Worked Problems on AC Circuits

EEEN20090 – Electrical Energy Systems

Problem 1

Given that the current in a given circuit is $3.90 - j6.04$ mA and the impedance is $5.16 + j1.14$ kΩ, find the magnitude of the voltage.

Problem 2

A resistor, an inductor and a capacitor are connected in series across an ac voltage source. A voltmeter measures 12.0 V, 15.5 V and 10.5 V respectively, when placed across each element separately. What is the magnitude of the voltage of the source?

Problem 3

Calculate the total impedance of the circuit in Figure 1.

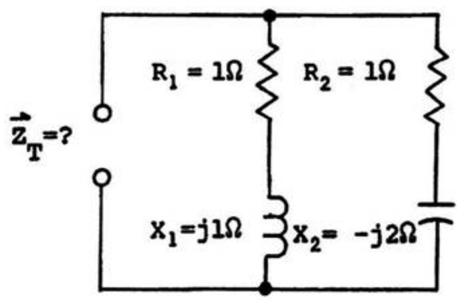

Figure 1

Problem 4

Find the Thevenin and Norton equivalents of the circuit shown in Figure 2.

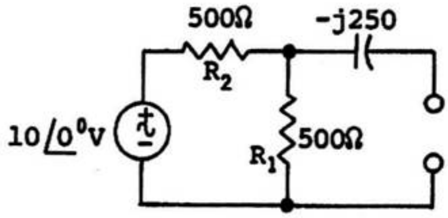

Figure 2

Problem 5

Use phasor techniques to find $\bar{V}_L(t)$ in the circuit of Figure 3.

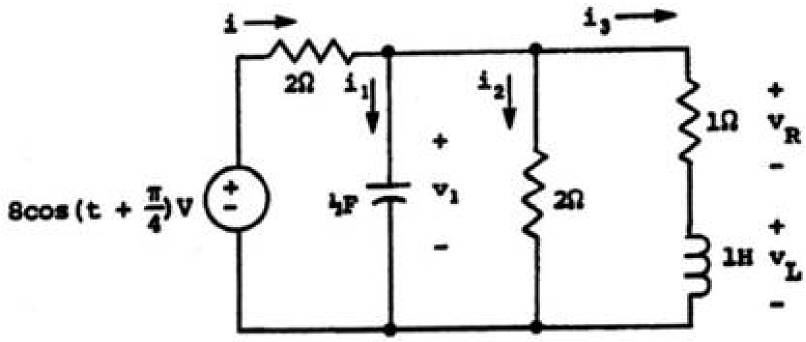

Figure 3

Solution of Problem 1

Note the units in the question: mA and kΩ. Applying the formula for voltage:

$$\begin{aligned} |\bar{V}| &= |\bar{I}| \times |\bar{Z}| \\ &= \left|(3.90 - j6.04) \times 10^{-3}\right| \times \left|(5.16 + j1.14) \times 10^{3}\right| \end{aligned}$$

We can move the powers of 10 outside the absolute value (magnitude) signs.

$$= |(3.90 - j6.04)| \times 10^{-3} \times |(5.16 + j1.14)| \times 10^{3}$$

The powers of 10 cancel out.

$$= |3.90 - j6.04| \times |5.16 + j1.14|$$

We now proceed to find the magnitude of each complex number:

$$\begin{aligned} &= \sqrt{(3.90)^2 + (-6.04)^2} \times \sqrt{(5.16)^2 + (1.14)^2} \\ &= 7.190 \times 5.284 \\ &= 38.0 \text{ V} \end{aligned}$$

So the voltage is 38 V.

Solution of Problem 2

The total reactance across a circuit with a resistance, an inductance and a capacitance in series is given by:

$$\bar{V}_{RLC} = \bar{V}_R + j(\bar{V}_L - \bar{V}_C)$$

Now

$$\begin{aligned} |V_R| &= 12 \text{ V} \\ |V_L| &= 15.5 \text{ V} \\ |V_C| &= 10.5 \text{ V} \end{aligned}$$

Now

$$\begin{aligned} \bar{V}_{RLC} &= \bar{V}_R + j(\bar{V}_L - \bar{V}_C) \\ &= 12 + j(15.5 - 10.5) \\ &= 12 + j5 \text{ V} \end{aligned}$$

