

Exercises on Complex Numbers

EEEN20090 – Electric Energy Systems

Exercises

Section 1

Write the following expressions in the form $a + ib$.

a) i^5	j) $i^6 + i^{16} + i^{26} + i^{36} + i^{46} + i^{56}$	s) $\frac{1}{i} + \frac{1}{1+i} + \frac{1}{1-i}$
b) i^{33}	k) $i \times i^2 \times i^3 \times \dots \times i^{99} \times i^{100}$	t) $\frac{5-3i}{i} + (1+2i)^2$
c) i^{148}	l) $(1+i)(-1+i) + (1+2i)^2$	u) $\frac{1+2i}{3-4i} - \frac{2-i}{5i^3}$
d) i^{999}	m) $\frac{2+4i}{1+i}(2-i)$	v) $\frac{i^3 + i^2 + i}{2+i} + (1+i)^2$
e) $i^3 + i^6 + i^9$	n) $\frac{2-4i}{1+i} + (1+2i)^2 i^3$	w) $\left(\frac{2-i}{1-3i}\right)^{-1} + \frac{i}{1-2i} \times i$
f) $i^{10} + i^{20} + i^{30} + i^{40}$	o) $\frac{i+1}{i} : \frac{i-1}{2i+3}$	x) $(-1+5i) : \left(2 - \frac{3+i}{2+i}\right)$
g) $i^{77} - i^{27} + i^{17} - i^{67}$	p) $1 - \frac{i}{i+1} + (i-1)i$	y) $5 \left(\frac{3+4i}{2+i}\right)^{-1} \left(\frac{2}{i-2}\right)^2 + \frac{1}{i}$
h) $i^2 - 2i^7 + 8i^3 - 6i + i^5$	q) $\frac{1+i}{1-i} + \frac{1-i}{1+i}$	z) $\frac{ 1+i }{1-i}$
i) $i^5 + i^{10} + i^{15} + i^{20} + i^{25} + i^{30}$	r) $\frac{2+i}{2-i} - \frac{2-i}{2+i}$	z) $\frac{ -1-i }{1+i\sqrt{3}}$

Section 2

Compute the absolute value of the following expressions.

- | | | |
|-----------------------------------------------------|----------------------------------------------|--------------------------------------------------|
| a) i | i) $-2\sqrt{3} + 2i$ | r) $(1+i)^3$ |
| b) $2-3i$ | j) $5-i5\sqrt{3}$ | s) $(3+i\sqrt{2})(3-i\sqrt{2})$ |
| c) $1+2i$ | k) $1+i$ | t) $(1+i\sqrt{2})(2-3i)(3+i)$ |
| d) $-3-4i$ | l) $\frac{2+i}{3}$ | u) $0.3563-0.9343i$ |
| e) $-\sqrt{2}+i\sqrt{7}$ | m) $6+8i$ | v) $\sin\frac{\pi}{4}-i\cos\frac{\pi}{4}$ |
| f) $\frac{3}{5}-\frac{4}{5}i$ | n) $-\frac{\sqrt{3}}{2}+\frac{1}{2}i$ | w) $\frac{(1+i)^3}{(1-i)^5}$ |
| g) $-\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i$ | o) $1+i\sqrt{3}$ | x) $\frac{i^{10}-i}{2i+1}$ |
| h) $\frac{1}{2}+\frac{\sqrt{3}}{2}i$ | p) $(1-2i)(3+i)$ | y) $1+2i-\frac{2-5i}{3-i}$ |

Section 3

Sum, subtract and multiply the following complex numbers.

