Energy Conversion

ELECTRICAL ENERGY SYSTEMS (EEEN20090)

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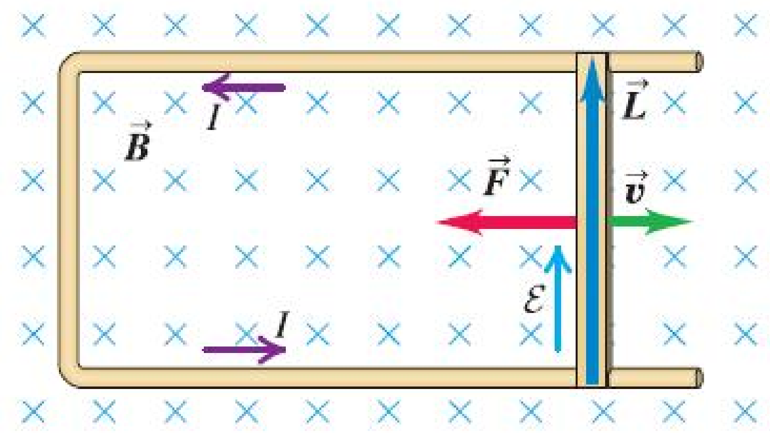
Energy Conversion in Magnetic Circuits

- Lorentz force
- Energy balance
- Energy and coenergy
- Examples:
  - Homopolar machine
  - Electromagnet
  - Reluctance motor

Linear Motor

- Consider a conductor (e.g. a copper rod) of length L resting on wires, carrying a current I, in the presence of a magnetic field whose flux density is in the direction shown, normal to the plane of the conductor.



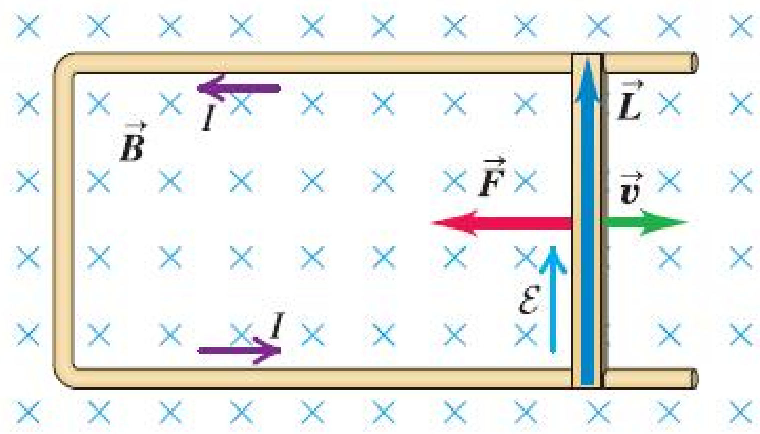
- Current leads to moving charges in the conductor, which experience an electromagnetic force, which is transmitted to the conductor itself.
- The conductor thus experiences a force (Lorentz force), given by the vector product

$$\vec{F} = i\vec{\ell} \times \vec{B}$$

- In this example F has magnitude BLI

Linear Generator

- The system considered in the previous slide can also be used as a generator. If the bar moves with constant speed $\vec{v}$, then the area spanned by the coil varies, which leads to a variation of magnetic flux:



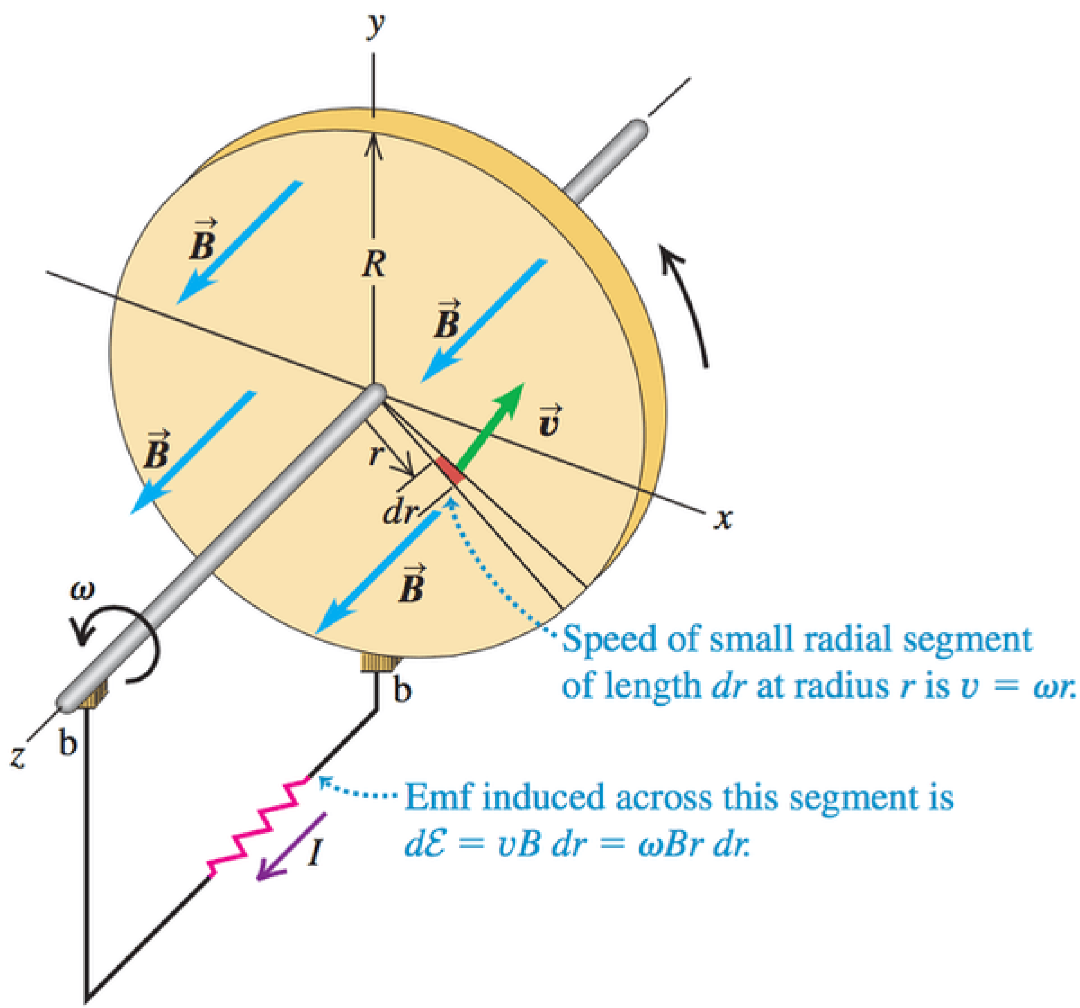
$$d\phi = BdA = BLdx$$

$$\Rightarrow \quad \frac{d\phi}{dt} = BL\frac{dx}{dt} = BLv$$

- This is a very simple example of electric machine which can work either as a motor or as a generator.
- The linear structure is impractical: how can such an uniform $\vec{B}$ be generated?
- That's why most electrical machines are rotating machines.

