EEEN20090 – Midterm Exam 2021/22

Section A

1. $\bar{V} = \bar{Z}_2 \cdot \bar{I}$ where $\bar{I} = \dfrac{\bar{V}_g}{\bar{Z}_1 + \bar{Z}_2}$

$$\bar{V}_g = 15/\sqrt{2}\,\angle 0 = (10.607\angle 0)\,V$$

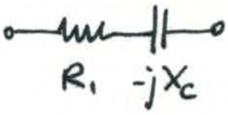
$$\bar{I} = \frac{10.607\angle 0}{(13.97 - j6.247) + (8.646 + j11.515)} = (0.445 - j\,0.104)\,A$$

$$\bar{V} = (8.646 + j11.515)(0.445 - j0.104) = (5.04 + j4.227)\,V$$

$$= (6.578\angle 0.698)\,V = (6.578\angle 39.986°)\,V$$

$$v(t) = \sqrt{2}\cdot 6.578\,\cos(20t + 0.698)\,V = 9.303\,\cos(20t + 0.698)\,V$$

2. $\bar{Z}_1$, series connection



$\bar{Z}_1 = Z_1\angle\varphi_1$

$R_1 = Z_1\cos\varphi_1$

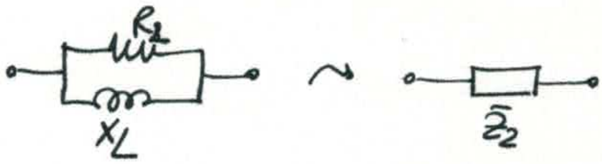
$X_C = Z_1|\sin\varphi_1|$

$R_1 = 13.966\ \Omega$

$X_C = 6.247\ \Omega$

$$X_C = \frac{1}{\omega C} \Rightarrow C = \frac{1}{\omega X_C} = \frac{1}{20\cdot 6.247} = 0.008\ F$$

$\bar{Z}_2$, parallel connection



$$\rightarrow \quad \bar{Y}_2 = \frac{1}{\bar{Z}_2} = (0.0417 - j\,0.0555)\,\Omega^{-1} = G_2 - jB_2$$

$$R_2 = \frac{1}{G_2} = 4973\ \Omega$$

$$X_L = \frac{1}{B_2} = 3733\ \Omega \qquad X_L = \omega L \Rightarrow L = \frac{X_L}{\omega} = 186.7\ H$$

3. $P_{R1} = R_1 \cdot I^2 = R_1 \cdot |\bar{I}|^2 = 2.915 \text{ W}$

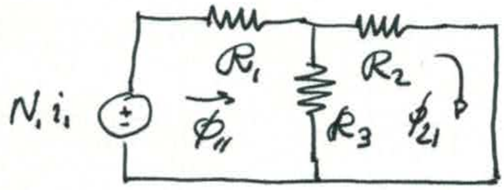
$P_{R2} = G_2 \cdot V^2 = G_2 \cdot |\bar{V}|^2 = 1.804 \text{ W}$

$\bar{S}_g = \bar{V}_g \cdot \bar{I}^* = (4.719 + j\,1.099) \text{ VA}$

Section B

4. $L_{11} = N_1 \dfrac{\phi_{11}}{i_1}$ where ϕ_{11} is determined by the following circuit:

(*)

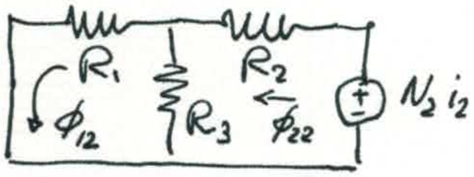


and, hence, one has: $$\phi_{11} = \frac{R_2 + R_3}{R_1R_2 + R_2R_3 + R_3R_1} N_1 i_1$$

Then: $$L_{11} = N_1^2 \frac{R_2 + R_3}{R_1R_2 + R_2R_3 + R_3R_1}$$

Equivalently $L_{22} = N_2 \dfrac{\phi_{22}}{i_2}$ where ϕ_{22} is given by:

(**)



$$\phi_{22} = \frac{R_1 + R_3}{R_1R_2 + R_2R_3 + R_3R_1} N_2 i_2$$

and, hence, one has: $$L_{22} = N_2^2 \frac{R_1 + R_3}{R_1R_2 + R_2R_3 + R_3R_1}$$

