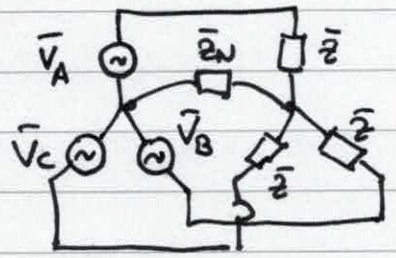
Section A

① 

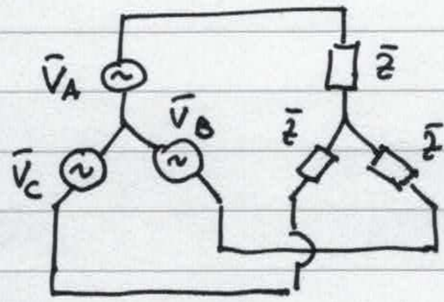
$\bar{Y} = 1/\bar{Z} = (0.04 - j0.02)\ \Omega^{-1}$
$\bar{Y}_N = 1/\bar{Z}_N = (-j0.2)\ \Omega^{-1}$

$$\bar{V}_N = \frac{\bar{Y}(\bar{V}_A + \bar{V}_B + \bar{V}_C)}{3\bar{Y} + \bar{Y}_N} = (3.3796 - j4.2245)\,V$$

$\bar{I}_A = \bar{Y}(\bar{V}_A - \bar{V}_N) = (4.3493 - j1.9634)\,A$

$\bar{I}_B = \bar{Y}(\bar{V}_B - \bar{V}_N) = (-0.2918 + j4.2543)\,A$

$\bar{I}_C = \bar{Y}(\bar{V}_C - \bar{V}_N) = (-4.9024 - j2.9668)\,A$

② 

In this case $\bar{Y}_N = 0$

hence: $\bar{V}_N = \dfrac{\bar{Y}(\bar{V}_A + \bar{V}_B + \bar{V}_C)}{3\bar{Y}} = \frac{1}{3}(\bar{V}_A + \bar{V}_B + \bar{V}_C)$

$= (0 - j11.547)\,V$

then:

$\bar{I}_A = \bar{Y}(\bar{V}_A - \bar{V}_N) = (4.6309 - j1.7381)\,A$

$\bar{I}_B = \bar{Y}(\bar{V}_B - \bar{V}_N) = (-0.0102 + j4.4796)\,A$

$\bar{I}_C = \bar{Y}(\bar{V}_C - \bar{V}_N) = (-4.6207 - j2.7414)\,A$

Section B

③ $$W = \int i\, d\lambda = \int K x^3 \lambda^2 d\lambda = \frac{1}{3} K x^3 \lambda^3$$

④ $$W' = \lambda i - W = K x^3 \lambda^3 - \frac{1}{3} K x^3 \lambda^3 = \frac{2}{3} K x^3 \lambda^3$$

where: $\lambda = K^{-1/2} x^{-3/2} i^{1/2}$ (from $i = K x^3 \lambda^2$)

hence: $$W' = \frac{2}{3} K x^3 K^{-3/2} x^{-9/2} i^{3/2}$$

$$= \frac{2}{3} K^{-1/2} x^{-3/2} i^{3/2}$$

Same result can be obtained from $W' = \int \lambda\, di = \int K^{-1/2} x^{-3/2} i^{1/2} di$

⑤ $$f = \left.\frac{\partial W'}{\partial x}\right|_{i\,const} = -\left.\frac{\partial W}{\partial x}\right|_{\lambda\,const}$$

$$f = \left.\frac{\partial W'}{\partial x}\right|_{i\,const} = \frac{2}{3} K^{-1/2} \left(-\frac{3}{2} x^{-5/2}\right) i^{3/2}$$

$$= -K^{-1/2} x^{-5/2} i^{3/2}$$

$$f = -\left.\frac{\partial W}{\partial x}\right|_{\lambda\,cons} = -\frac{1}{3} K\, 3 x^2 \lambda^3 = -K x^2 \lambda^3$$

and, substituting the expression of $\lambda = K^{-1/2} x^{-3/2} i^{1/2}$

$$f = -K x^2 K^{-3/2} x^{-9/2} i^{3/2} = -K^{-1/2} x^{-5/2} i^{3/2} \quad \#$$

Section C

(6) $$\omega_{S1} = \frac{60 \cdot 60}{4} = 900 \text{ rpm}$$

(7) $$\sigma_N = \frac{\omega_{S1} - \omega_{mN}}{\omega_{S1}} = \frac{900 - 850}{900} = 0{,}0556 = 5.56\ \%$$