Homopolar Machine – I

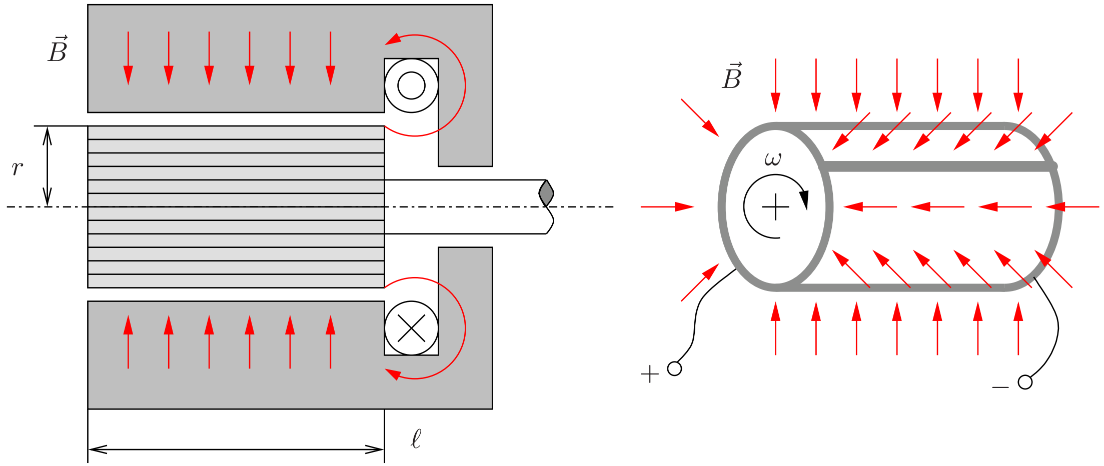
- It is possible though to implement the machine seen previously using an *cyclical* structure (known as *homopolar machine*, which is a modern version of the *Faraday's disk*).
- The Faraday's disk is illustrated in the figure.



Homopolar Machine – II

- The magnetic induction $\vec{B}$ is radial and time-invariant. While rotating, the coil spanned area changes its relative position with respect to the magnetic induction:

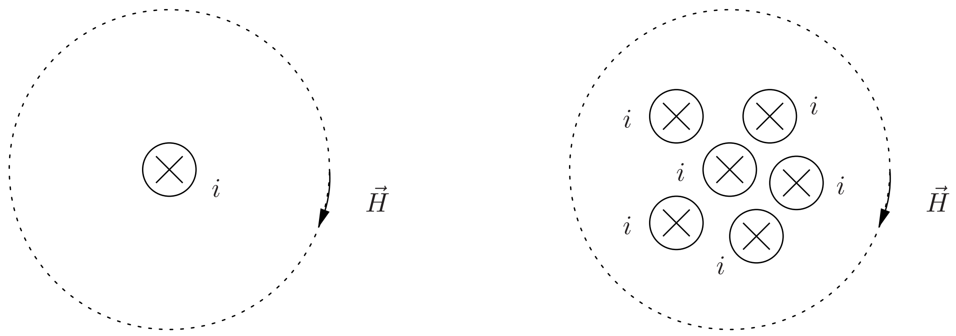
$$e = \frac{d\phi}{dt} = B\ell\omega r = B\ell v$$



- The homopolar machine is characterized by a continuous (low) voltage and extremely high currents (order of kA).

Motivation of Coils with High N

- The homopolar machine has no relevant practical applications.
- This is mainly because the coil on the rotor has just one turn $\Rightarrow$ inefficient!



$H = i/2\pi r$ $\qquad$ $H = Ni/2\pi r$

- The advange of using N turns is an easy way to *increase* the magnetic field $\vec{H}$ while keeping relatively small the current of the winding.

Force in Magnetic Field - I

- The power balance of a coil:

$$p = vi = ri^2 + i\frac{d\lambda}{dt}$$

- Neglecting losses for simplicity, but without loss of generality:

$$pdt = id\lambda$$

- The total energy absorbed when the flux linkage changes from 0 to λ is:

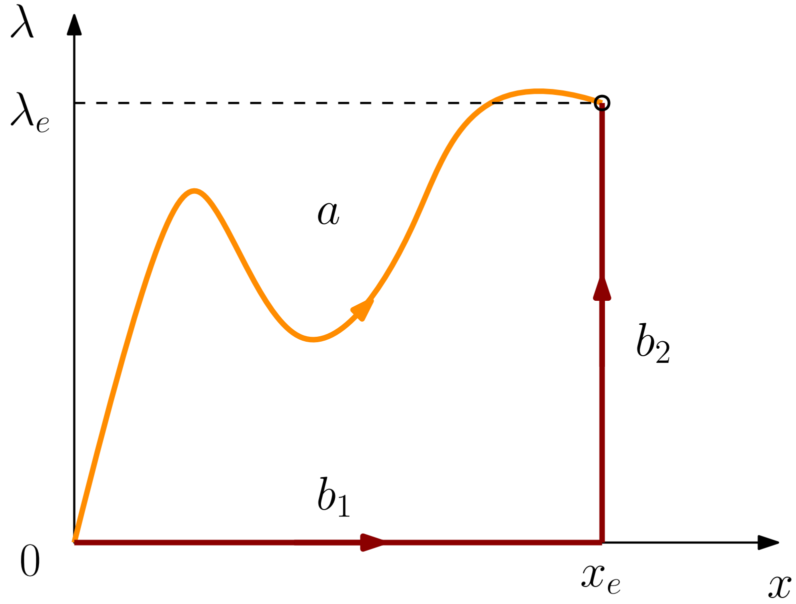
$$W_e = \int pdt = \int_0^{\lambda} id\lambda$$

Motivation

- Let's consider the variation of energy from $(0,0)$ to (x_e, λ_e).
- If there are no losses, any path can be used:

$$\Delta W_e = \Delta W_a = \Delta W_{b1} + \Delta W_{b2}$$

- However, path b_1-b_2 is much simpler.



- Along b_1, $\lambda = 0$ and $d\lambda = 0$, hence $i = 0$ and $f = 0$, hence: $\Delta W_{b1} = 0$
- Along b_2, $x = x_e = \text{const.}$, $dx = 0$, hence:

$$\Delta W_e = \Delta W_{b2} = \int_0^{\lambda_e} i(x_e, \lambda) d\lambda$$

Force in Magnetic Field – II

- In the case of movable parts, the current is a function of λ and the position x:

$$i = i(\lambda, x)$$

- Then also the magnetic field energy W_f depends on λ and x:

$$W_f = W_f(\lambda, x)$$

- The incremental change of magnetic field energy gives:

$$dW_f(\lambda, x) = \frac{\partial W_f}{\partial \lambda} d\lambda + \frac{\partial W_f}{\partial x} dx$$

Force in Magnetic Field – III

- On the other hand, neglecting losses, the variation of electric energy has to be equal to the variation of magnetic energy plus the variation of mechanical energy:

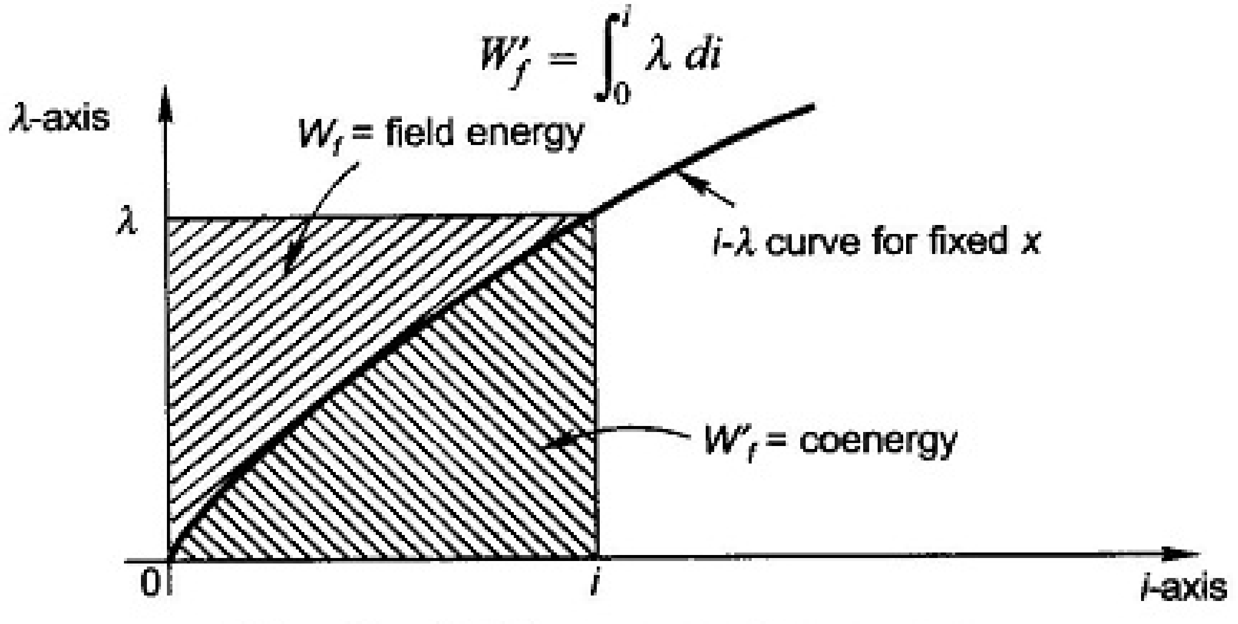
$$dW_e = dW_f + f\,dx = i\,d\lambda \qquad dW_f = i\,d\lambda - f\,dx$$

- Hence, we have:

$$i = \frac{\partial W_f}{\partial \lambda}\Big|_{dx=0} \qquad f = -\frac{\partial W_f}{\partial x}\Big|_{d\lambda=0}$$

Coenergy

- For fixed x, the field energy is the area of the upper side of the λ-i curve:
  $dW_f = \int_0^\lambda i d\lambda$
- Alternatively, for fixed x, one can define **coenergy** as the area of the lower side:
  $dW'_f = \int_0^i \lambda di$



Mechanical Force in terms of Coenergy

- Remember that:

$$dW_e = dW_f + dW_m = id\lambda$$

- Then, by definition of coenergy:

$$W_f + W_f' = i\lambda \qquad dW_f + dW_f' = id\lambda + \lambda di$$

- Subsitituing into the expression of the energy balance:

$$id\lambda + \lambda di - dW_f' + dW_m = id\lambda$$

$$\Rightarrow \quad dW_f' = \lambda di + dW_m = \lambda di + f dx$$

- Which leads to:

$$f = \frac{\partial W_f'}{\partial x}\Big|_{di=0}$$

Linear Case with Time-invariant Inductance

- Let's assume the linear case (constant inductance):

$$\lambda = Li$$

- Then, the field energy is:

$$W_f = \int_0^\lambda i d\lambda = \int_0^\lambda \frac{\lambda}{L} d\lambda = \frac{1}{2}\frac{\lambda^2}{L} = \frac{1}{2}Li^2$$

- The coenergy is:

$$W_f' = \int_0^i \lambda di = \int_0^i Lidi = \frac{1}{2}Li^2 = \frac{1}{2}\frac{\lambda^2}{L}$$

- Hence, $W_f = W_f'$, as it can be deduced straightforwardly from the curve on the λ-i plane.

Alternative Expressions

- The inductance can be expressed as:

$$L = N\frac{\phi}{i}$$

- Then, taking into account the Hopkinson law ($\mathcal{F} = \mathcal{R}\phi$, the inductance can be rewritten as:

$$L = N^2\frac{\phi}{Ni} = N^2\frac{\phi}{\mathcal{R}\phi} = \frac{N^2}{\mathcal{R}} = N^2\mathcal{P}$$

where $\mathcal{P}$ is the permeance, $\mathcal{P} = 1/\mathcal{R}$

- So, in the linear case, one has:

$$W_f = W_f' = \frac{1}{2}\mathcal{F}\phi = \frac{1}{2}\mathcal{R}\phi^2 = \frac{1}{2}\frac{\phi^2}{\mathcal{P}} = \frac{1}{2}\frac{\mathcal{F}^2}{\mathcal{R}} = \frac{1}{2}\mathcal{P}\mathcal{F}^2$$

Special Case 1: Constant Flux

- The assumption is that the movement of the movable iron core parts is sufficiently fast to prevent the magnetic flux to vary.
- In this case, the energy absorbed from the feeder is zero:

$$dW_e = i d\lambda = 0 \quad \Rightarrow \quad 0 = dW_f + dW_m$$

- Hence:

$$dW_m = -dW_f \quad \Rightarrow \quad f = -\frac{dW_f}{dx}|_{d\lambda=0}$$

i.e., *the mechanical work is obtained by varying the stored magnetic energy*.

Special Case 2: Constant Current

- The assumption is that the movement of the movable iron core parts is sufficiently slow to prevent the current in the coil to vary.

- In this case, we have:

$$dW_e = id\lambda = dW_f + dW_m = id\lambda + \lambda di - dW'_f + dW_m$$

- Hence:

$$dW_m = dW'_f \quad \Rightarrow \quad f = \frac{dW'_f}{dx}|_{di=0}$$

i.e., *the mechanical work is obtained by varying the coenergy of the magnetic circuit*.