- | | | |
|------------------------------|-----------------------------|----------------------------------|
| a) $x=2+5i, y=3-2i$ | g) $x=-7+3i, y=-2+i$ | m) $x=8-i, y=-6-9i$ |
| b) $x=4-i, y=1+7i$ | h) $x=4+2i, y=-2+5i$ | n) $x=i, y=-10+5i$ |
| c) $x=1+2i, y=1+4i$ | i) $x=1+i, y=2+i$ | o) $x=-2+7i, y=-3$ |
| d) $x=-2+i, y=-3-2i$ | j) $x=2+i, y=2-i$ | p) $x=12+15i, y=16-22i$ |
| e) $x=-5+2i, y=-3+3i$ | k) $x=1-2i, y=3+7i$ | r) $x=42-6i, y=-51+99i$ |
| f) $x=-2-2i, y=-1+3i$ | l) $x=6+3i, y=-5+2i$ | s) $x=1+2.3i, y=4.5-0.2i$ |

Section 4

Write the following expressions in the form $a+ib$.

a) $\frac{3+5i}{2-i}$	g) $\frac{5+3i}{-2-i}$	m) $\frac{4+5i}{3+2i}$
b) $\frac{1-i}{3+2i}$	h) $\frac{-4-i}{-2-4i}$	n) $\frac{3+i}{1-2i}$
c) $\frac{2+3i}{1-2i}$	i) $\frac{7-2i}{-3+2i}$	o) $(2-12i) : 4i$
d) $\frac{3+i}{3-i}$	j) $\frac{1+i}{1+2i}$	p) $\frac{1}{\sqrt{3}+i\sqrt{2}}$
e) $\frac{1-2i}{2+i}$	k) $\frac{5+i\sqrt{5}}{5-i\sqrt{5}}$	r) $\frac{2+3i}{-3+2i}$
f) $\frac{-3-i}{1+i}$	l) $\frac{1+2i}{3+4i}$	s) $\frac{1-2i}{3+7i}$

Section 5

Write the following expressions in the form $\rho(\cos(\theta) + i \sin(\theta))$.

a) -5	g) $-1-i\sqrt{3}$	m) $-2+2i$
b) $-7i$	h) $5-i5\sqrt{3}$	n) $\frac{1}{4}+\frac{1}{4}i$
c) $7i$	i) $-\sqrt{2}+i\sqrt{6}$	o) $3-i\sqrt{3}$
d) $-1+i$	j) $-2-i\sqrt{12}$	p) $\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i$
e) $1-i$	k) $\frac{3}{2}+\frac{\sqrt{3}}{2}i$	r) $\frac{1+i}{1-i}$
f) $\sqrt{3}+i$	l) $-\sqrt{3}-i$	s) $\frac{1}{1+i}+\frac{1}{-1+i}$

Section 6

Write the following expressions in the form $a + ib$.

a)	$\sqrt{2}(\cos 315^\circ + i \sin 315^\circ)$	g)	$10(\cos 50^\circ + i \sin 50^\circ)$	m)	$\frac{\sqrt{2}}{2} \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right)$
b)	$5(\cos 210^\circ + i \sin 210^\circ)$	h)	$\sqrt{2} \left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right)$	n)	$7(\cos 180^\circ + i \sin 180^\circ)$
c)	$2 \left(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2} \right)$	i)	$4 \left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} \right)$	o)	$\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}$
d)	$4 \left(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6} \right)$	j)	$7\sqrt{2} \left(\cos \frac{5\pi}{4} + i \sin \frac{5\pi}{4} \right)$	p)	$2e^{\frac{i\pi}{4}}$
e)	$2 \left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right)$	k)	$10\sqrt{2} \left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4} \right)$	r)	$7e^{\frac{i\pi}{3}}$
f)	$\cos 135^\circ + i \sin 135^\circ$	l)	$2 \left(\cos \frac{5\pi}{2} + i \sin \frac{5\pi}{2} \right)$	s)	$4e^{\frac{i\pi}{2}}$

Section 7

Write the following expressions in the form $a + ib$.

