

1st term, year 2017/18.

Section A

This 3-phase, Y-connected system has a conductor between the generator and the load neutral.

Because of this, the current in each line can be calculated as if we had ~~the~~ single-phase circuits.

The generator terminals will be unprimed and the load terminals primed. The line currents are:

$$\bar{I}_{11'} = \bar{V}_{1'n'} / \bar{Z}_1 = 220 \angle 0^\circ / 10 \angle 0^\circ = 22 \angle 0^\circ \text{ A}$$

$$\bar{I}_{22'} = \bar{V}_{2'n'} / \bar{Z}_2 = 220 \angle 120^\circ / 20 \angle 20^\circ = 11 \angle 100^\circ \text{ A}$$

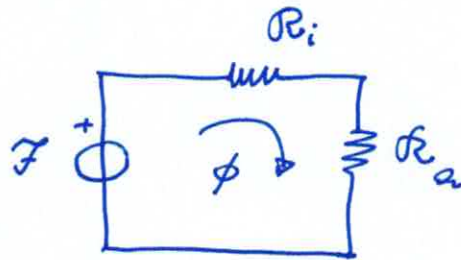
$$\bar{I}_{33'} = \bar{V}_{3'n'} / \bar{Z}_3 = 220 \angle -120^\circ / 12 \angle -35^\circ = 18.3 \angle -85^\circ \text{ A}$$

The current in the neutral line is found by summing the above three currents:

$$\bar{I}_{nn'} = \bar{I}_{11'} + \bar{I}_{22'} + \bar{I}_{33'} = 229 \angle -18.8^\circ \text{ A}$$

Section B

The equivalent magnetic circuit for the system is shown below:



where R_i is the reluctance of the core and R_a is that of the air gap.

$$\begin{aligned} a) \quad R_i &= (700 \times 10^3) / (1850 \cdot 4\pi \cdot 10^{-7} \cdot 100^2 \cdot 10^{-6}) \\ &= 38.42 \cdot 10^3 \text{ A/Wb} \end{aligned}$$

$$\begin{aligned} R_a &= (4\pi \cdot 10^3) / (4\pi \cdot 10^{-7} \cdot 100^2 \cdot 10^{-6}) = \\ &= 318.3 \cdot 10^3 \text{ A/Wb} \end{aligned}$$

The total reluctance of the flux path is:

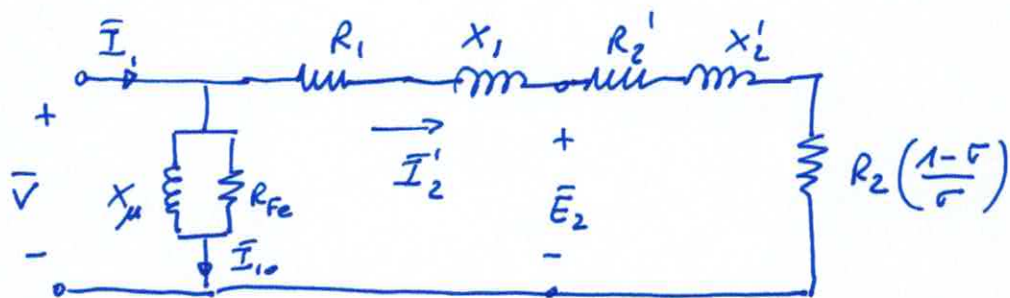
$$R = R_i + R_a = 356.7 \cdot 10^3 \text{ A/Wb}$$

The proportion of the total mmf required to overcome the air-gap reluctance is

$$R_a / R = \frac{318.3}{356.7} = 0.892$$

Section C

Let's consider the following approximated circuit:



a) - slip at which maximum power occurs:

$$\sigma_M = \frac{R_2'}{\sqrt{R_1^2 + (X_1 + X_2')^2}} = \frac{0.18}{\sqrt{0.15^2 + 0.62^2}} = 0.282$$

b) - Maximum torque:

$$\begin{aligned} T_{max} &= \frac{3P}{\omega_n} \cdot \frac{V_1^2}{\left(R_1 + \frac{R_2'}{\sigma_M}\right)^2 + (X_1 + X_2')^2} \cdot \frac{R_2'}{\sigma_M} \\ &= \frac{3 \cdot 2}{\frac{3600}{60} \cdot 2\pi} \cdot \frac{127^2}{\left(0.15 + \frac{0.18}{0.282}\right)^2 + (0.62)^2} \cdot \frac{0.18}{0.282} \\ &= 17.06 \text{ Nm} \cdot \frac{60}{2\pi} = 162.9 \text{ Nm} \end{aligned}$$

where 3600 is the synchronous angular speed of the feeder in rpm and $\frac{3600 \cdot 2\pi}{60} \cdot \frac{1}{2}$ is the mechanical synchronous speed of the motor in $\frac{\text{rad}}{\text{s}}$.