Summary

- **ENERGY** (W_f) is useful if we know the function $i(x, \lambda)$:

$$W_f = \int_\lambda i(x, \lambda) d\lambda$$

- **COENERGY** (W'_f) is useful if we know the function $\lambda(x, i)$:

$$W'_f = \int_i \lambda(x, i) di$$

Expressions of Mechanical Force (or Torque)

- From the differentiation of the energy, we obtain:

$$dW_f = id\lambda - f dx = \left.\frac{\partial W_f}{\partial \lambda}\right|_{x=\mathrm{const.}} \cdot d\lambda + \left.\frac{\partial W_f}{\partial x}\right|_{\lambda=\mathrm{const.}} \cdot dx$$

- From the differentiation of the coenergy, we obtain:

$$dW'_f = \lambda di + f dx = \left.\frac{\partial W'_f}{\partial i}\right|_{x=\mathrm{const.}} \cdot di + \left.\frac{\partial W'_f}{\partial x}\right|_{i=\mathrm{const.}} \cdot dx$$

- Hence, there two expressions of the force (torque):

$$f = -\left.\frac{\partial W_f}{\partial x}\right|_{\lambda=\mathrm{const.}} = \left.\frac{\partial W'_f}{\partial x}\right|_{i=\mathrm{const.}}$$

Example

- An electromechanical system has the following relation between the total magnetic flux and the current:

$$\lambda = \frac{4 \cdot 10^{-4}}{x^2} \cdot i^{1/3}$$

- Determine the mechanical force.

Example - Coenergy-based Solution

- We determine W'_f first and the we calculate $f = \frac{\partial W'_f}{\partial x}$.
- Since the function $\lambda(x,i)$ is given, we have:

$$W'_f = \int \lambda(x,i)di = \int \frac{4 \cdot 10^{-4}}{x^2} \cdot i^{1/3} di$$
$$= \frac{3 \cdot 10^{-4}}{x^2} i^{4/3}$$

- Then, the force is obtained as:

$$f = \left.\frac{\partial W'_f}{\partial x}\right|_{i=\text{const.}}$$
$$= -\frac{2 \cdot 3 \cdot 10^{-4}}{x^3} \cdot i^{4/3} = -\frac{6 \cdot 10^{-4}}{x^3} \cdot i^{4/3}$$

Example - Energy-based Solution

- We determine W_f first and the we calculate $f = -\frac{\partial W_f}{\partial x}$.
- We first compute $i(x, \lambda)$, then we have:

$$W_f = \int i(x,\lambda)d\lambda = \int \frac{x^6}{(4 \cdot 10^{-4})^3} \cdot \lambda^3 d\lambda$$
$$= \frac{x^6}{(4 \cdot 10^{-4})^3} \cdot \frac{1}{4} \cdot \lambda^4$$

- Then, the force is obtained as:

$$f = - \left. \frac{\partial W_f}{\partial x} \right|_{\lambda=\mathrm{const.}} = -6 \cdot \frac{x^5}{(4 \cdot 10^{-4})^3} \cdot \frac{1}{4} \cdot \lambda^4 \,,$$

- which, substituting back the original expression of $\lambda(x, i)$, gives the same solution obtained with the coenergy approach.

Linear System with Two Windings – I

- Let us consider a linear system with two mutually coupled windings.
- The links between total flues and currents are as follows:

$$\lambda_1 = L_1(\alpha)\, i_1 + M(\gamma)\, i_2$$
$$\lambda_2 = M(\gamma)\, i_1 + L_2(\beta)\, i_2$$

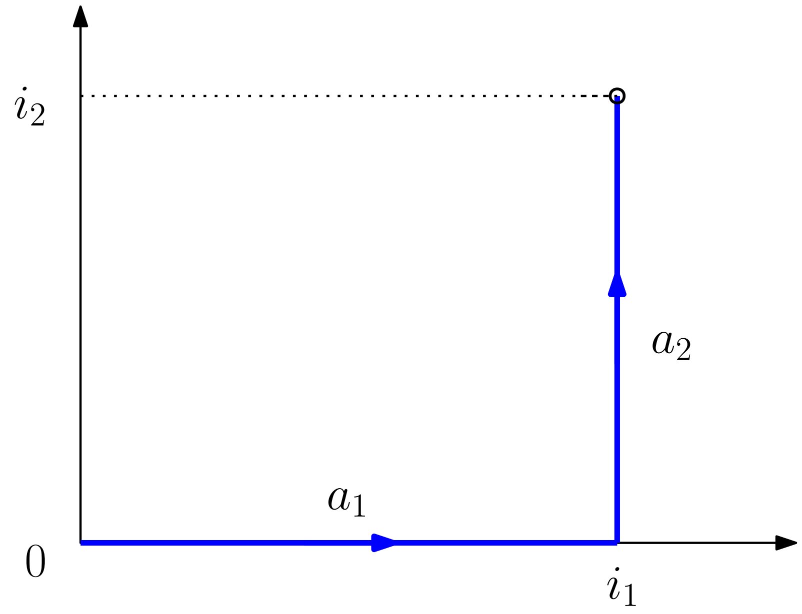
  where the self and mutual inducatnces are assumed to be a function of linear or angular positions α, β and γ.

- We are interested in determining the total magnetic energy and the forces (torques) originated by the systems.

Linear System with Two Windings – II

- We already know that the forces are null for zero current if there are no permanent magnets.
- Let's then consider the variation of energy from $(0, 0)$ to (i_1, i_2).
- If there are no losses, any path can be used:

$$W_f = W_{f,a1} + W_{f,a2}$$



- Since the system is linear, the magnetic energy is equal to the coenergy:

$$W_f = W'_f = \int_{a1} \lambda_1 di_1 + \int_{a1} \lambda_2 di_2 + \int_{a2} \lambda_1 di_1 + \int_{a2} \lambda_2 di_2$$

Linear System with Two Windings – III

- Along a_1, $i_2 = di_2 = 0$, hence:

$$W_{a1} = \int_0^{i_1} \lambda_1 di_1 = \frac{1}{2} L_1(\alpha)\, i_1^2$$

- Along a_2, $i_1 = \text{const.}$, $di_1 = 0$, hence:

$$W_{a2} = \int_0^{i_2} \lambda_2 di_2 = M(\gamma)\, i_1 i_2 + \frac{1}{2} L_2(\beta)\, i_2^2$$

- The total energy is thus:

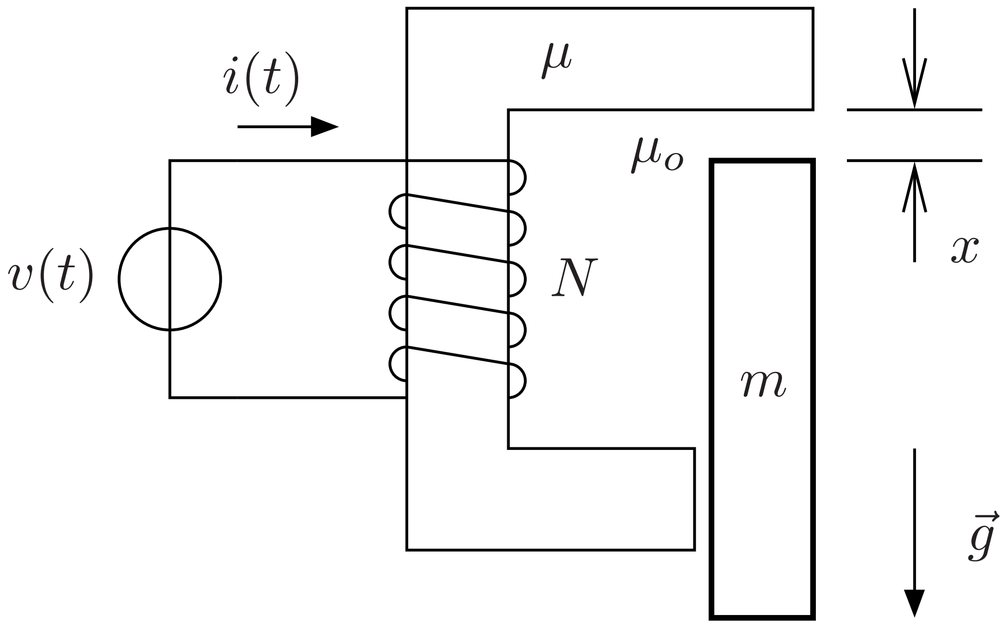
$$W_f = \frac{1}{2} L_1(\alpha)\, i_1^2 + \frac{1}{2} L_2(\beta)\, i_2^2 + M(\gamma)\, i_1 i_2$$