a)	$(1+i)^4$	g)	$(-1+i)^6$	m)	$(1+i\sqrt{3})^{14}$
b)	$(1-i)^8$	h)	$(-2\sqrt{3}-2i)^{12}$	n)	$\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i \right)^9$
c)	$(2+i)^3$	i)	$(-\sqrt{3}-i)^8$	o)	$\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i \right)^4$
d)	$(\sqrt{2}+i)^4$	j)	$(-3+3i)^4$	p)	$\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i \right)^8$
e)	$(1+i)^{10}$	k)	$(1-i\sqrt{3})^{12}$	r)	$\left(\frac{1-i}{1+i} \right)^{20}$
f)	$(1-i\sqrt{3})^5$	l)	$(-2+i\sqrt{12})^6$	s)	$\left(\frac{\sqrt{3}}{2} - \frac{1}{2}i \right)^{100}$

Section 8

Solve the following equations for x .

a)	$x^2 + 5 = 0$	i)	$2x^2 - 11x + 14 = 0$	r)	$x^3 + 1 = 0$
b)	$2x^2 + 7 = 0$	j)	$x^2 - 2ix - 5 = 0$	s)	$x^4 - 1 = 0$
c)	$3x^2 + 4 = 0$	k)	$x^2 - (2+i)x - 1 + 7i = 0$	t)	$27x^3 + 125 = 0$
d)	$5x^2 - 4x + 1 = 0$	l)	$x^2 - (3+2i)x + 5 + i = 0$	u)	$x^4 + 1 = 0$
e)	$x^2 - 4x + 6 = 0$	m)	$\frac{1}{x-2} + \frac{1}{x-11} = \frac{1}{x-8}$	v)	$\sqrt{x^2 + 8ix + 9} = -x + i$
f)	$5x^2 - 6x + 2 = 0$	n)	$\frac{52}{x-10} + 10 + x = \frac{52}{10-x}$	w)	$\sqrt{x^2 + 8ix - 16} = 2x - 5i$
g)	$x^2 - 2x + 5 = 0$	o)	$x - \frac{1}{x} = \frac{2}{i} - \frac{i}{2}$	x)	$x^2 + 9 + \sqrt{x^2 + 9} = 0$
h)	$x^2 - 4x + 5 = 0$	p)	$x + i = \frac{1}{x} + \frac{1}{i}$	y)	$32x^5 - 1 = 0$

Solutions

Section 1 – Solutions

a) i	j) 0	s) $1-i$
b) i	k) -1	t) $-6-i$
c) 1	l) $-5+4i$	u) $-\frac{2}{5}$
d) $-i$	m) $7-i$	v) $-\frac{2}{5}+\frac{11}{5}i$
e) -1	n) 3	w) $-\frac{3}{5}-\frac{1}{5}i$
f) 0	o) $-3-2i$	x) $1+8i$
g) $4i$	p) $-\frac{1}{2}-\frac{3}{2}i$	y) $\frac{8}{5}-\frac{1}{5}i$
h) $-1-11i$	q) 0	z) $\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i$
i) $-1+i$	r) $\frac{8}{5}i$	Z) $\frac{\sqrt{2}}{4}-\frac{\sqrt{6}}{4}i$

Section 2 – Solutions

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|-----------------------------------|--------------------------------------------|-----------------------------------------------|
| a) 1 | i) 4 | r) $2\sqrt{2} \doteq 2.83$ |
| b) $\sqrt{13} \doteq 3.61$ | j) 10 | s) 11 |
| c) $\sqrt{5} \doteq 2.24$ | k) $\sqrt{5} \doteq 1.41$ | t) $\sqrt{390} \doteq 19.75$ |
| d) 5 | l) $\frac{\sqrt{5}}{3} \doteq 0.75$ | u) 1 |
| e) 3 | m) 10 | v) 1 |
| f) 1 | n) 1 | w) $\frac{1}{2}$ |
| g) 1 | o) 2 | x) $\frac{\sqrt{10}}{5} \doteq 0.63$ |
| h) 1 | p) $\sqrt{50} \doteq 7.07$ | y) $\sqrt{\frac{109}{10}} \doteq 3.30$ |

