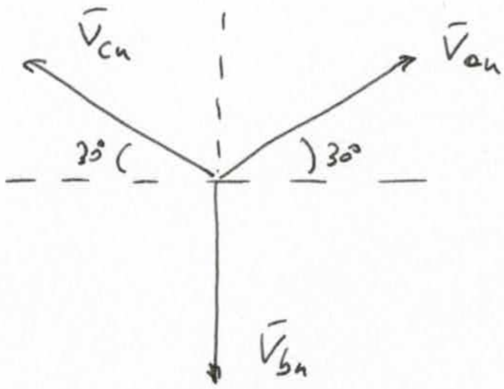
EEEN 20090 - Final Exam - May 2019

Section A

The source is symmetrical, hence:

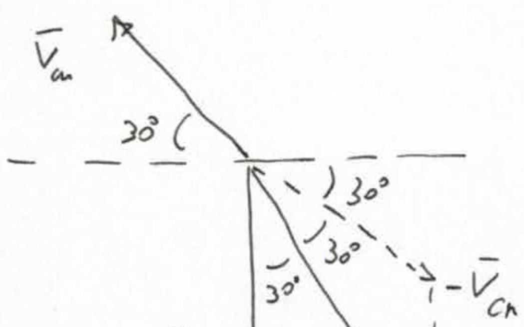
$$|\bar{V}_{an}| = |\bar{V}_{bn}| = |\bar{V}_{cn}| = 390 \text{ V}$$



1 $\quad \bar{V}_{bn} = \bar{V}_{an} \angle -120° = V_{an} \angle 30° - 120° = 390 \angle -90° \text{ V}$

$\quad \bar{V}_{cn} = \bar{V}_{an} \angle 120° = V_{an} \angle 30° + 120° = 390 \angle 150° = 390 \angle -210 \text{ V}$

2 $\quad \bar{V}_{bc} = \bar{V}_{bn} - \bar{V}_{cn} = \sqrt{3} \cdot 390 \cdot \left(\cos(-90°) + j\sin(-90°) - \cos(-240°) - j\sin(240°)\right)$



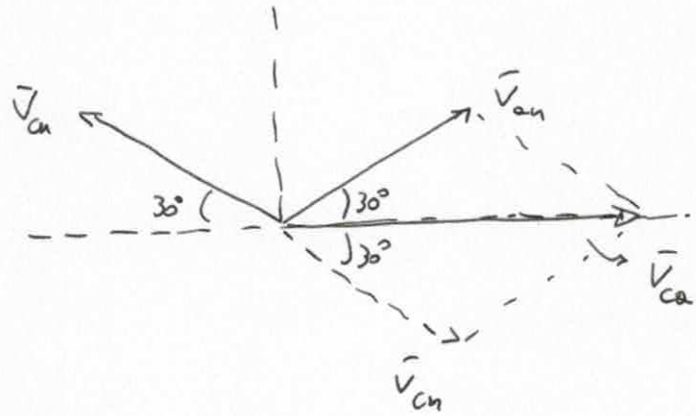
$\angle \bar{V}_{bc} = -60°$

hence:

$\bar{V}_{bc} = \sqrt{3} \cdot 390 \angle -60°$

$= 676 \angle -60° \text{ V}$

3 $\bar{V}_{ac} = \bar{V}_{an} - \bar{V}_{cn}$



hence: $\bar{V}_{ca} = 676 \angle 0 \quad V$

SECTION B

4 The coenergy of the system is given by:

$$W' = \int \lambda_1 \, di_1 + \int \lambda_2 \, di_2$$

Then:

$$W' = \int x^2 i_1^2 \, di_1 + \int \left(x^2 i_2^2 + x i_1 \right) di_2$$

$$= \frac{1}{3} x^2 \left(i_1^3 + i_2^3 \right) + x i_1 i_2$$

5 By definition, the magnetic energy can be obtained from the coenergy:

$$W = \lambda_1 i_1 + \lambda_2 i_2 - W'$$

hence:

$$W = \frac{2}{3} x^2 \left(i_1^3 + i_2^3 \right) + x i_1 i_2$$

6 The mechanical force is given by:

$$f = \left. \frac{\partial W'}{\partial x} \right|_{i_1, i_2 \text{ const.}}$$

$$= \frac{2}{3} x \left(i_1^3 + i_2^3 \right) + i_1 i_2$$

SECTION C

7 → Rotor current for $\sigma = 0.03$:

$$I_2' = \frac{V_1}{\sqrt{\left(R_1 + R_2'/\sigma\right)^2 + \left(X_1 + X_2'\right)^2}} = 24.7 \text{ A}$$

where $V_1 = 220/\sqrt{3}$ (phase-to-neutral voltage magnitude)

→ Torque for $\sigma = 0.03$:

$$T = \frac{3\, I_2'^{\,2}\, R_2'}{\omega_s\, \sigma} = \frac{3 \cdot (24.7)^2 \cdot 0.144}{125.7 \cdot 0.03} = 69.9 \text{ N.m}$$

where $\omega_s = 2 \cdot \pi \cdot 60 / 3 = 2 \cdot \pi \cdot 20 = 125.7$ rad/s

→ Mechanical power for $\sigma = 0.03$:

$$P_{mech} = P_2 = 3\, I_2'^{\,2} \cdot R_2' \left(\frac{\sigma}{1-\sigma}\right)^{-1} = 8.5 \text{ kW}$$

8 The slip factor that leads to the maximum ~~power~~ torque is given by:

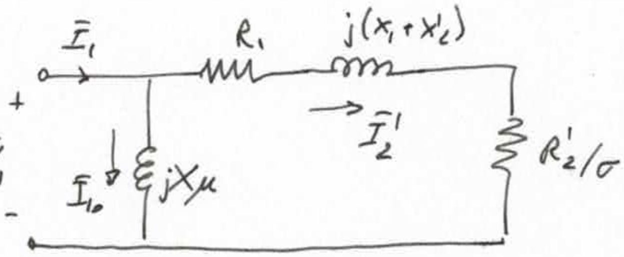
$$\sigma_M = \frac{R_2'}{\sqrt{R_1^2 + \left(X_1 + X_2'\right)^2}} = 0.187$$

then:

$$T_{max} = \frac{3}{\omega_s} \frac{V_1^2}{\left(R_1 + R_2'/\sigma_M\right)^2 + \left(X_1 + X_2'\right)^2} \frac{R_2'}{\sigma_M} = 181 \text{ N.m}$$

9 At the start-up, the slip factor is : $\sigma_{su} = 1$.

Let us consider the following approximated model of the motor:



The start-up torque is :

$$T_{su} = \frac{3\ I'^{2}_{2,su} \cdot R'_2}{\omega_s} = 79.3\ N{\cdot}m$$

where:

$$I'_{2,su} = \frac{V_1}{\sqrt{(R_1+R'_2)^2 + (X_1+X'_2)^2}} = 152\ A$$

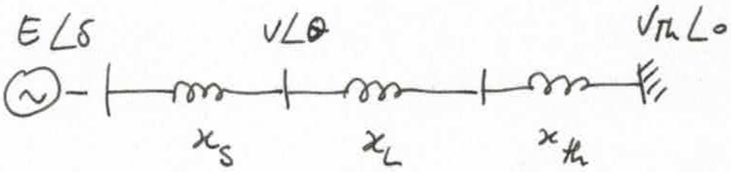
To calculate the stator current at start-up, let us consider the equivalent equivalent circuit of the motor and $\bar{V}_1 = V_1 \angle 0$, then:

$$\bar{I}_{1,su} = \frac{\bar{V}_1}{(R_1+R'_2)+j(X_1+X'_2)} + \frac{\bar{V}_1}{jX_\mu} = \bar{I}'_{2,su} + \bar{I}_{10}$$

Finally: $|\bar{I}_{1,su}| = 160\ A$.

