EEEN20090 - Electrical Energy Systems

Final Exam - Spring Term 2019-2020 - Solutions

Section A

1. Since the source is symmetrical, $V_L = \sqrt{3} \cdot 120 = 208\ V$, and, with the notation of Figure 1:

$$\begin{cases} \bar{V}_{L1} = 208 \angle -30^\circ\ V = (180 - j104)\ V \\ \bar{V}_{L2} = 208 \angle 90^\circ\ V = j208\ V \\ \bar{V}_{L3} = 208 \angle -150^\circ\ V = (-180 - j104)\ V \end{cases}$$

2. Phase currents can be found using Ohm's Law:

$$\bar{I}_{P1} = \bar{V}_{L1} / 40\angle 0^\circ = 5.2 \angle -30^\circ\ A = (4.5 - j2.6)\ A$$
$$\bar{I}_{P2} = \bar{V}_{L2} / 20\angle -60^\circ = 10.4 \angle 150^\circ\ A = (-9 + j5.2)\ A$$
$$\bar{I}_{P3} = \bar{V}_{L3} / 15\angle 45^\circ = 13.8 \angle -195^\circ\ A = (-13.4 + j3.6)\ A$$

3. Line currents can be found using KCL:

$$\bar{I}_{L1} = \bar{I}_{P1} - \bar{I}_{P3} = (17.9 - j6.2)\ A$$
$$\bar{I}_{L2} = \bar{I}_{P2} - \bar{I}_{P1} = (-13.5 + j7.8)\ A$$
$$\bar{I}_{L3} = \bar{I}_{P3} - \bar{I}_{P2} = (-4.4 - j1.6)\ A$$

Note that: $\bar{I}_{L1} + \bar{I}_{L2} + \bar{I}_{L3} \neq 0$ #

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Section B

4. Reluctance in the airgap (with $x = 3.5$ mm):

$$\mathcal{R}_1 = \frac{x}{\mu_0 A} = \frac{0.0035}{4\cdot\pi\cdot10^{-7}\cdot\pi\cdot0.015^2} = 3{,}940{,}268 \ \frac{A\text{-}t}{Wb}$$

Hence: $L_1 = \frac{N^2}{\mathcal{R}_1} = \frac{4000^2}{\mathcal{R}_1} = 4.06 \ H$

and the Force is: $f = \frac{1}{2}\frac{\partial L}{\partial x} i^2 = -\frac{1}{2}\frac{\mu_0 A N^2}{x^2} i^2 = -0.5221 \ N$

5. Reluctance in the airgap with $x = 5$ mm:

$$\mathcal{R}_2 = \frac{x}{\mu_0 A} = \frac{0.005}{4\cdot\pi\cdot10^{-7}\cdot\pi\cdot0.015^2} = 5{,}628{,}955 \ A\text{-}t/Wb$$

Hence: $L_2 = \frac{N^2}{\mathcal{R}_2} = \frac{4000^2}{\mathcal{R}_2} = 2.84 \ H$

$$W_f(3.5 \text{ mm}) = \frac{1}{2} i^2 L_1 = 0.001827 \ J$$

$$W_f(5 \text{ mm}) = \frac{1}{2} i^2 L_2 = 0.001279 \ J$$

$$\Delta W_f = W_f(5 \text{ mm}) - W_f(3.5 \text{ mm}) = -0.000548 \ J$$

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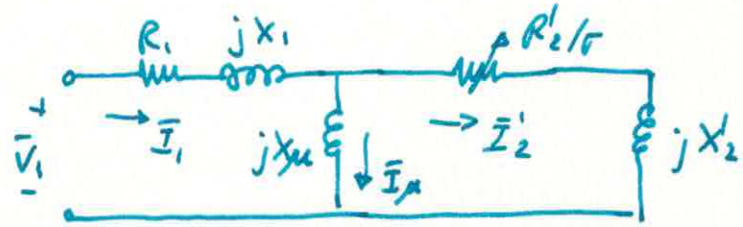
Section C

6. Let assume the following equivalent circuit of the motor:



where $X_\mu = 1/B_\mu$

The equivalent impedance of the circuit is:

$$\bar{Z}_{eq} = (R_1 + jX_1) + \underbrace{jX_\mu \,//\, (R'_2/\sigma + jX'_2)}_{R_f + jX_f} =$$

$$= (9.40 + j4.64)\,\Omega = (10.5\angle 26.3^\circ)\,\Omega$$

Let $\bar{V}_1 = 127\angle 0$ V, then: $\bar{I}_1 = \dfrac{\bar{V}_1}{\bar{Z}_{eq}} = 12.1\angle -26.3^\circ$ A

$\Rightarrow \cos\varphi_1 = \cos(-26.3^\circ) = 0.90$ (lagging)

7. The airgap power is $P_{C1} = 3R'_2/\sigma\,(I'_2)^2 = 3R_f I_1^2 = 3{,}920$ W

Mechanical power $P_m = (1-\sigma)\,P_{C1} = 0.96 \cdot 3920 = 3{,}760$ W

Synchronous speed $\omega_{C1} = 4\pi f/p = 188.5$ rad/s

mechanical speed $\omega_m = (1-\sigma)\,\omega_{C1} = 181$ rad/s

Mechanical torque $T_m = P_m/\omega_m = 20.8$ Nm #

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Section D

8. $V = \frac{4600}{\sqrt{3}} = 2,660\ V$ phase voltage

$I = \frac{1000 \cdot 10^3}{3V} = 125\ A$ current in each phase

At unity power factor:

$\bar{E} = \bar{V} + IR_a + jIX_s$ where $\bar{V} = V\angle 0$

$= 2,660 + 125 \cdot 2 + j\,125 \cdot 20$

$= 2,910 + j\,2500$

$|E| = 3,845\ V$

9. At 0.75 power factor lagging:

$\bar{E} = (V\cos\theta + IR_a) + j(V\sin\theta + IX_s)$

$= (2,660 \cdot 0.75 + 125 \cdot 2) + j\,(2660 \cdot 0.676 + 2500)$

$= 2250 + j\,4270$

$|E| = 4,820\ V$

→ we have assumed $\bar{V} = V\cos\theta + jV\sin\theta$ #

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Section E

10. Let's use the single phase equivalent.

Phase voltage: $E = 440/\sqrt{3} = 254$ V

Current in the lighting load: $I_L = E/15\,\Omega = 16.9$ A

$P_L = 3\,E\,I_L = 3 \cdot 254 \cdot 16.9 = 12.9$ kW

11. Current in the motor: $\bar{I}_M = E\angle 0 / (10 + j5) = 22.68\angle -26.57$ A

$= (20.28 - j10.14)$ A

$P_M = 3\,E\,Re\{\bar{I}_M\}^* = 3 \cdot 254 \cdot 20.28 = 15.4$ kW

12. Power delivered to both loads (per phase)

$P_S = (P_L + P_M)/3 = 5.1$ kW $+ 4.3$ kW $= 9.4$ kW