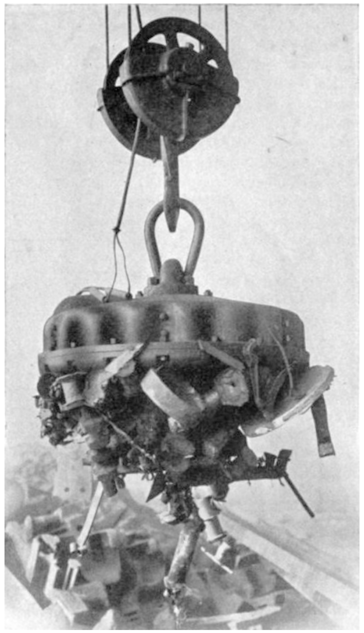
- There are thus three forces:

$$f_\alpha = \frac{1}{2} \frac{\partial L_1(\alpha)}{\partial \alpha}\, i_1^2, \qquad f_\beta = \frac{1}{2} \frac{\partial L_2(\beta)}{\partial \beta}\, i_2^2, \qquad f_\gamma = \frac{\partial M(\gamma)}{\partial \gamma}\, i_1 i_2$$

Electromagnet - I

- Let consider a simple electro-mechanical system: the electromagnet (on the right, industrial electromagnet lifting scrap iron, 1914).





- What is the force acting on the armature?

Electromagnet – II

- The electrical equation is:

$$v = Ri + e$$

where R is the resistance of the coil and e is the mmf induced by the magnetic circuit:

$$e = \frac{d\lambda}{dt}$$

and ϕ is the magnetic flux in the iron core, which depends on the current and on the position x:

$$\lambda(x, i) = L(x)i = \frac{N^2 i}{\mathcal{R}_{\mathrm{Fe}} + \mathcal{R}_0(x)} \approx \mu_0 \frac{N^2 A i}{x}$$

where L is the inductance of the coil, A is the iron core section area and we have assumed that the reluctance in the iron core is negligible with respect to the one in the air gap ($\mathcal{R}_{\mathrm{Fe}} \ll \mathcal{R}_0$).

Electromagnet – III

- The time derivative of the magnetic flux gives:

$$\frac{d\lambda}{dt} = L(x)\frac{di}{dt} + \frac{\partial L}{\partial x}\frac{dx}{dt}i$$

- The mechanical equation is:

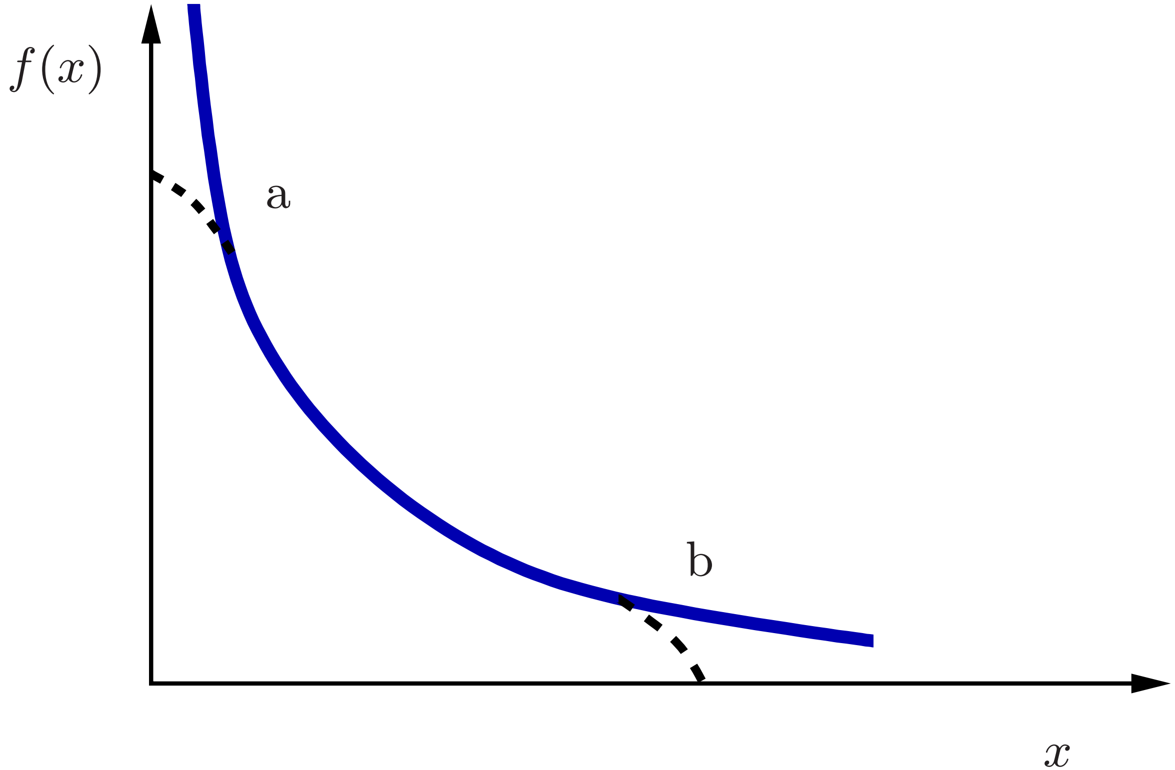
$$m\frac{d^2x}{dt^2} = f(x) + b\frac{dx}{dt} + mg$$

where b is a viscous friction coefficient, $f(x)$ is the force generated by the magnetic circuit:

$$f(x) = \frac{1}{2}\frac{\partial L(x)}{\partial x}i^2 = -\frac{1}{2}\frac{N^2}{x^2}\mu_0 A i^2$$

Electromagnet – IV

- The function of the force is not valid for *all* values of x.



- In region a, the force does not become infinite due to the iron core reluctance.
- In region b, the force goes to zero as the magnetic flux field disperses in the air.