c) - Slip at which maximum power occurs:

$$\sigma_{mp} = \frac{R_2'}{R_2' + Z_e'} = \frac{0,18}{0,18 + 0,703} = 0,206$$

where $Z_e' = \sqrt{(R_1 + R_2')^2 + (X_1 + X_2')^2}$

d) - Maximum power:

$$P_{max} = \frac{3 V^2}{\left(R_1 + \frac{R_2'}{\sigma_{mp}}\right)^2 + (X_1 + X_2')^2} \cdot \frac{R_2'}{\sigma_{mp}} = 29,4 \text{ kW}$$

e) - Starting torque ($\sigma_{su} = 1$)

$$T_{su} = \frac{3P}{\omega_n} \cdot \frac{V^2}{(R_1 + R_2')^2 + (X_1 + X_2')^2} \quad R_2' = 93,7 \text{ Nm}$$

f) - Starting current:

$$I_{su} = \frac{V}{\sqrt{(R_1 + R_2')^2 + (X_1 + X_2')^2}} = \frac{V}{Z_e} = 181 \text{ A}$$

Section D

$$a) \quad f = \frac{\text{poles}}{2} \cdot \text{rpm} / 60 = \frac{6}{2} \cdot 1200 \frac{1}{60} = 60 \text{ Hz}$$

$$b) \quad \text{rpm}_{25} = \frac{120 \cdot 25}{6} = \frac{2 \cdot 60}{\text{poles}} \cdot 25 = 500 \text{ rpm}$$

$$\text{rpm}_{50} = \frac{2 \cdot 60}{\text{poles}} \cdot 50 = \frac{120}{6} \cdot 50 = 1000 \text{ rpm}$$

$$c) \quad \text{poles} = \frac{2 \cdot 60 \cdot f}{\text{rpm}} = \frac{2 \cdot 60 \cdot 60}{240} = 30 \text{ poles}$$

Section E

a) Total resistance $R = 5 \cdot 0.078 = 0.39 \Omega$

Let I be the current and V the voltage at the load:

$$VI = 60,000 \text{ W} \quad (*)$$

$$V = 600 - 0.39 I \quad (**)$$

Substituting V from $(**)$ in $(*)$:

$$60,000 = (600 - 0.39 I) I \quad (***)$$

$$0.39 I^2 - 600 I = -60,000$$

Dividing $(***)$ by 0.39 :

$$I^2 - 1,538 I = -153,800$$

$$\Rightarrow \text{solutions: } \begin{cases} 1,430.5 \text{ A} \\ 107.5 \text{ A} \end{cases}$$

\rightarrow Lower current (107.5 A) is the sought solution as it minimizes losses (max. efficiency).

hence: $I = 107.5 \text{ A} \quad \#$

$$b) \quad V = 600 - (0.39 \cdot 107.5) = 600 - 41.9 = 558.1 \text{ V}$$

$$P = 558.1 \cdot 107.5 = 60,000 \text{ W} \quad (\text{check!})$$

$$c) \quad \eta = \frac{558.1 \cdot \cancel{107.5}}{600 \cdot \cancel{107.5}} = 93 \%$$

d) The maximum power is obtained for a load whose impedance is the same as the transmission line:

$$I = \frac{V}{2R} = 769.23 \text{ A} \quad V_L = 600 - RI = 300 \text{ V}$$

hence: $P = V_L I = 230.1 \text{ kW}$

Note that in this case $\eta = 50\%$ (too high!)

e) The maximum current occurs when the load is short circuited:

$$I = \frac{V}{R} = \frac{600}{0.39} = 1,538 \text{ A}$$

In this case the voltage on the load (V_L) is zero. No power is delivered to the load.