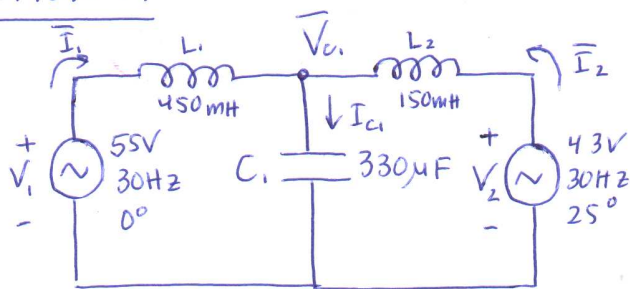


SECTION A



$f = 30\text{Hz}$

$L_1 = 450\text{mH}$

$L_2 = 150\text{mH}$

$C_1 = 330\mu\text{F}$

$\bar{V}_1 = 55\angle 0^\circ$

$\bar{V}_2 = 43\angle 25^\circ$

$\omega = 2\pi f$

①

$X_{L1} = \omega L_1 = 2\pi f L_1 = 450 \cdot 10^{-3} \cdot 2\pi \cdot 30 = \underline{\underline{84,8\ \Omega}}$

$X_{L2} = \omega L_2 = 2\pi f L_2 = 150 \cdot 10^{-3} \cdot 2\pi \cdot 30 = \underline{\underline{28,3\ \Omega}}$

$X_{C1} = \frac{1}{\omega C_1} = \frac{1}{2\pi f C_1} = \frac{1}{330 \cdot 10^{-6} \cdot 2\pi \cdot 30} = 16,1\ \Omega$

②

KLC:  $I_{L1} + I_{L2} - I_{C1} = 0$  for node 2

where:  $I_{L1} = \frac{V_1 - V_c}{jX_{L1}}$  (\*)  $I_{L2} = \frac{V_2 - V_c}{jX_{L2}}$  (\*)  $I_{C1} = \frac{V_c}{(-jX_{C1})}$

so:

$\frac{V_1 - V_c}{jX_{L1}} + \frac{V_2 - V_c}{jX_{L2}} - \frac{V_c}{-jX_{C1}} = 0$

$\frac{V_1}{jX_{L1}} + \frac{V_2}{jX_{L2}} = V_c \left( \frac{1}{jX_{L1}} + \frac{1}{jX_{L2}} - \frac{1}{jX_{C1}} \right)$

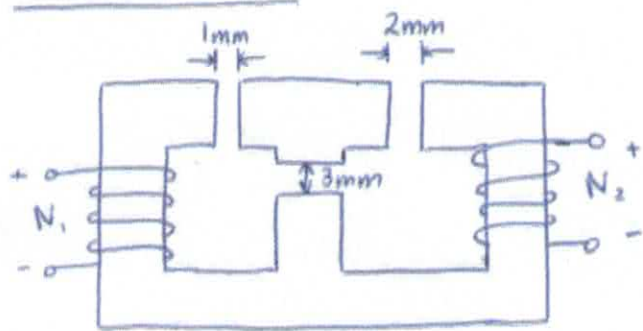
solve for  $V_c$ :  $V_c = \frac{\frac{V_1}{jX_{L1}} + \frac{V_2}{jX_{L2}}}{\frac{1}{jX_{L1}} + \frac{1}{jX_{L2}} - \frac{1}{jX_{C1}}} = \underline{\underline{141,8\angle 162,4^\circ\ \text{V}}}$

From (\*)  $\underline{\underline{I_{L1} = 2,29\angle -77,3^\circ\ \text{A}}}$

From (\*)  $\underline{\underline{I_{L2} = 6,5\angle -70,68^\circ\ \text{A}}}$

## SECTION B

2/4



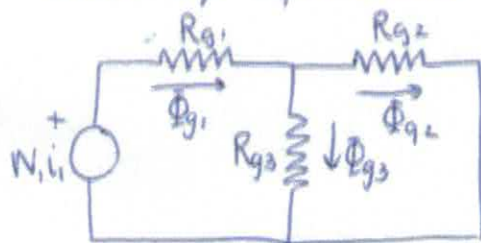
The iron core has a permeability  $\mu \rightarrow \infty \Rightarrow$  ignore core losses.

There are three air gaps ( $\mu_0 = 4\pi \cdot 10^{-7}$ )  
 $l_{g1} = 1\text{mm}$ ,  $l_{g2} = 2\text{mm}$  &  $l_{g3} = 3\text{mm}$   
 $A_c = 10\text{cm}^2 = 10 \cdot 10^{-4}\text{m}^2$

$N_1 = 200$  &  $N_2 = 100$

③ If  $i_1 = 10\text{A}$  and  $i_2 = 0\text{A}$ , determine  $\Phi_{g1}$ ,  $\Phi_{g2}$ ,  $\Phi_{g3}$ .

