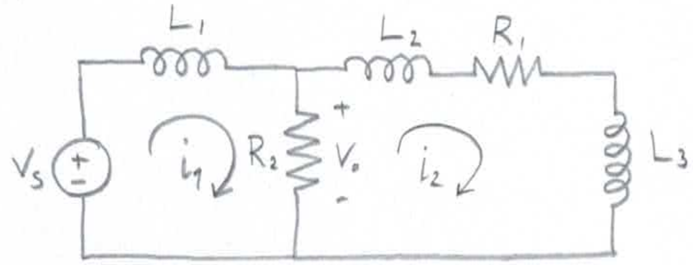
MIDTERM EXAMINATION

EEEN20090 - ELECTRICAL ENERGY SYSTEMS

SECTION A



$L_1 = 8H \qquad R_1 = 50\Omega$

$L_2 = 6H \qquad R_2 = 400\Omega$

$L_3 = 3H$

$V_S(t) = 25\cos(40t + 45°)\,V$

(1.) $i_1(t)$ & $i_2(t)$?

$\overline{V}_s = \frac{25}{\sqrt{2}} \angle 45°$ ← RMS and $\omega = 40$

and

$X_1 = \omega L_1 = 40 \cdot 8 = 320\Omega$

$X_2 = \omega L_2 = 40 \cdot 6 = 240\Omega$

$X_3 = \omega L_3 = 40 \cdot 3 = 120\Omega$

$\overline{I}_1 = \frac{\overline{V}_s}{\overline{Z}_{total}}$ where $\overline{Z}_{total} = jX_1 + R_2 \| (R_1 + j(X_2 + X_3))$

$= jX_1 + \frac{R_2(R_1 + j(X_2+X_3))}{R_2 + (R_1 + j(X_2+X_3))}$

$= j320 + \frac{400(50 + j(240+120))}{400 + 50 + j(240+120)}$

$= 526{,}11 \angle 69{,}7°\,\Omega$

$\overline{I}_1 = \frac{\frac{25}{\sqrt{2}}\angle 45°}{526{,}11\angle 69{,}7°}$

$= 33{,}6 \angle -24{,}7°\,mA$

$i_1(t) = 33{,}6 \cdot \sqrt{2}\cos(40t - 24{,}7°)$

$= 47{,}5\cos(40t - 24{,}7°)\,mA$

$\overline{I}_2 = \overline{I}_1 \cdot \frac{R_2}{R_1 + R_2 + j(X_2+X_3)} = 23{,}32 \angle -63{,}35°\,mA$

$i_2(t) = 32{,}98\cos(40t - 63{,}35°)\,mA$

SECTION A

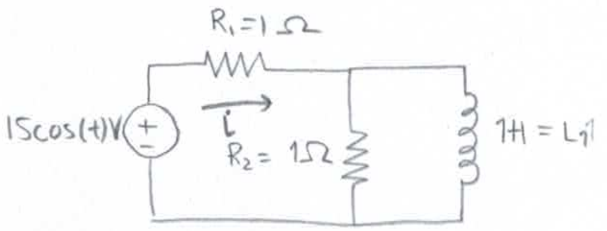
(2.) $V_o(t)$?

$$\bar{V_o} = \bar{V_s} - \bar{I_1} \cdot X_1 = \frac{25}{\sqrt{2}} \angle 45° - 33{,}6 \cdot 10^{-3} \angle -24{,}7° \cdot 320 \angle 90°$$

$$= 8{,}461 \angle 18{,}8°$$

$$V_o(t) = \sqrt{2} \cdot 8{,}461 \cos(40t + 18{,}8°)$$

$$= \underline{\underline{11{,}97 \cos(40t + 18{,}8°)}}$$



$\omega = 1 \Rightarrow X_1 = \omega L_1 = 1\Omega$

$\bar{V} = \frac{15}{\sqrt{2}} \angle 0°$

(3.) The average power delivered by the source to the circuit?

First find current from source

$$\bar{I} = \frac{\bar{V}}{Z_{eq}} \quad \text{where } Z_{eq} = R_1 + R_2 \| jX_1 = 1 + \frac{1 \cdot j1}{1 + j1} = \frac{\sqrt{10}}{2} \angle 18{,}4°$$

$$\Rightarrow \bar{I} = \frac{\frac{15}{\sqrt{2}} \angle 0°}{\frac{\sqrt{10}}{2} \angle 18{,}4°} = 3\sqrt{5} \angle -18{,}4°$$

changes sign of angle

$$\text{Find } \bar{S} = \bar{V} \cdot \bar{I}^* = \frac{15}{\sqrt{2}} \angle 0° \cdot 3\sqrt{5} \angle 18{,}4° = 71{,}15 \angle 18{,}4° \text{VA}$$

The active power delivered:

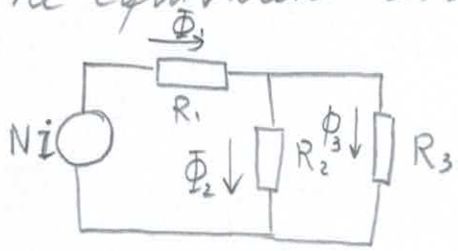
$$P = Re\{\bar{S}\} = 71{,}15 \cos(18{,}4°) = \underline{\underline{67{,}5W}}$$

(4.) The power absorbed by the resistor R_1.

$$P_1 = R_1 I^2 = \underline{\underline{45W}}$$

SECTION B

The equivalent circuit for the magnetic system:



where $l_1 = l_3 = 300mm$

$l_2 = 100mm$

$A_1 = A_3 = 200mm^2$

$A_2 = 400mm^2$

$\mu_{r_1} = \mu_{r_3} = 2250$

$N = 25$

(5.) ϕ_1, ϕ_2 & ϕ_3 ?
for $i = 0,5A$

Find reluctances:

$$R_1 = R_3 = \frac{l_1}{\mu_{r_1}\mu_0 A_1} = \frac{300\cdot10^{-3}}{2250\cdot4\pi\cdot10^{-7}\cdot200\cdot10^{-6}} = 0,531\cdot10^6 \, A/Wb$$

$$R_2 = \frac{100\cdot10^{-3}}{1350\cdot4\pi\cdot10^{-7}\cdot400\cdot10^{-6}} = 0,148\cdot10^6 \, A/Wb$$

$$R_t = R_1 + R_2 // R_3 = 0,6458\cdot10^6 \, A/Wb$$

$$\phi_1 = \frac{N\cdot i}{R_t} = \underline{\underline{19,36\cdot10^{-6} \, Wb}}$$

$$\phi_2 = \frac{R_3}{R_2 + R_1}\phi_1 = \underline{\underline{15,15\cdot10^{-6} \, Wb}}$$

$$\phi_3 = \phi_1 - \phi_2 = \underline{\underline{4,21\cdot10^{-6} \, Wb}}$$

(6.) B_1, B_2 & B_3 ? for $i = 0,5A$

$$B_1 = \phi_1/A_1 = 19,36\cdot10^{-6}/200\cdot10^{-6} = \underline{\underline{0,0968\,T}}$$

$$B_2 = \phi_2/A_2 = 15,15\cdot10^{-6}/400\cdot10^{-6} = \underline{\underline{0,03788\,T}}$$

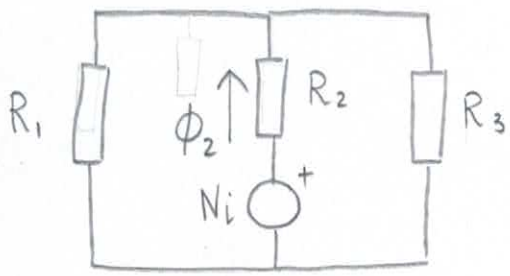
$$B_3 = \phi_3/A_3 = 4,21\cdot10^{-6}/200\cdot10^{-6} = \underline{\underline{0,02105\,T}}$$

SECTION B

(7.) Self-inductance of the magnetic circuit?

$$L = \frac{N\phi_1}{i} = \frac{25 \cdot 19{,}36 \cdot 10^{-6}}{0{,}5} = \underline{\underline{0{,}000968\ H}}$$

(8.) The self-inductance of the magnetic circuit if the coil is moved to the central part of the magnetic circuit.



$$R_{total} = R_2 + R_3 \parallel R_1$$
$$= R_2 + \frac{R_3 R_1}{R_3 + R_1} = 0{,}413 \cdot 10^6\ A/Wb$$

$$\phi_2 = \frac{Ni}{R_{total}} = \frac{25 \cdot 0{,}5}{0{,}413 \cdot 10^6} = 30{,}3 \cdot 10^{-6}\ Wb$$

$$L = \frac{N \cdot \phi_2}{i} = \frac{25 \cdot 30{,}3 \cdot 10^{-6}}{0{,}5} = \underline{\underline{0{,}001515\ H}}$$

SECTION C

Nominal parameters of transformer:

$S_N = 200\,kVA$, $V_{1N} = 13{,}200\,V$, $V_{2N} = 2{,}200\,V$

Open-circuit test: 2.200 V, 3,1 A & 1.550 W

Short-circuit test: 210 V, 90,9 A & 2.500 W

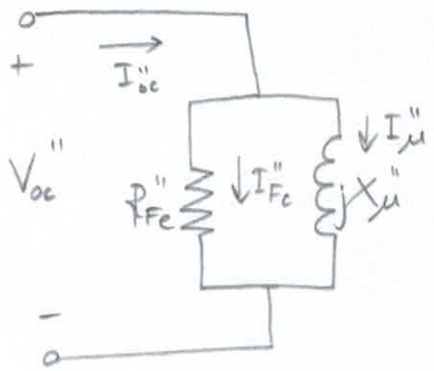
Transformer is feeding a load consuming:

$S_2 = 180\,kVA$, pf $\cos(\phi_2) = 0{,}9$ leading, $V_2 = V_{2N}$

9. G''_{Fe} & B''_μ ?

Use the results from the open circuit test:

$$R''_{Fe} = \frac{V''^2}{P''} = \frac{2{,}200^2}{1550} = 3123\,\Omega$$



$$\Rightarrow G''_{Fe} = \frac{1}{R''_{Fe}} = \underline{\underline{0{,}00032}}$$

$$I''_{Fe} = \frac{2200}{R''_{Fe}} = 0{,}7045\,A$$

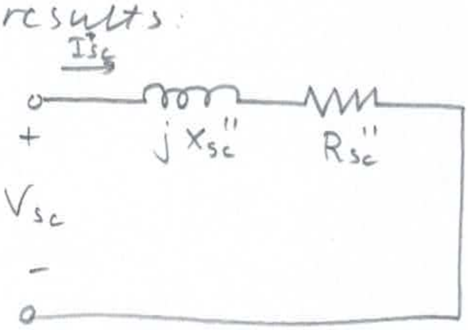
$$I''_\mu = \sqrt{3{,}1^2 - 0{,}7045^2} = 3{,}0189\,A$$

$$X''_\mu = \frac{2200}{3{,}0189} = 728{,}7\,\Omega \Rightarrow B''_\mu = \frac{1}{X''_\mu} = \underline{\underline{0{,}001372}}$$

10. R''_{sc} & X''_{sc} ?

Use the short circuit test results:

$$R''_{sc} = \frac{P_{sc}}{I_{sc}^2} = \frac{2500}{90{,}9^2} = \underline{\underline{0{,}3026\,\Omega}}$$



$$Z''_{sc} = \frac{210}{90{,}9} = 2{,}31\,\Omega$$

$$X''_{sc} = \sqrt{Z''^2_{sc} - R''^2_{sc}}$$

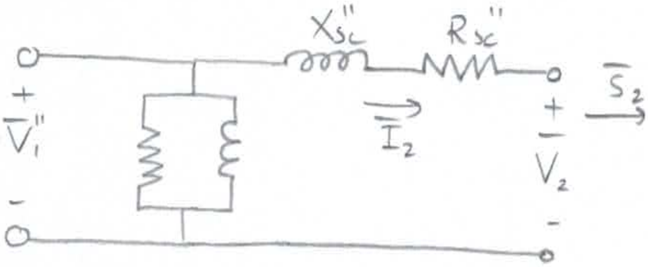
$$= \sqrt{2{,}31^2 - 0{,}3026^2} = \underline{\underline{2{,}29\,\Omega}}$$

SECTION C

(11.) The voltage regulation at the given operating condition ?

$$\mathcal{E}_v = 100 \frac{|V_1'' - V_2|}{|V_1''|}$$ ← known

Need to find V_1''. Circuit model:



$\cos(\varphi_2) = 0,9$ leading $\Rightarrow \varphi_2 = -25,84°$

↑ negative because is leading

$\Rightarrow \sin(\varphi_2) = -0,436$

Then we have: $|\overline{I}_2| = \frac{|\overline{S}_2|}{|\overline{V}_2|} = \frac{180 \cdot 10^3}{2200} = 81,82 \text{ A}$

and: $V_2 = V_1'' - I_2(R_{sc}'' \cos(\varphi_2) + X_{sc}'' \sin(\varphi_2))$

$$\Rightarrow V_1'' = V_2 + I_2(R_{sc}'' \cos(\varphi_2) + X_{sc}'' \sin(\varphi_2))$$
$$= 2140,6 \text{ V}$$

and calculate:

$$\mathcal{E}_v = 100 \frac{|V_1'' - V_2|}{|V_1''|} = \underline{\underline{27,75}}$$

(12.) Efficiency ?

$$\eta = \frac{P_2}{P_2 + P_{Fe} + P_{\mu}}$$

$P_{Fe} = V_1''^2 G_{Fe} = 2140,6^2 \cdot 0,00032 = 1,46 \text{ kW}$

$P_{\mu} = R_{sc}'' \cdot I_2^2 = 0,3026 \cdot 81,82^2 = 2.026 \text{kW}$

$P_2 = S_2 \cos(\phi) = 180 \cdot 0,9 = 162 \text{ kW}$

$$\eta = \frac{162}{162 + 1,46 + 2,026} = 0,979 \simeq \underline{\underline{98\%}}$$