Section 3 – Solutions

- | | | | | | |
|-----------|-----------------------------------------------------------------|-----------|------------------------------------------------------------------|-----------|------------------------------------------------------------------------------|
| a) | $x + y = 5 + 3i$
$x - y = -1 + 7i$
$x \cdot y = 16 + 11i$ | g) | $x + y = -9 + 4i$
$x - y = -5 + 2i$
$x \cdot y = 11 - 13i$ | m) | $x + y = 2 - 10i$
$x - y = 14 + 8i$
$x \cdot y = -57 - 66i$ |
| b) | $x + y = 5 + 6i$
$x - y = 3 - 8i$
$x \cdot y = 11 + 27i$ | h) | $x + y = 2 + 7i$
$x - y = 6 - 3i$
$x \cdot y = -18 + 16i$ | n) | $x + y = -10 + 6i$
$x - y = 10 - 4i$
$x \cdot y = -5 - 10i$ |
| c) | $x + y = 2 + 6i$
$x - y = -2i$
$x \cdot y = -7 + 6i$ | i) | $x + y = 3 + 2i$
$x - y = -1$
$x \cdot y = 1 + 3i$ | o) | $x + y = -5 + 7i$
$x - y = 1 + 7i$
$x \cdot y = 6 - 21i$ |
| d) | $x + y = -5 - i$
$x - y = 1 + 3i$
$x \cdot y = 8 + i$ | j) | $x + y = 4$
$x - y = 2i$
$x \cdot y = 5$ | p) | $x + y = 28 - 7i$
$x - y = -4 + 37i$
$x \cdot y = 522 - 24i$ |
| e) | $x + y = -8 + 5i$
$x - y = -2 - i$
$x \cdot y = 9 - 21i$ | k) | $x + y = 4 + 5i$
$x - y = -2 - 9i$
$x \cdot y = 17 + i$ | r) | $x + y = -9 + 93i$
$x - y = 93 - 105i$
$x \cdot y = -1,548 + 4,464i$ |
| f) | $x + y = -3 + i$
$x - y = -1 - 5i$
$x \cdot y = 8 - 4i$ | l) | $x + y = 1 + 5i$
$x - y = 11 + i$
$x \cdot y = -36 - 3i$ | s) | $x + y = 5.5 + 2.1i$
$x - y = -3.5 + 2.5i$
$x \cdot y = 4.96 + 10.15i$ |

Section 4 – Solutions

a)	$\frac{1}{5} + \frac{13}{5}i$	g)	$-\frac{13}{5} - \frac{1}{5}i$	m)	$\frac{22}{13} + \frac{7}{13}i$
b)	$\frac{1}{13} - \frac{5}{13}i$	h)	$\frac{3}{5} - \frac{7}{10}i$	n)	$\frac{1}{5} + \frac{7}{5}i$
c)	$-\frac{4}{5} + \frac{7}{5}i$	i)	$-\frac{25}{13} - \frac{8}{13}i$	o)	$-3 - \frac{1}{2}i$
d)	$\frac{4}{5} + \frac{3}{5}i$	j)	$\frac{3}{5} - \frac{1}{5}i$	p)	$\frac{\sqrt{3}}{5} - \frac{\sqrt{2}}{5}i$
e)	$-i$	k)	$\frac{2}{3} + \frac{\sqrt{5}}{3}i$	r)	$-i$
f)	$-2 + i$	l)	$\frac{11}{25} + \frac{2}{25}i$	s)	$-\frac{11}{58} - \frac{13}{58}i$

Section 5 – Solutions

a)	$5(\cos \pi + i \sin \pi)$	g)	$2\left(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3}\right)$	m)	$\sqrt{8}\left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}\right)$
b)	$7\left(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2}\right)$	h)	$10\left(\cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3}\right)$	n)	$\frac{\sqrt{2}}{4}\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right)$
c)	$7\left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right)$	i)	$\sqrt{8}\left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}\right)$	o)	$2\sqrt{3}\left(\cos \frac{11\pi}{6} + i \sin \frac{11\pi}{6}\right)$
d)	$\sqrt{2}\left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}\right)$	j)	$4\left(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3}\right)$	p)	$\cos \frac{7\pi}{8} + i \sin \frac{7\pi}{8}$
e)	$\sqrt{2}\left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4}\right)$	k)	$\sqrt{3}\left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6}\right)$	r)	$\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}$
f)	$2\left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6}\right)$	l)	$2\left(\cos \frac{7\pi}{6} + i \sin \frac{7\pi}{6}\right)$	s)	$\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2}$