Electromagnet – V

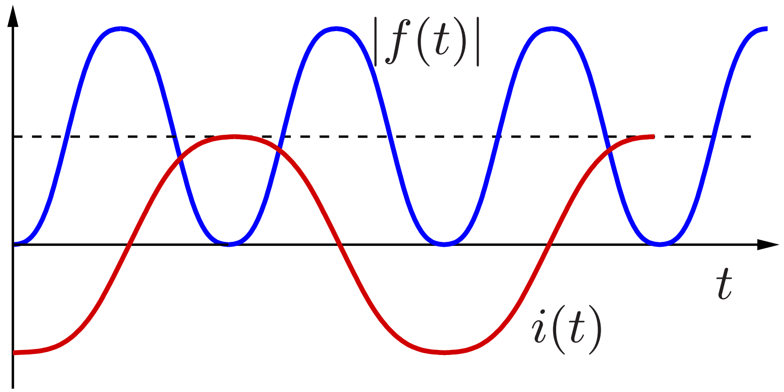
- If the current is ac, we have:

$$f = -\frac{1}{2}\frac{\partial L}{\partial x} i^2(t) = -\frac{1}{2}\frac{\partial L}{\partial x} I_M^2 \sin^2(\omega t)$$

- Since $\sin^2 \alpha = 0.5(1 - \cos(2\alpha))$, the previous expression can be rewritten as:

$$f = -\frac{1}{2}\frac{\partial L}{\partial x} I^2 + \frac{1}{2}\frac{\partial L}{\partial x} I^2 \cos(2\omega t)$$

where $I = I_M/\sqrt{2}$ is the rms value of the current.



Electromagnet – VI

- For the particular case of ac voltage, stationary conditions can be described by static phasors:

$$I = \frac{V}{\sqrt{R^2 + X^2}}$$

where I and V are the rms values of the current and the voltage, respectively, and X is the system reactance:

$$X = \omega L(x) = \omega \mu_0 \frac{N^2 A}{x}$$

- Assuming $R \ll X$:

$$I \approx \frac{V}{X} = \frac{Vx}{\mu_0 \omega N^2 A}$$

Electromagnet – VII

- Note that the term that depends on the electrical pulsation ω is filtered by the inertia of the mobile iron core, i.e., the mobile iron core does not oscillate!

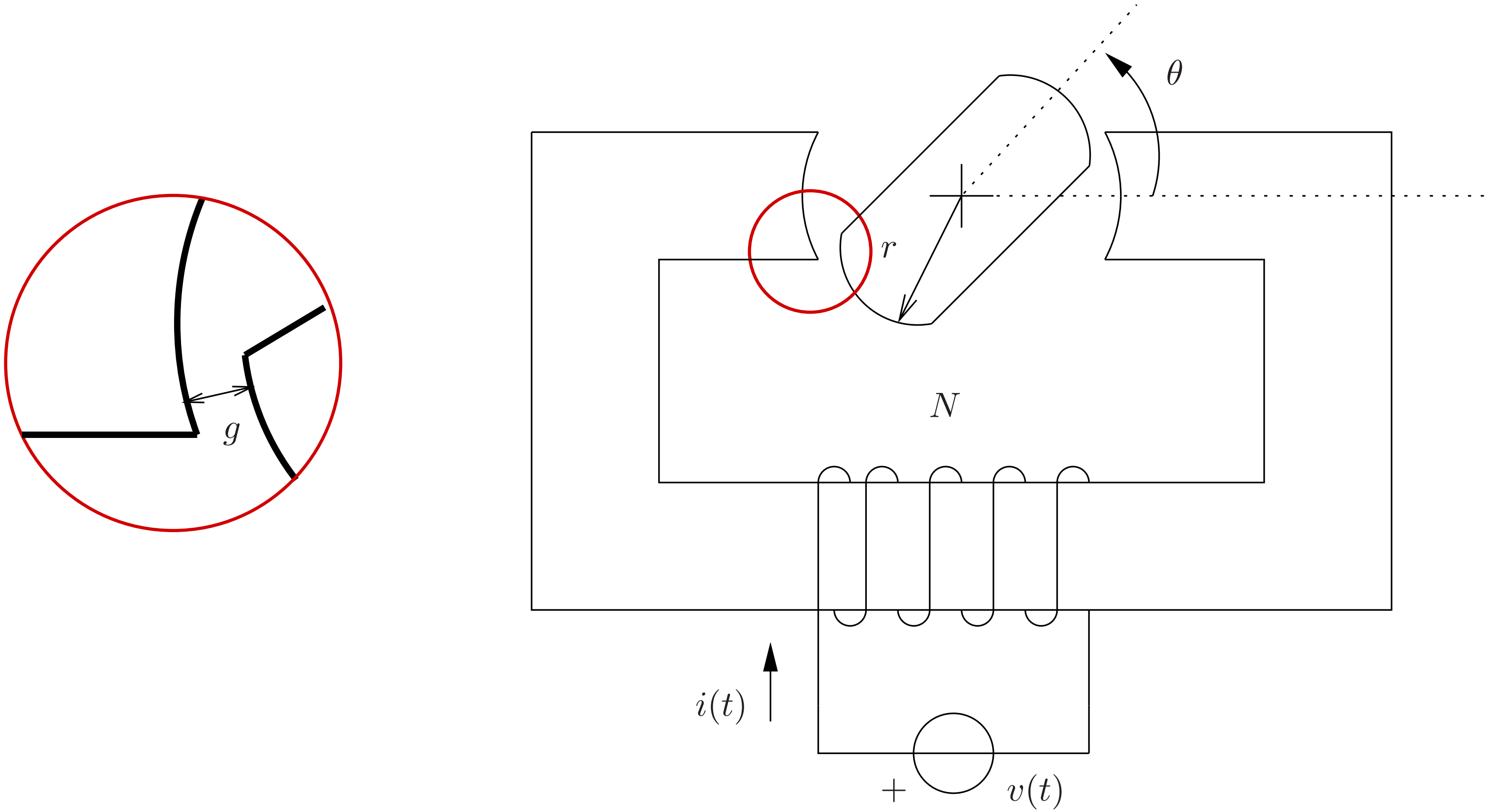
- Observe also that, assuming ac stationary conditions, i.e., no dynamic interactions between electromagnetic and mechanic dynamics, the electromagnetic force becomes:

$$f \approx -\frac{1}{2}\frac{\partial L}{\partial x}I^2 = -\frac{1}{2}\frac{AN^2}{x^2}I^2 \approx -\frac{1}{2}\frac{V^2}{\mu_0\omega^2N^2A}$$

- The force is only a function of the voltage rms value, not of the position of the mobile iron core.

Reluctance Machine

- Let's consider the following magnetic circuit:



Steady-State Reluctance Machine

- The reluctance of the two airgaps is:

$$\mathcal{R} = \frac{1}{\mu_0} \cdot \frac{2g}{r\theta\ell}$$

where ℓ is the width of the magnetic core and we have neglected the reluctances of the fixed and mobile iron cores.

