Section A

1. $\bar{V}_{ab} = (400\angle 30°)\,V \Rightarrow \bar{V}_a = (230.94\angle 0)\,V$

$\bar{V}_b = (230.94\angle -120°)\,V$

$\bar{V}_c = (230.94\angle +120°)\,V$

$$\bar{Z}_A = \bar{Z}_B = \bar{Z}_C = (9 + j12)\,\Omega$$

$$\bar{I}_A = \frac{\bar{V}_a}{\bar{Z}_A} = \bar{Y}_A \bar{V}_a = (0.04 - j0.0533)(230.94\angle 0)$$

$$= (9.2376 - j12.3168)\,A = (15.396\angle -53.13°)\,A$$

$$\bar{I}_B = \bar{I}_A \angle -120° = (-15.2855 - j1.8416)\,A$$

$$= (15.396\angle -173.13°)\,A$$

$$\bar{I}_C = \bar{I}_A \angle +120° = (6.048 + j14.158)\,A$$

$$= (15.396\angle 66.87°)\,A$$

$$\bar{S} = 3\,\bar{V}_a \bar{I}_a^{*} = (6400 + j8533)\,VA$$

2.

$$\bar{Y}_A = \frac{1}{\bar{Z}_A} = (0.04 - j\,0.0533)\ \Omega^{-1}$$

$$\bar{Y}_B = \frac{1}{\bar{Z}_B} = (0.111 + j0)\ \Omega^{-1}$$

$$\bar{Y}_C = \frac{1}{\bar{Z}_C} = (0.04 + j\,0.0533)\ \Omega^{-1}$$

$$\bar{V}_N = \frac{\bar{Y}_A \bar{V}_a + \bar{Y}_B \bar{V}_b + \bar{Y}_C \bar{V}_c}{\bar{Y}_A + \bar{Y}_B + \bar{Y}_C} = (-98.78 - j\,171.09)\ V$$

$$\bar{I}_A = \bar{Y}_A (\bar{V}_a - \bar{V}_N) = (22.314 - j\,10.741)\ A$$

$$= (24.764 \angle -25.7^\circ)\ A$$

$$\bar{I}_B = \bar{Y}_B (\bar{V}_b - \bar{V}_N) = (-1.855 - j\,3.212)\ A$$

$$= (3.709 \angle -119.9^\circ)\ A$$

$$\bar{I}_C = \bar{Y}_C (\bar{V}_c - \bar{V}_N) =$$

$$(-20.46 + j\,13.953)\ A$$

$$= (24.764 \angle 145.7^\circ)\ A$$

$$\bar{S} = \bar{V}_a \bar{I}_A^* + \bar{V}_b \bar{I}_B^* + \bar{V}_c \bar{I}_C^* = (11\,162.8 + j0)\ VA$$

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Section B

3. $\begin{cases} i_1(t) = 10 \sin(200t) \\ i_2(t) = 0 \end{cases}$

$$e_2 = \frac{d\lambda_2}{dt} = \frac{d}{dt}\left(L_{21} i_1 + L_{22} i_2\right) =$$

$$= \frac{d}{dt}\left[(0.05 \cos\theta)(10 \sin 200t)\right]$$

since $\theta = \omega t \overset{+\delta}{=} 200t + \delta$

then

$$e_2 = \frac{d}{dt}\left[\left(0.05 \cos(200t + \delta)\right)(10 \sin 200t)\right] = 100 \cos(200t + \delta)$$

where δ is an angular position that depends on the initial position of the rotor w.r.t. the angular reference of the current i_1.

4. The expression of the mechanical torque is:

$$T = \frac{1}{2} i_1^2 \frac{dL_{11}}{d\theta} + \frac{1}{2} i_2^2 \frac{dL_{22}}{d\theta} + i_1 i_2 \frac{dL_{12}}{d\theta}$$

$$= (-0.05 \sin\theta)\, i_1 i_2$$

since $i_1 = i_2 = 10 \sin 200t$ and $\theta = \omega t + \delta$

one obtains: $T = -0.05 \sin(\omega t + \delta)\, 100 \sin^2(200t) =$

$$= -2.5 \sin(\omega t + \delta) + 2.5 \cos(400t) \sin(\omega t + \delta)$$

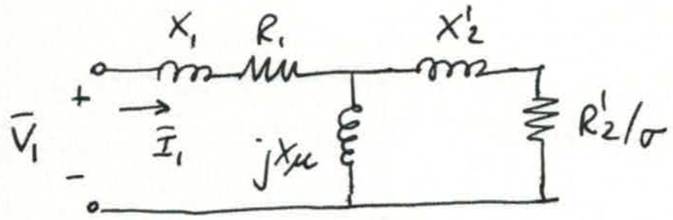
→ the average T is not null, thus, is $\omega = 400$ rad/s #

