Proposed Problems on Complex Numbers

EEEN20090 – Electric Energy Systems

1. Compute real and imaginary part of
$$\bar{z} = \frac{j-4}{2j-3} \ .$$

2. Compute the absolute value and the conjugate of
$$\bar{z} = (1+j)^6; \qquad \bar{w} = j^{17} \ .$$

3. Write in the rectangular form $(a+jb)$ the following complex numbers:
$$\bar{z} = j^5 + j + 1; \qquad \bar{w} = (3+3j)^8 \ .$$

4. Write in the trigonometric form $(\rho(\cos\theta + j\sin\theta))$ the following complex numbers:
$$8; \qquad 6j; \qquad \left(\cos\frac{\pi}{3} - j\sin\frac{\pi}{3}\right)^7$$

5. Simplify the following expressions:
$$\frac{1+j}{1-j} - (1+2j)(2+2j) + \frac{3-j}{1+j} \ ;$$
$$2j(j-1) + \left(\left(\sqrt{3}+j\right)^*\right)^3 + (1+j)(1+j)^* \ .$$

6. Compute the square roots of $\bar{z} = -1 - j$.

7. Compute the cube roots of $\bar{z} = -8$.

8. Prove that there is no complex number such that $|\bar{z}| - \bar{z} = j$.

9. Find $\bar{z} \in \mathbb{C}$ such that:
$$\bar{z} = j(\bar{z}-1); \qquad \bar{z}^2 \cdot \bar{z}^* = \bar{z}; \qquad |\bar{z}+3j| = 3|\bar{z}| \ .$$

10. Find $\bar{z} \in \mathbb{C}$ such that $\bar{z}^2 \in \mathbb{C}$.

11. Fing $\bar{z} \in \mathbb{C}$ such that:
$$\begin{aligned} &\mathrm{Re}\{\bar{z}(1+j)\} + \bar{z}\cdot\bar{z}^* = 0 \ ; \\ &\mathrm{Re}\{\bar{z}^2\} = j\mathrm{Im}\{\bar{z}^*(1+2j)\} = -3 \ ; \\ &\mathrm{Im}\{(2-j)\bar{z}\} = 1 \ . \end{aligned}$$

12. Find $a \in \mathbb{R}$ such that $\bar{z} = -j$ is a root for the polymnomial $P(\bar{z}) = \bar{z}^3 - \bar{z}^2 + \bar{z} + 1 + a$. Furthermore, for such value of a, find the factors of $P(\bar{z})$ in $\mathbb{R}$ and in $\mathbb{C}$.