Draw up equivalent circuit.



$$R_{g1} = \frac{l_{g1}}{\mu_0 A_c} = \frac{1 \cdot 10^{-3}}{4\pi \cdot 10^{-7} \cdot 10 \cdot 10^{-4}} = 795,8 \cdot 10^3 \text{ H}^{-1}$$

$$R_{g2} = \frac{l_{g2}}{\mu_0 A_c} = 2 \cdot R_{g1} = 1591,5 \cdot 10^3 \text{ H}^{-1}$$

$$R_{g3} = \frac{l_{g3}}{\mu_0 A_c} = 3 \cdot R_{g1} = 2387,3 \cdot 10^3 \text{ H}^{-1}$$

Find the total resistance:

$$R_{\text{total}} = R_{g1} + R_{g2} \parallel R_{g3} = R_{g1} + \frac{R_{g2} R_{g3}}{R_{g2} + R_{g3}} = 1750,7 \cdot 10^3 \text{ H}^{-1}$$

Now the magnetic fluxes can be computed:

$$\Phi_{g1} = \frac{N_1 i_1}{R_{\text{total}}} = \frac{200 \cdot 10}{1750,7 \cdot 10^3} = \underline{\underline{1,142 \text{ mWb}}}$$

$$\Phi_{g2} = \frac{R_{g3}}{R_{g2} + R_{g3}} \cdot \Phi_{g1} = 0,4 \cdot 1,142 \text{ mWb} = \underline{\underline{0,4570 \text{ mWb}}}$$

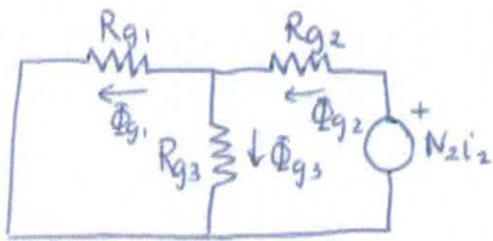
$$\Phi_{g3} = \frac{R_{g2}}{R_{g2} + R_{g3}} \cdot \Phi_{g1} = 0,4 \cdot 1,142 \text{ mWb} = \underline{\underline{0,4570 \text{ mWb}}}$$

④ Determine the self-induction coefficient ( $L_{11}$ ) and the mutual induction ( $M_{12}$ ).

$$L_{11} = \frac{\lambda}{i} = \frac{N_1 \Phi_1}{i_1} = \underline{\underline{22,84 \text{ mH}}}$$

$$M_{12} = \frac{N_2 \Phi_2}{i_1} = \underline{\underline{6,852 \text{ mH}}}$$

- ⑤ If  $i_1 = 0A$  and  $i_2 = 20A$  the equivalent circuit can be redrawn:



In this case

$$R_{total} = R_{g2} + R_{g1} \parallel R_{g3} = R_{g2} + \frac{R_{g1} R_{g3}}{R_{g1} + R_{g3}}$$

$$= 2188,3 \cdot 10^3 \text{ H}^{-1}$$

Determine  $\Phi_{g2} = \frac{N_2 i_2}{R_{total}} = \underline{\underline{0,914 \text{ mWb}}}$

and  $\Phi_{g1} = \underline{\underline{0,685 \text{ mWb}}}$  &  $\Phi_{g3} = \underline{\underline{0,2285 \text{ mWb}}}$

⑥  $L_{22} = \frac{\lambda}{i_2} = \frac{N_2 \Phi_{g2}}{i_2} = \underline{\underline{4,57 \text{ mH}}}$

$M_{21} = \frac{N_1 \cdot \Phi_{g1}}{i_2} = \underline{\underline{6,85 \text{ mH}}}$

# SECTION C

4/4

$S_n = 75 \text{ kVA}$   
 $V_{in} = 3000 \text{ V}$   
 $V_{2n} = 220 \text{ V}$   
 $f_n = 50 \text{ Hz}$

Short circuit test: primary voltage (200V)  
 consumed power (2kW)  
 Current ( $I_{nominal}$ )  
 open circuit test: power consumed (1,5kW)

⑦ Short-circuit impedance of transformer?

$I_{sc} = \frac{S_n}{V_{in}} = 25 \text{ A}$  (The nominal current)

The short-circuit resistance:

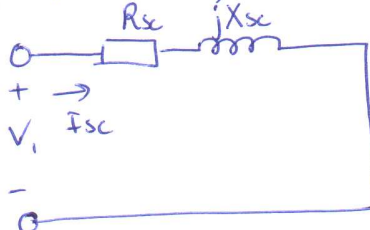
$R_{sc} = \frac{V}{I} = \frac{P}{I^2} = \frac{2000}{25^2} = 3,2 \Omega$

The active power consumed during short circuit test:

$P_{sc} = V I \cos(\theta) \Rightarrow \cos(\theta) = \frac{P_{sc}}{V_1 I_{sc}} = \frac{2000}{200 \cdot 25} = 0,4$   
 (powerfactor)

$\Rightarrow \theta = 66,42^\circ (\arccos(0,4))$

$I_p X_{sc} = V_p \sin(\theta) \Rightarrow X_{sc} = 7,332 \Omega$



Short-circuit test: second winding shorted

⑧ Voltage drop ~~on secondary winding~~ <sup>under full load conditions</sup> for p.f : 0,8 lagging  $\Rightarrow \cos(\theta) = 0,8$   
 $\sin(\theta) = 0,6$

$I_{full\ load} = I_{sc} = 25 \text{ A}$

The voltage drop in the short circuit impedance

$V_{loss} = I_p (R_{sc} \cos(\theta) + X_{sc} \sin(\theta)) = 25(3,2 \cdot 0,8 + 7,332 \cdot 0,6)$   
 $= 174 \text{ V}$

$\Delta V = \frac{V_{loss}}{V_{in}} = 0,058 \approx 5,8\%$

⑨ pf: 0,8 leading  $\Rightarrow \cos(\theta) = 0,8$   
 $\sin(\theta) = -0,6$

$V_{loss} = 25(3,2 \cdot 0,8 + 7,332(-0,6)) = -45,98 \text{ V}$

$\Delta V = \frac{-45,98}{3000} = -1,53\%$

⑩ ~~Efficiency~~  $\eta = \frac{S_n \cos(\theta)}{S_n \cos(\theta) + \text{sc losses} + \text{oc losses}} = 0,945 = 94,5\%$