Section 6 – Solutions

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|----|-------------------------------------------|----|-----------------|----|------------------------------------|
| a) | $1-i$ | g) | $7.660+6.428i$ | m) | $\frac{1}{2}+\frac{1}{2}i$ |
| b) | $-\frac{5\sqrt{3}}{2}-\frac{5}{2}i$ | h) | $-1+i$ | n) | -7 |
| c) | $-2i$ | i) | $-2+2i\sqrt{3}$ | o) | $\frac{1}{2}+\frac{\sqrt{3}}{2}i$ |
| d) | $-2\sqrt{3}+2i$ | j) | $-7-7i$ | p) | $\sqrt{2}+i\sqrt{2}$ |
| e) | $\sqrt{3}+i$ | k) | $10-10i$ | r) | $\frac{7}{2}+\frac{7\sqrt{3}}{2}i$ |
| f) | $-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i$ | l) | $2i$ | s) | $4i$ |

Section 7 – Solutions

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|----|------------------|----|---------------------|----|------------------------------------|
| a) | -4 | g) | $8i$ | m) | $2^{13}(-1+i\sqrt{3})$ |
| b) | 16 | h) | 2^{24} | n) | -1 |
| c) | $2+11i$ | i) | $-128(1+i\sqrt{3})$ | o) | -1 |
| d) | $-7+i4\sqrt{2}$ | j) | -324 | p) | i |
| e) | $32i$ | k) | 2^{12} | r) | 1 |
| f) | $16+16i\sqrt{3}$ | l) | 4^6 | s) | $-\frac{1}{2}-\frac{\sqrt{3}}{2}i$ |

Section 8 – Solutions

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|----------------------------------------------------|--------------------------------------------|-------------------------------------------------------------------------------------------|
| a) $x_{1,2} = \pm i\sqrt{5}$ | i) $x_1 = 2; x_2 = \frac{7}{2}$ | r) $x_1 = -1; x_{2,3} = \frac{1}{2} \pm \frac{\sqrt{3}}{2}i$ |
| b) $x_{1,2} = \pm i\sqrt{\frac{7}{2}}$ | j) $x_{1,2} = \pm 2 + i$ | s) $x_{1,2} = \pm 1; x_{3,4} = \pm i$ |
| c) $x_{1,2} = \pm \frac{2\sqrt{3}}{3}i$ | k) $x_1 = 3 - i; x_2 = -1 + 2i$ | t) $x_1 = -\frac{5}{3}; x_{2,3} = \frac{5}{6} \pm \frac{5\sqrt{3}}{6}i$ |
| d) $x_{1,2} = \frac{2}{5} \pm \frac{1}{5}i$ | l) $x_1 = 1 - i; x_2 = 2 + 3i$ | u) $x_{1,2,3,4} = \pm \frac{\sqrt{2}}{2} \pm \frac{\sqrt{2}}{2}i$ |
| e) $x_{1,2} = 2 \pm i\sqrt{2}$ | m) $x_{1,2} = 8 \pm i3\sqrt{2}$ | v) $x = i$ |
| f) $x_{1,2} = \frac{3}{5} \pm \frac{1}{5}i$ | n) $x_{1,2} = \pm 2i$ | w) $x_1 = 9i; x_2 = \frac{1}{3}i$ |
| g) $x_{1,2} = 1 \pm 2i$ | o) $x_1 = -\frac{1}{2}i; x_2 = -2i$ | x) $x_{1,2} = \pm 3i$ |
| h) $x_{1,2} = 2 \pm i$ | p) $x = -i$ | y) $x_{1,2} = 0.1545 \pm 0.4755i$
$x_{3,4} = -0.4045 \pm 0.294i$
$x_5 = 0.5$ |