5. $L_{21} = N_2 \dfrac{\phi_{21}}{i_1}$ where ϕ_{21} is determined from circuit (*) as:

$$\phi_{21} = \phi_{11} \frac{R_3}{R_2 + R_3}$$

then: $$L_{21} = N_1 N_2 \frac{R_3}{R_1R_2 + R_2R_3 + R_3R_1}$$

Equivalently, one can determine $L_{12} = N_1 \dfrac{\phi_{12}}{i_2}$ where, from circuit (**), one has:

$$\phi_{12} = \phi_{22} \frac{R_3}{R_1 + R_3}$$

and, finally:

$$L_{12} = N_1 N_2 \frac{R_3}{R_1 R_2 + R_2 R_3 + R_3 R_1}$$

6. From the solution of 4. and 5., we observe that

$$M = L_{12} = L_{21} \neq \sqrt{L_{11} L_{22}}$$

This result is as expected because:

a. No flux leakage is assumed

b. $\phi_{11} \neq \phi_{21}$ and $\phi_{22} \neq \phi_{12}$

The condition $M = \sqrt{L_{11} L_{22}}$ holds only in circuits for which the flux in both windings is the same.

One can define a "coupling coefficient" as follow:

$$\left.\begin{aligned} K_1 &= \frac{\phi_{21}}{\phi_{11}} = \frac{R_3}{R_2 + R_3} \\ K_2 &= \frac{\phi_{12}}{\phi_{22}} = \frac{R_3}{R_1 + R_3} \\ K &= \sqrt{K_1 K_2} = \frac{R_3}{\sqrt{(R_2 + R_3)(R_1 + R_3)}} \end{aligned}\right\} \Rightarrow M = K\sqrt{L_{11} L_{22}}$$

hence $M = \sqrt{L_{11} L_{22}}$ only if the coupling coefficient $K = 1$ #

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Section C

7. $V_1 = 2000\ V$, $I_{10} = 1\ A$, $P_0 = 1000\ W$

$$P_0 = V_1 I_{10} \cos\varphi_0 \Rightarrow \cos\varphi_0 = 0.5$$
$$\sin\varphi_0 = 0.866$$

$$\Rightarrow I_{Fe} = I_{10} \cos\varphi_0 = 0.5\ A$$
$$I_\mu = I_{10} \sin\varphi_0 = 0.866\ A$$

$$\Rightarrow R_{Fe} = \frac{V_1}{I_{Fe}} = \frac{2000}{0.5} = 4000\ \Omega$$
$$X_\mu = \frac{V_1}{I_\mu} = \frac{2000}{0.866} = 2309.5\ \Omega$$

8. $V_{1SC} = K_T V_{2SC} = \frac{2000}{200} \cdot 8 = 80\ V$

$$I_{1SC} = I_{1N} = \frac{S_N}{V_{1N}} = \frac{160000}{2000} = 80\ A$$

$P_{SC} = 2560\ W$

$$P_{SC} = V_{1SC} \cdot I_{1N} \cos\varphi_{SC} \Rightarrow \cos\varphi_{SC} = 0.4$$
$$\sin\varphi_{SC} = 0.917$$

$$Z'_{SC} = \frac{V_{1SC}}{I_{1N}} = 1\ \Omega \Rightarrow R'_{SC} = Z'_{SC} \cdot \cos\varphi_{SC} = 0.4\ \Omega$$
$$X'_{SC} = Z'_{SC} \cdot \sin\varphi_{SC} = 0.917\ \Omega$$

9. From Kapp's formula, one has:

$$V_1 = V'_2 + R'_{SC} I'_2 \cos\varphi_2 + X'_{SC} I'_2 \sin\varphi_2$$

where:

$$V_1 = 2000\ V$$

$$I_2 = 400\ A \quad \Rightarrow \quad I_2' = \frac{400}{K_T} = \frac{400}{10} = 40\ A$$

$$\cos\varphi_2 = 0.8 \ (\text{lagging}) \quad \Rightarrow \quad \sin\varphi_2 = 0.6$$

hence:

$$2000 = V_2' + 0.4 \cdot 40 \cdot 0.8 + 0.917 \cdot 40 \cdot 0.6$$

$$\Rightarrow V_2' = 1965.2\ V \qquad \Rightarrow \quad V_2 = \frac{V_2'}{K_T} = 196.52\ V$$

10. The efficiency is maximum when the losses in the iron are equal to the losses in the windings. This is equivalent to impose a power consumption equal to:

$$S_{\eta max} = \sqrt{\frac{P_0}{P_{SC}}} \cdot S_N = \sqrt{\frac{1000}{2560}} \cdot 160 = 100\ KVA$$

Then, the maximum efficiency is given by:

$$\eta_{max} = \frac{S_{\eta max} \cos\varphi}{S_{\eta max}\cos\varphi + 2P_0} = \frac{100 \cdot 0.8}{100 \cdot 0.8 + 2 \cdot 1} = \frac{80}{82} = 97.56\ \%$$

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