Section C

5. Let us first refer the rotor parameters to the stator:

$$R'_2 = K_T^2 R_2 = 4^2 \cdot 0.025 = 0.4\ \Omega$$

$$X'_2 = K_T^2 X_2 = 4^2 \cdot 0.05 = 0.8\ \Omega$$



where $\sigma = 0.035$

$$\bar{I}_1 = \bar{V}_1 / \bar{Z}_{eq} \quad \text{where } \bar{V}_1 = \left(\frac{230}{\sqrt{3}} \angle 0\right) V$$

$$= (10.720 - j3.747)\,A$$

$$\bar{Z}_{eq} = \bar{Z}_1 + \frac{\bar{Z}'_2(\sigma)\,\bar{Z}_\mu}{\bar{Z}'_2 + \bar{Z}_\mu} = (11.038 + j3.858)\,\Omega$$

with $\bar{Z}_1 = R_1 + jX_1 = (0.5 + j0.7)\Omega$

$\bar{Z}'_2 = R'_2/\sigma + jX'_2 = (11.43 + j0.8)\Omega$

$\bar{Z}_\mu = 0 + jX_\mu = (50j)\,\Omega$

$$\bar{S}_1 = 3\,\bar{V}_1 \cdot \bar{I}_1^* = (4270.7 + j1492.8)\,VA$$

$\cos\varphi_1 = 0.944$ (lagging)

6. $$\sigma_M = \frac{R'_2}{\sqrt{R_1^2 + (X_1 + X'_2)^2}} = 0.253 \Rightarrow I_{1M} = (52.172)\,A$$

$$T_{max} = \frac{3p}{\omega_1} I_{1M}^2 \frac{R'_2}{\sigma_M} = 123.3\ Nm$$

where $p = 3$, $\omega_1 = 2\pi \cdot 50 = 314.16$ rad/s and I_{1M} is calculated as I_1 above but for $\sigma = \sigma_M$.

At startup, $\sigma = 1$, hence:

$$I_{start} = \frac{\bar{V}_1}{\bar{Z}_{eq}(\sigma=1)} = 76.55 \text{ A}$$

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Section D

7. Phase voltage at the stator of the generator:

$$\bar{V} = \frac{1100}{\sqrt{3}} = (6350.85\angle 0)\ V$$

The internal emf is given by:

$$\bar{E} = \bar{V} + jX_s\bar{I} = 6350.85\angle 0 + j10 \cdot 220\angle 0$$
$$= (6721.11\angle 19.11^\circ)\ V$$

The increased emf is $E' = 1.25 \cdot 6721.11 = 8401.39\ V$

The initial mechanical power is:

$$P_m = P_e = \sqrt{3} \cdot 1100 \cdot 220 = 4191563\ W$$

Since P_m is assumed to remain unchanged:

$$P_m = \frac{3E'V}{X_s}\sin\delta \Rightarrow \sin\delta = 0.2618 \Rightarrow \delta = 15.18^\circ$$

hence $\bar{E}' = E'\angle\delta$. Then, one has:

$$\bar{E}_0' = \bar{V} + jX_s\bar{I}_s' \Rightarrow 8401.4\angle 15.18^\circ = 6350.85\angle 0 + \bar{I}_s' \cdot j10$$

$$\Rightarrow \bar{I}_s' = (281.5\angle 38.61^\circ)\ A$$

$I_s' = 281.5\ A$

$\varphi_s = 38.61^\circ$

8. The maximum power is obtained for a load angle of 90°.

Hence:

$$P_{max} = 3\frac{E'V}{X_S}\sin 90° = 3\,\frac{8401.39\cdot 6350.85}{10} = 16\ MW$$

Then:

$$\bar{E}'_{\phi} = \bar{V} + jX_S\bar{I}_{S\,max} \Rightarrow 8401.39\angle 90° = 6350.85\angle 0° + j10\,\bar{I}_{S\,max}$$

$$\Rightarrow \quad \bar{I}_{S\,max} = (1053.17\ \angle 37.09°)\ A$$

$I_{S\,max} = 1053.17\ A$

$\varphi_{S\,max} = 37.09°$

Section E

9. Let us calculate first the complex power for each load:

$$\begin{cases} \bar{S}_1 = 700 \angle 36.9^\circ = (560 + j420)\ \text{KVA} \\ \bar{S}_2 = 1000 \angle 60^\circ = (500 + j866)\ \text{KVA} \\ \bar{S}_3 = 800 \angle 25.8^\circ = (720 + j349)\ \text{KVA} \end{cases}$$

The total complex power is:

$$\bar{S}_T = \bar{S}_1 + \bar{S}_2 + \bar{S}_3 = (1780 + j1635)\ \text{kVA}$$

$$= (2417 \angle 42.57^\circ)\ \text{kVA}$$

We also have that $|\bar{S}_T| = \sqrt{3}\ V_S\ I_S$

Hence:
$$I_S = \frac{2417 \cdot 10^3}{\sqrt{3}\ 13.8 \cdot 10^3} = 101.1\ \text{A}$$

The combined power factor is:

$$\cos \varphi_S = \frac{1780}{2417} = 0.7365\ (\text{lagging})$$

10. a power factor equal to 0.92 lagging corresponds to $\cos\varphi = 0.92 \Rightarrow \varphi = 23.07°$.

Therefore the new complex power at the source is:

$$\overline{S}_{T\,new} = 1780 + j\,1780 \tan(23.07°)$$

$$= (1780 + j\,758.28)\ \text{kVA}$$

$$\overline{S}_{cap} = \overline{S}_{T\,new} - \overline{S}_T = j\,Q_c$$

$$\Rightarrow jQ_c = j(758.28 - 1635) = (-j\,876.72)\ \text{kVA}$$

$$-j\omega C V_s^2 = -j\frac{876.72 \cdot 10^3}{3}$$

Assuming a frequency of 50 Hz and $V_s = 13.8$ kV (line-to-line)

then $\omega = 2\pi \cdot 50 = 314.16$ rad/s

$\Rightarrow C = 4.885\ \mu F$

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