SECTION D

(10)



The active power that can be transferred from the generator to the grid is:

$$P = \frac{3\,E \cdot V_{th}}{X_{tot}} \sin(\delta)$$

where $E = \frac{25}{\sqrt{3}}$ kV , $V_{th} = \frac{20}{\sqrt{3}}$ kV $\Rightarrow$ $E = \cancel{}\ 14.43$ kV

$V_{th} = 11.55$

$$X_{tot} = X_S + X_L + X_{th} = 2.5 + 1 + 0.3 = 3.8\ \Omega/\text{phase}$$

The maximum power is obtained for $\delta = 90°$, hence:

$$P_{max} = \frac{3\,E\,V_{th}}{X_{tot}} = 131.6\ \text{MW}$$

(11) $\delta = 10° = 0.1745$ rad.

then:

$$P = \frac{3\,E\,V_{th}}{X_{tot}} \sin(\delta) = 22.8\ \text{MW}$$

$$\bar{V} = \bar{V}_{th} + \frac{\bar{E} - \bar{V}_{th}}{jX_{tot}} \cdot j(X_L + X_{th})$$

$$= 11.55\angle 0 + \left(14.43\angle 0.1745 - 11.55\angle 0\right)\frac{1.3}{3.8} =$$

$$= (12.46 + j\,0.857)\ V = (12.49\angle 0.0687\ \text{rad})\ V = (12.49\angle 3.94°)\ V$$

12 The voltage regulation is:

$$\Delta V_{\%} = \frac{V - V_{th}}{V_{th}} \cdot 100 = \frac{12.49 - 11.55}{11.55} \cdot 100 = 8.16\ \%$$

Finally:

$$Q = 3\frac{V^2}{X_L + X_{th}} - 3\frac{V\,V_{th}}{X_L + X_{th}} \cos(\angle \bar{V})$$

$$= 27.93 \quad MVAr$$

SECTION E

(13) $\bar{S}_1 = P_1 + jQ_1 = 30 \cdot 0.8 + j\,30 \cdot 0.6 = (24 + j18)$ KVA

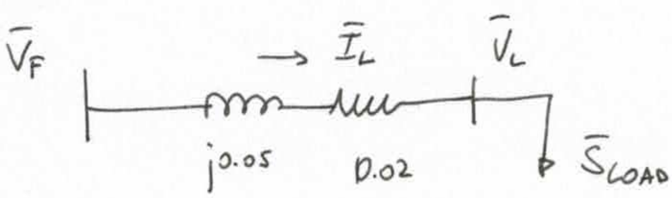
$$\bar{S}_2 = P_2 + jQ_2 = 24 - j\,24\,\frac{0.8}{0.6} = (24 - j32) \text{ KVA}$$

$$\bar{S}_{LOAD} = 120 + j0 = \bar{S}_1 + \bar{S}_2 + \bar{S}_3$$

$$P_3 = P_{LOAD} - P_1 - P_2 = 72 \text{ KW}$$

$$Q_3 = Q_{LOAD} - Q_1 - Q_2 = 14 \text{ KVAr}$$

(14)



$$\bar{I}_L = \frac{\bar{S}^*_{LOAD}}{3\,\bar{V}_L^*} = \frac{120 - j0}{3\,(208/\sqrt{3})} = (0.333 + j0) \text{ KA}$$

where: $\bar{V}_L = \frac{208}{\sqrt{3}} \angle 0$ V

Finally: $\bar{V}_F = \bar{V}_L + (0.02 + j0.05)\,\bar{I}_L$

$$= (126.75 + j16.65) \text{ V}$$

The phase-to-phase voltage magnitude at the feeder is:

$$\sqrt{3}\,|\bar{V}_F| = 221.4 \text{ V}$$