- The resulting torque is:

$$T = \frac{1}{2}\frac{\partial L}{\partial \theta} i^2 = \frac{1}{2}\frac{\mu_0 r\ell}{2g} N^2 i^2$$

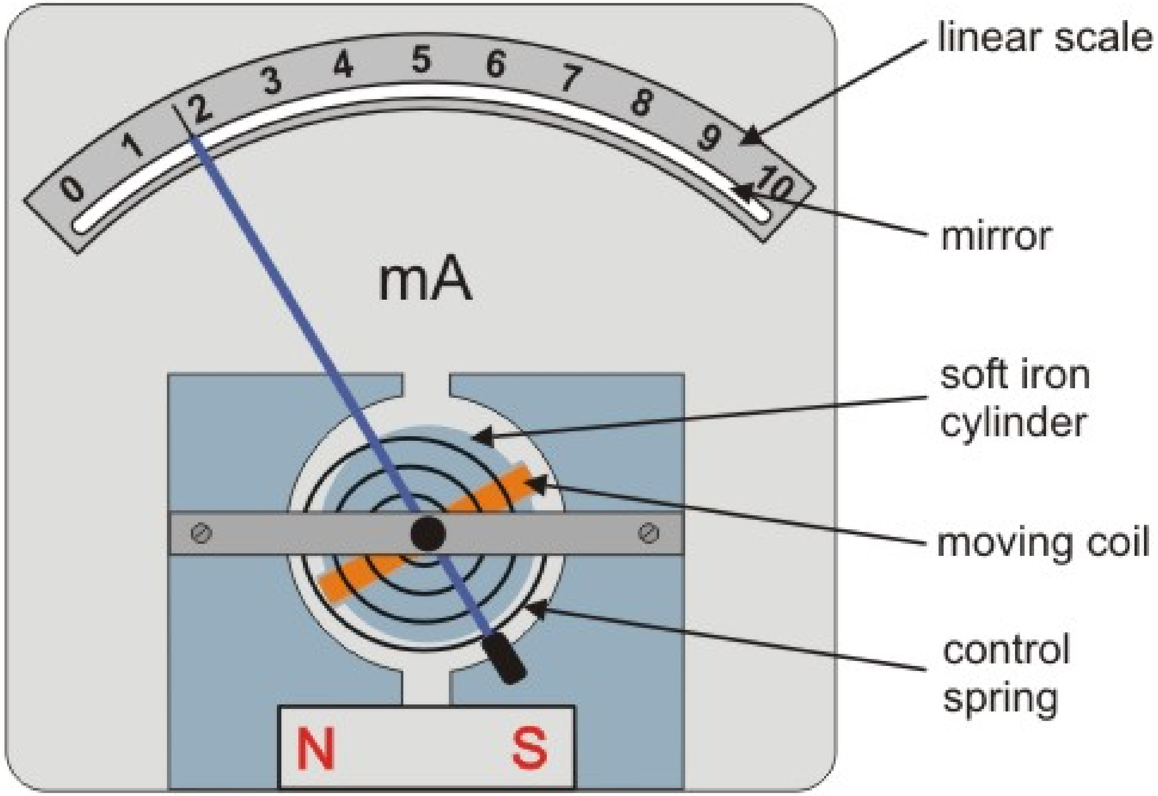
- If the circuit is AC and in steady state:

$$T \approx \frac{1}{2}\frac{\mu_0 r\ell}{2g} N^2 I^2 = \text{const.}$$

- The torque is thus constant for a given current.

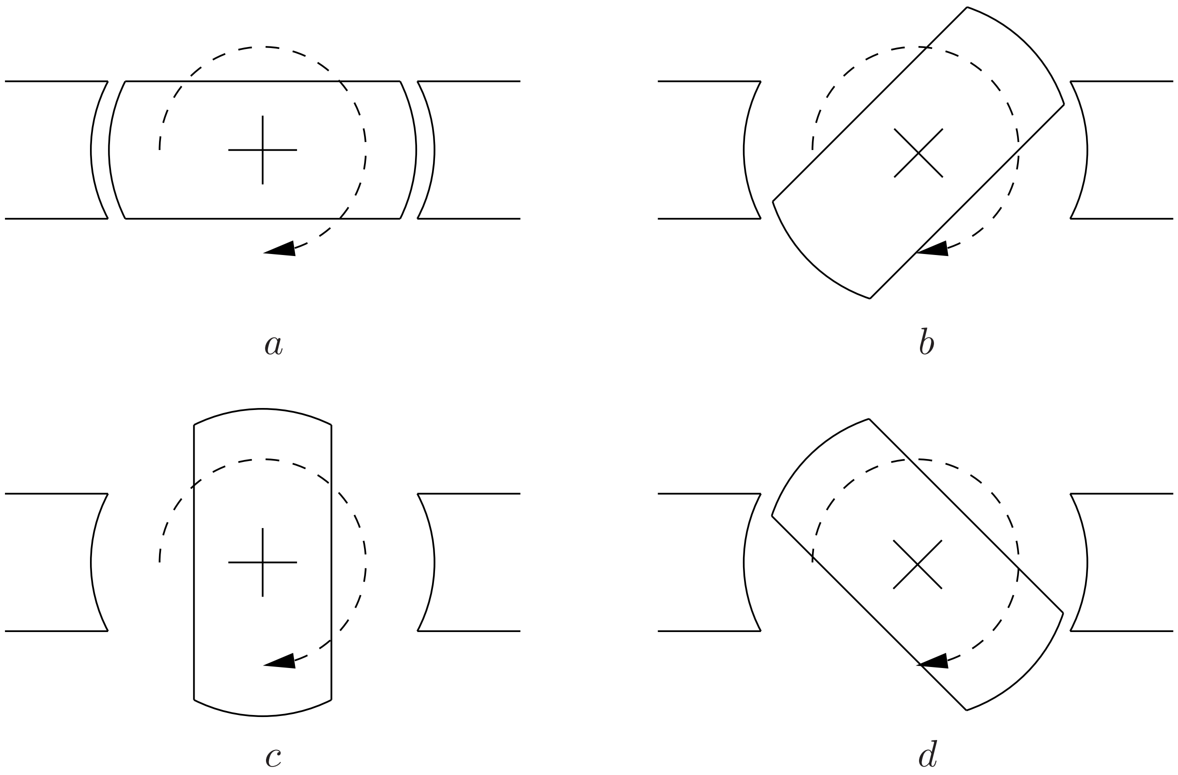
Galvanometer

- This electro-magnetic circuit is the base for instruments to measure the current, voltage and power. For example, the figure below shows a galvanometer.



