Solutions of the resit exam for module EEEN 20090.

1^{st} term, year 2017/18.

Section A

This 3-phase, Y-connected system has a conductor between the generator and the load neutral. Because of this, the current in each line can be calculated as if we had ~~th~~ single-phase circuits. The generator terminals will be unprimed and the load terminals primed. The line currents are:

$$\bar{I}_{11'} = \bar{V}_{1'n'} / \bar{Z}_1 = 220\angle 0° / 10\angle 0° = 22\angle 0° \text{ A}$$

$$\bar{I}_{22'} = \bar{V}_{2'n'} / \bar{Z}_2 = 220\angle 120° / 20\angle 20° = 11\angle 100° \text{ A}$$

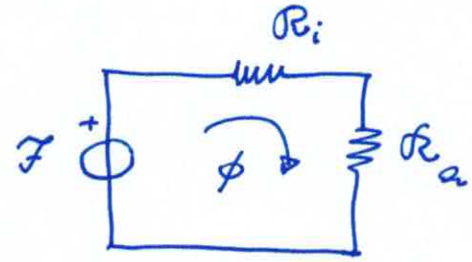
$$\bar{I}_{33'} = \bar{V}_{3'n'} / \bar{Z}_3 = 220\angle -120° / 12\angle -35° = 18.3\angle -85° \text{ A}$$

The current in the neutral line is found by summing the above three currents:

$$\bar{I}_{nn'} = \bar{I}_{11'} + \bar{I}_{22'} + \bar{I}_{33'} = 229\angle -18.8° \text{ A}$$

Section B

The equivalent magnetic circuit for the system is shown below:



where $\mathcal{R}_i$ is the reluctance of the core and $\mathcal{R}_a$ is that of the air gap.

a)

$$\mathcal{R}_i = (700 \times 10^{3}) / (1450 \cdot 4\pi \cdot 10^{-7} \cdot 100^2 \cdot 10^{-6})$$

$$= 38.42 \cdot 10^3 \ A/Wb$$

$$\mathcal{R}_a = (4\pi \cdot 10^3) / (4\pi \cdot 10^{-7} \cdot 100^2 \cdot 10^{-6} =$$

$$= 318.3 \cdot 10^3 \ A/Wb$$

The total reluctance of the flux path is:

$$\mathcal{R} = \mathcal{R}_i + \mathcal{R}_a = 356.7 \cdot 10^3 \ A/Wb$$

The proportion of the total mmf required to overcome the air-gap reluctance is

$$\mathcal{R}_a / \mathcal{R} = \frac{318.3}{356.7} = 0.892$$

b) $$\phi = \mathcal{F}/\mathcal{R} = \left(10^3 \cdot 1.5\right) / \left(356.7 \cdot 10^3\right) = 4.205 \cdot 10^{-3}\ \mathrm{Wb}$$

$$B = \phi / S = \left(4.205 \cdot 10^{-3}\right) / \left(100^2 \cdot 10^{-6}\right) = 0.4205\ \mathrm{T}$$

c) Ratio: $$\left(0.4205\right) / \left(10.77 \cdot 10^{-3}\right) = 39.0$$

d) In the air gap,

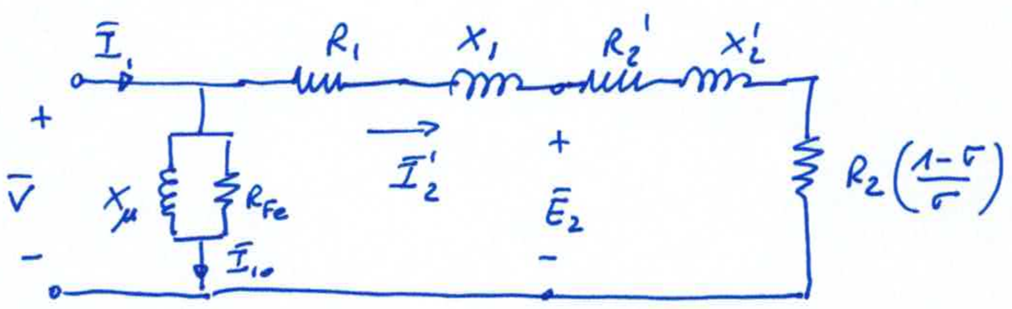
$$H_a = B/\mu_0 = \frac{0.4205}{4\pi \cdot 10^{-7}} = 0.3346 \cdot 10^{6}\ \mathrm{A/m}$$

In the core:

$$H_i = B/\mu_r \mu_0 = \frac{0.4205}{1450 \cdot 4 \cdot \pi \cdot 10^{-7}} = 230.8\ \mathrm{A/m}$$