The electrical power required by the motor is $P_{el} = \frac{P_m}{0{,}86} = 6081.4 \text{ W}$

then : $P_{el} = 3\, V_N\, I_L \cos\varphi$ where V_N is the phase-to-neutral voltage

$$= 3 \cdot 380 \cdot I_L\ 0{,}7 \quad \Rightarrow I_L = \frac{6081.4}{3 \cdot 380 \cdot 0{,}7} = 7.621 \text{ A}$$

(8) $P_{j2} = \sigma_N P_{S1}$ where $P_{S1} = P_{el} - P_{j1} - P_{Fe}$

$$P_{S1} = 6081{,}4 - 410 = 5671.4 \text{ W}$$

$$P_{j2} = 0.0556\ P_{S1} = 315 \text{ W}$$

Finally $P_{ml} = P_{el} - P_{j1} - P_{Fe} - P_{j2} - P_m = 126.4 \text{ W}$

Section D

(9) in open circuit $E = V$, hence $\varepsilon_v = 1$

(10) $V = \dfrac{22000}{\sqrt{3}} = 12701\ V$

$I_N = \dfrac{S_N}{\sqrt{3}\,V_N} = \dfrac{2000000}{\sqrt{3}\cdot 22000} = 52.49\ A$

$\bar{E} = V\angle 0 + (R_a + jX_s)\, 0.75\, I\angle 0 = (13095 + j\,1968)\,V$

unity power factor

$E = |\bar{E}| = 13242\ V$

$\varepsilon_v = \dfrac{E-V}{V} = 0.0426 = 4.26\,\%$

(11) $\bar{E} = V\angle 0 + (R_a + jX_s)\, I\,(\cos\varphi + j\sin\varphi) = (11359 + j\,2315)\,V$

leading power factor

$E = |\bar{E}| = 11593\ V$

$\varepsilon_v = \dfrac{E-V}{V} = -0.087 = -8.7\,\%$

Section E

(12) $P_1 = 20 \cdot 0.8 = 16 \quad KW$

$Q_1 = 20 \cdot 0.6 = 12 \quad KVAr$

$P_2 = 10 \quad kW$

$Q_2 = -10 \cdot \frac{\sqrt{1-0.7^2}}{0.7} = -10.2 \quad KVAr$

$P_3 = 10 \quad KW$

$Q_3 = 0$

$P_4 = 8 \frac{0.6}{0.8} = 6 \quad KW$

$Q_4 = 8 \quad KVAr$

$P_L = P_1 + P_2 + P_3 + P_4 = 42 \quad kW$

$Q_L = Q_1 + Q_2 + Q_3 + Q_4 = 9.8 \quad KVAr \qquad (Q_L > 0)$

$\cos\varphi_L = \frac{P_L}{\sqrt{P_L^2 + Q_L^2}} = 0.9738$ lagging

(13) $\bar{V}_L = \frac{380}{\sqrt{3}} \angle 0 = 219.4 \angle 0 \, V$

$\bar{I}_L = \frac{\bar{S}_L^*}{\bar{V}_L^*} = (0.0638 - j0.0149) \, A$

$\bar{V}_S = \bar{V}_L + \bar{Z}\,\bar{I}_L = (223.65 + j12.46) \, V$

$\bar{S}_S = 3 \bar{V}_S \bar{I}_L^* = (42.258 + j12.374) \, KVA = P_S + jQ_S$

$\cos\varphi_S = \frac{P_S}{\sqrt{P_S^2 + Q_S^2}} = 0.9597$ lagging