0 = \frac{6600}{\sqrt{3}} \angle 0^\circ + (0.4 + j6)\, 101.01 \angle 30^\circ = 3584.16 \angle 8.75^\circ \text{ V}$$

$$\Rightarrow \quad \varepsilon_{\%} = \frac{3584.16 - 3810.51}{3810.51} = -5.94\ \%$$

Section E



E ⇒ phase-to-ground voltage
V ⇒ phase-to-phase voltage
I ⇒ line current

13. $$\bar{I}_S = \frac{(\bar{S}_1 + \bar{S}_2)^*}{3\,\bar{E}_L^*} \qquad \bar{E}_L = \frac{480}{\sqrt{3}} \angle 0^\circ$$

$$\bar{S}_1 = 40 + j\,40 \cdot \frac{0.6}{0.8} = (40 + j\,30) \text{ kVA}$$

$$\bar{S}_2 = 30\,(\cos\varphi + j \sin\varphi) = 30\,(0.9 - j\,0.436)$$

$$= (27 - j\,13) \text{ kVA}$$

$$|\bar{I}_S| = 83.11 \text{ A}$$

14. Combined power factor of the load:

$$\bar{S}_L = \bar{S}_1 + \bar{S}_2 = (40 + j30) + (27 - j13) = (67 + j17) \text{ kVA}$$
$$(P_L + j\,Q_L)$$

$$\cos\varphi_L = \frac{P_L}{S_L} = \frac{P_L}{\sqrt{P_L^2 + Q_L^2}} = 0.9696 \text{ lagging}$$

15. $$\bar{E}_S = (0.1 + j0.2)\,\bar{I}_S + \bar{E}_L$$

$$\bar{E}_L = \frac{480}{\sqrt{3}} \angle 0^\circ \text{ V} \quad , \quad \bar{I}_S = (80.6 - j20.4) \text{ A}$$

$$\Rightarrow \bar{E}_S = (289.24 + j\,14.1) \text{ V}$$

$$V_S = \sqrt{3}\,|\bar{E}_S| = 501.6 \text{ V (rms)}$$

16. $$\cos\phi_S = \frac{P_S}{S_S}$$

where:
$$\bar{S}_S = 3\,\bar{E}_S\,\bar{I}_S^*$$
$$P_S = Re\{\bar{S}_S\}$$

$$\Rightarrow \cos\phi_S = 0.9566 \text{ lagging}$$