Section C

Let's consider the following approximated circuit:



a) – Slip at which maximum power occurs:

$$\sigma_M = \frac{R_2'}{\sqrt{R_1^2 + (X_1 + X_2')^2}} = \frac{0{,}18}{\sqrt{0{,}15^2 + 0.62^2}} = 0{,}282$$

b) – Maximum torque:

$$T_{max} = \frac{3p}{\omega_n} \cdot \frac{V_1^2}{\left(R_1 + R_2'/\sigma_M\right)^2 + (X_1 + X_2')^2} \cdot \frac{R_2'}{\sigma_M}$$

$$= \frac{3 \cdot 2}{\frac{3600}{60} \cdot 2\pi} \cdot \frac{127^2}{\left(0{,}15 + \frac{0.18}{0{,}282}\right)^2 + (0.62)^2} \cdot \frac{0.18}{0{,}282}$$

$$= 17{,}06 \ Nm \cdot \frac{60}{2\pi} = 162{,}9 \ Nm$$

where 3600 is the synchronous angular speed of the feeder in rpm and $\frac{3600}{60} \cdot 2\pi \cdot \frac{1}{2}$ is the mechanical synchronous speed of the motor in $\frac{rad}{s}$.

c)- Slip at which maximum power occurs:

$$\sigma_{mp} = \frac{R_2'}{R_2' + Z_e'} = \frac{0,18}{0,18 + 0,703} = 0.204$$

where $Z_e' = \sqrt{(R_1 + R_2')^2 + (X_1 + X_2')^2}$

d)- Maximum power:

$$P_{max} = \frac{3\,V^2}{\left(R_1 + \frac{R_2'}{\sigma_{mp}}\right)^2 + (X_1 + X_2')^2} \cdot \frac{R_2'}{\sigma_{mp}} = 29,4 \text{ kW}$$

e)- Starting torque ($\sigma_{su} = 1$)

$$T_{su} = \frac{3p}{\omega_n} \cdot \frac{V^2}{(R_1 + R_2')^2 + (X_1 + X_2')^2} R_2' = 93,7 \text{ Nm}$$

f)- Starting current:

$$I_{su} = \sqrt{\frac{V^2}{(R_1 + R_2')^2 + (X_1 + X_2')^2}} = \frac{V}{Z_e} = 181 \text{ A}$$

Section D

a) $$f = \frac{poles}{2} \cdot rpm/60 = \frac{6}{2} \cdot 1200 \frac{1}{60} = 60 \text{ Hz}$$

b) $$rpm_{25} = \frac{120 \cdot 25}{6} = \frac{2 \cdot 60}{poles} \cdot 25 = 500 \text{ rpm}$$

$$rpm_{50} = \frac{2 \cdot 60}{poles} \cdot 50 = \frac{120}{6} \cdot 50 = 1000 \text{ rpm}$$

c) $$poles = \frac{2 \cdot 60 \cdot f}{rpm} = \frac{2 \cdot 60 \cdot 60}{240} = 30 \text{ poles}$$

Section E

a) Total resistance $R = 5 \cdot 0.078 = 0.39\ \Omega$

Let I be the current and V the voltage at the load:

$$V I = 60{,}000 \text{ W} \qquad (*)$$

$$V = 600 - 0.39\, I \qquad (**)$$

Substituting V from $(**)$ in $(*)$:

$$60{,}000 = (600 - 0.39\, I)\, I \qquad (***)$$

$$0.39\, I^2 - 600\, I = -60{,}000$$

Dividing $(***)$ by 0.39:

$$I^2 - 1{,}538\, I = 153{,}800$$

$\Rightarrow$ solutions: $\begin{cases} 1{,}430.5 \text{ A} \\ 107.5 \text{ A} \end{cases}$

→ Lower current (107.5 A) is the sought solution as it minimizes losses (max. efficiency).

hence: $I = 107.5$ A #

b) $V = 600 - (0.39 \cdot 107.5) = 600 - 41.9 = 558.1 \; V$

$P = 558.1 \cdot 107.5 = 60{,}000 \; W$ (check!)

c) $\eta = \dfrac{558.1 \cdot \cancel{107.5}}{600 \cdot \cancel{107.5}} = 93\,\%$

d) The maximum power is obtained for a load whose impedance is the same as the transmission line:

$I = \dfrac{V}{2R} = 769.23 \; A$ $\qquad V_L = 600 - RI = 300 \; V$

hence: $P = V_L I = 230.1 \; kW$

Note that in this case $\eta = 50\,\%$ (too high!)

e) The maximum current occurs when the load is short circuited:

$I = \dfrac{V}{R} = \dfrac{600}{0.39} = 1{,}538 \; A$

In this case the voltage on the load (V_L) is zero. No power is delivered to the load.