The magnitude of this voltage is:

$$|12 + j5| = 13 \text{ V}$$

Solution of Problem 3

Circuit elements form two branch impedances

$$\bar{Z}_1 = R_1 + jX_1$$
$$\bar{Z}_2 = R_2 + jX_2 \ .$$

Hence:

$$\bar{Z}_T = \frac{\bar{Z}_1\bar{Z}_2}{\bar{Z}_1+\bar{Z}_2} = \frac{(R1+jX1)(R2+jX2)}{(R1+R2)+j(X1+X2)}$$
$$\bar{Z}_T = [(1+j)(1-j^2)/(2-j)] = [(3-j)/(2-j)]$$
$$\bar{Z}_T = [(3-j)(2+j)/(22+1)] = [(7+j)/5] = (7/5)+j(14/5) \ .$$

In polar coordinates, we have,

$$\bar{Z}_T = \sqrt{(7/5)^2+(1/5)^2}\cdot\angle\tan^{-1}\frac{1}{7}$$
$$\bar{Z}_T = \sqrt{\frac{49}{25}+\frac{1}{25}}\cdot\angle 8.1^\circ$$
$$\bar{Z}_T = \sqrt{\frac{50}{25}}\cdot\angle 8.1^\circ$$
$$\bar{Z}_T = \sqrt{2}\cdot\angle 8.1^\circ \ .$$

Solution of Problem 4

Thevenin's equivalent voltage is the open circuit voltage shown in Figure 2. If the terminals are open, there will be no current flowing in the capacitive reactance of $-j250\ \Omega$ and therefore no voltage drop across it. The open circuit voltage is equal to the voltage across the 500 Ω resistor R_1. Using the voltage division principle, the source voltage $10\angle 0^\circ$ V will be divided into the voltages across two 500 Ω resistors. Therefore the Thevenin equivalent voltage $\bar{V}_{\text{th}}$ is given by:

$$\bar{V}_{\text{th}} = 10\angle 0^\circ \cdot \frac{500}{500+500} = 5\angle 0^\circ \text{ V}$$

The Norton equivalent internal impedance can be obtained by turning off the $10\angle 0^\circ$ V source and short circuiting the source terminals. Then the internal impedance of the network presented across the output terminals is the series impedance of the capacitive reactance of $-j250\ \Omega$, and the parallel resistance of two 500 Ω resistors. Therefore, the Norton equivalent internal impedance

$$\bar{Z}_{\text{no}} = \frac{500}{2} - j250 = 250 - j250 = 250(1-j1) = 250\angle-45^\circ\ \Omega.$$

Solution of Problem 5

The impedance of the R-L branch is

$$1 + j1\ \Omega \qquad (\omega = 1)$$

and its admittance is, therefore,

$$[1/(\sqrt{2}\angle 45^\circ)] = .707\angle - 45^\circ = (1/2) - j(1/2)\ \Omega^{-1}\ .$$

The capacitor and resistor parallel branches have admittances

$$\bar{Y}_C = \omega C = 0 + j(1/2)\ \Omega^{-1}$$
$$\bar{Y}_R = (1/2) + j0\ \Omega^{-1}\ .$$

The equivalent admittance of the parallel branches is

$$\bar{Y}_p = [(1/2) + j0] + [0 + j(1/2)] + [(1/2) - j(1/2)] = 1 + j0\ \Omega^{-1}\ ,$$

and, consequently, the total impedance driven by the source is

$$\bar{Z}_t = 2 + j0 + (1/\bar{Y}_p) = 2 + j0 + 1 + j0 = 3 + j0\ \Omega\ .$$

The current out of the source is

$$\bar{I}_t = [(8\angle 45^\circ)/(3\angle 0^\circ)] = (8/3)\angle 45^\circ\ \text{A}\ .$$

This current produces a voltage across the parallel branches of

$$\bar{V}_p = (8/3)\angle 45^\circ(1\angle 0^\circ) = (8/3)\angle 45^\circ\ \text{V}\ .$$

By voltage division, then,

$$\begin{aligned}\bar{V}_L &= [j/(1 + j1)](8/3)\angle 45^\circ = [1\angle 90^\circ(8/3)\angle 45^\circ/(\sqrt{2}\angle 45^\circ)] \\ &= [8/(3\sqrt{2})]\angle 90^\circ\ \text{V}\ .\end{aligned}$$

In the time domain $v_L(t) = 1.89 cos(t + 90^\circ)$ V.