Solutions of the resit exam of MODULE EEEN20090

1st term, year 2018/19

SECTION A

The equations of the two meshes shown in Fig 1 are:

$$(10+j0)\,\bar{I}_1 + (10+j10)\,\bar{I}_1 - (10+j10)\,\bar{I}_2 = 212\angle 90°$$

$$-(10+j10)\,\bar{I}_1 + (10+j10)\,\bar{I}_2 + (0-j20)\,\bar{I}_2 = 212\angle -150°$$

Substituting and solving for each current, gives:

$$\bar{I}_1 = 3.66\angle 15°\ A$$

$$\bar{I}_2 = 11.96\angle -113.8°\ A$$

From the circuit, one has:

$$\bar{I}_{a'a} = \bar{I}_1$$

$$\bar{I}_{c'c} = -\bar{I}_2 = 11.96\angle 66°\ A$$

$$\bar{I}_{b'b} = -\bar{I}_{a'a} + \bar{I}_{c'c} = -\bar{I}_1 + \bar{I}_2 = 14.54\angle -125.1°\ A$$

Section B

(1) The stored energy is:

$$W = W' = \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 + M i_1 i_2$$

Substituting the expressions of L_1, L_2 and M, and the values of i_1 and i_2, one has:

$$W = \frac{1}{2}(5 + 2\cos 2\theta)\, 1^2 + \frac{1}{2}(3 + \cos 2\theta)\, 0.5^2 + 10 \cos\theta \cdot 1 \cdot 0.5$$

$$= 2.5 + \cos 2\theta + 0.375 + 0.125 \cos 2\theta + 5\cos\theta$$

$$= 2.875 + 1.125 \cos 2\theta + 5\cos\theta$$

(2) The mechanical torque is:

$$T = \left.\frac{\partial W'}{\partial \theta}\right|_{i = \text{const.}} = -2.25 \sin 2\theta - 5 \sin\theta$$

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SECTION C

3. Synchronous speed :
$$n_1 = \frac{60 \cdot 50}{2} = 1500 \text{ rpm}$$

Nominal speed :
$$n = n_1 (1 - \sigma_N) = 1500(1 - 0.04) =$$
$$= 1440 \text{ rpm}$$

4. Start-up current :
$$I_{sup} = \left| \frac{380/\sqrt{3} \angle 0}{(0.1 + j0.4) + (0.1 + j0.3)} \right| = 301.4 \text{ A}$$

Nominal current :
$$I'_{2N} = I_{1N} = \frac{380/\sqrt{3} \angle 0}{\left| (0.1 + j0.4) + \left(\frac{0.1}{0.04} + j0.3\right) \right|} = 81.48 \text{ A}$$

5. Nominal power:
$$P_N = 3 R'_2 \left(\frac{1}{\sigma_N} - 1 \right) I_{1N}^2 =$$
$$= 3 \cdot 2.4 \cdot 81.48^2 = 47.8 \text{ kW}$$

Nominal torque:
$$T_N = \frac{P_N}{\omega_n} = \frac{47800}{2\pi \frac{1440}{60}} = 317 \text{ N.m}$$

6. Total power in nominal conditions:
$$P_1 = P_N + 1200 \text{ W} + \underbrace{3 \cdot (0.1 + 0.1) \cdot 81.48^2}_{3 \cdot (R'_2 + R_1)(I'_2)^2} = 52983 \text{ W}$$

efficiency :
$$\eta = \frac{P_N}{P_1} = \frac{47800}{52983} \cong 90 \%$$

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Section D

7. 10. Full load current

$$I = \frac{P}{\sqrt{3}\, V \cos\varphi} = \frac{2 \cdot 10^6}{\sqrt{3} \cdot 15 \cdot 10^3 \cdot 1} = 94.28\ A$$

$$\bar{I} = I \angle 0 = 94.28 \angle 0\ A$$

→ f.e.m $\bar{E}_o = \frac{15000}{\sqrt{3}} + (0.5 + j10)\ 94.28 \angle 0 = (8707 + j\ 942.8)\,V$

$$\varepsilon\% = \frac{8758 - 8660}{8660} = 1.13\ \%$$

$\left(\text{where } E = 8660 = \frac{15000}{\sqrt{3}}\right)$

8. Full load current: $I = \frac{P}{\sqrt{3}\, V \cos\varphi} = \frac{2 \cdot 10^6}{\sqrt{3} \cdot 15 \cdot 10^3\ 0.9} = 104.8\ A$

$$\bar{I} = 104.8\ (0.9 - j0.436)\ A$$

$$\bar{E}_o = \frac{15000}{\sqrt{3}} \angle 0^\circ + (0.5 + j10)\ 104.8 (0.9 - j0.436) = (9164 + j\ 920)\,V$$

$$\varepsilon\% = \frac{9210 - 8660}{8660} = 6.35\ \%$$

9. Full load current $\bar{I} = 104.8\ (0.9 + j\ 0.436)\ A$

$$\bar{E}_o = \frac{15000}{\sqrt{3}} + (0.5 + j\ 10)\ 104.8 (0.9 + j\ 0.436) = (8250 + j\ 966)\ V$$

$$\varepsilon\% = \frac{8307 - 8660}{8660} = -4.08\ \%$$

SECTION D

10. the complex power of the three loads is:

$$\bar{S}_1 = 700 \angle 36.9^\circ = (560 + j\,420)\ \text{KVA}$$

$$\bar{S}_2 = 1000 \angle 60^\circ = (500 + j\,866)\ \text{KVA}$$

$$\bar{S}_3 = 800 \angle 25.8^\circ = (720 + j\,349)\ \text{KVA}$$

The Total power is:

$$\bar{S}_T = \bar{S}_1 + \bar{S}_2 + \bar{S}_3 = (1780 + j\,1635)$$

$$= 2417 \angle 42.57^\circ\ \text{KVA}$$

Since $\bar{S}_T = \sqrt{3}\,\bar{V}_L \bar{I}_L^*$ and $S_T = \sqrt{3}\,V_L I_L$

the line current is: $I_L = \dfrac{2417 \cdot 10^3}{\sqrt{3} \cdot 13.8 \cdot 10^3} = 101.1$ A (rms)

11. The combined power factor for the three loads is found for the expression

$$\cos\theta = pf = \frac{1780}{2417} = 0.7365 \text{ lagging}$$

12. the new complex power with power factor 0.92 lagging is

$$\bar{S}_{new} = 1780 + j\,1780 \tan(23.07^\circ), \quad \text{where } \cos(23.07^\circ) = 0.92$$

$$= (1780 + j\,758.28)\ \text{KVA}$$

the reactive power of the capacitor is: $\bar{S}_{cap} = j\,Q_C = \bar{S}_{new} - \bar{S}_T$

Assuming $V_{rms} = 15000/\sqrt{3}$ we obtain: $\bar{S}_{cap} = -j\,876.72$ KVA

$$j\omega C V_{rms}^2 = \frac{-j\,876.72}{3} \Rightarrow C = 12.2\ \mu F \quad \#$$