Rotating Reluctance Machine - I

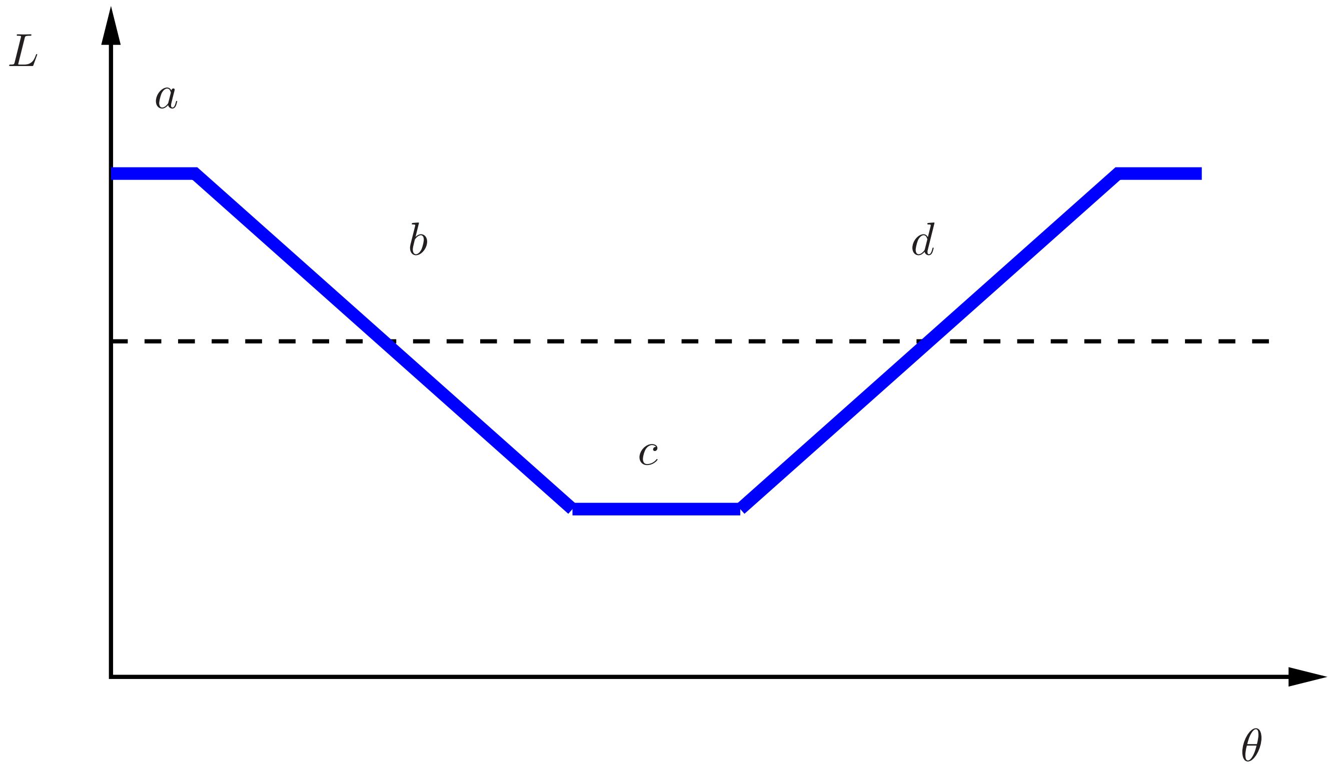
- The inductance of the machine changes as the rotor changes position.



- Position a: L_{max}; Position c: L_{min}

Rotating Reluctance Machine - II

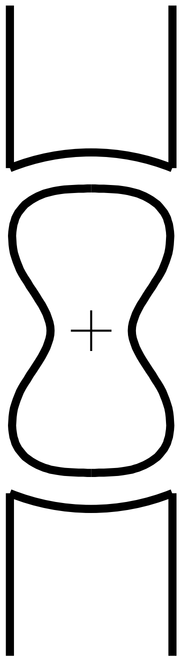
- The inductance L varies periodically as a function of the position θ.

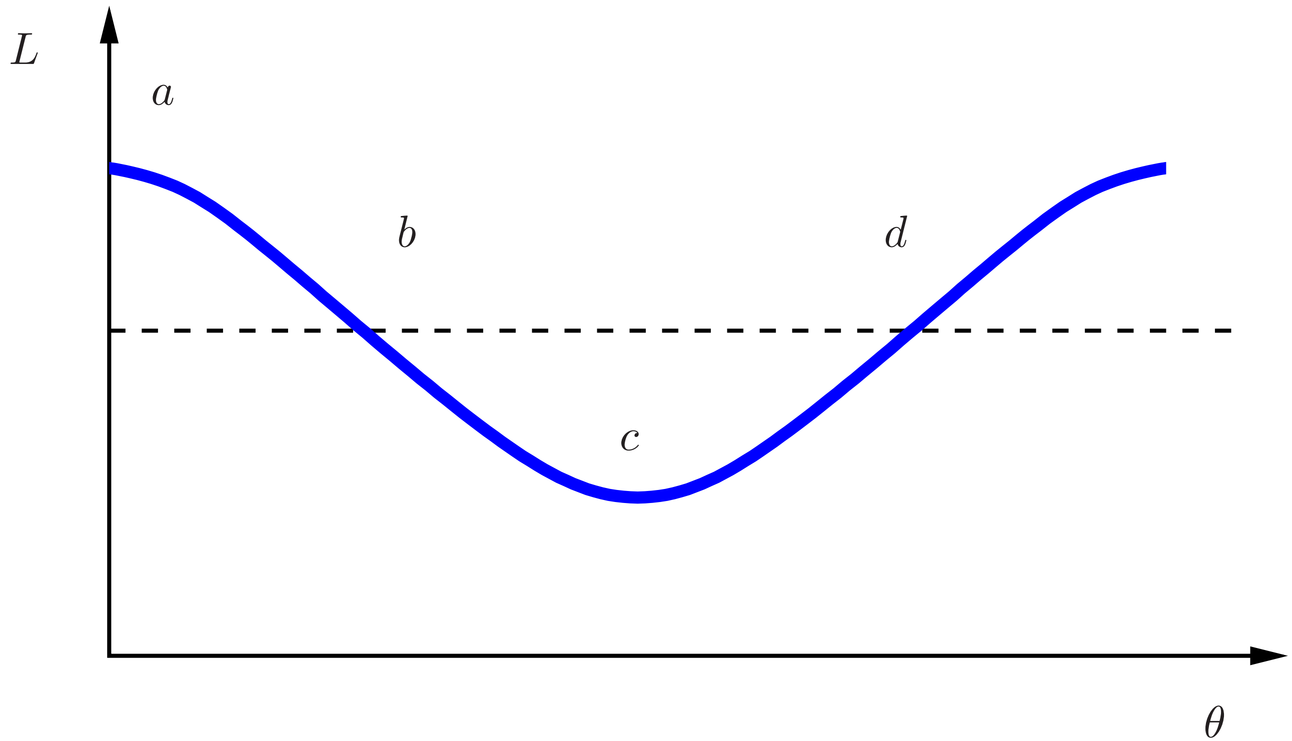


Rotating Reluctance Machine - III

- It is possible to shape the rotor so that the inductance L varies sinusoidally:

$$L(\theta) = \frac{L_{\max} + L_{\min}}{2} + \frac{L_{\max} - L_{\min}}{2} \sin(2\theta)$$





Rotating Reluctance Machine - IV

- The resulting torque is:

$$T = -\frac{1}{2} \cdot i^2 \cdot \frac{\partial L}{\partial \theta} = -\frac{1}{2} \cdot i^2 \cdot \frac{L_{\max} - L_{\min}}{2} \cdot 2 \cdot \cos(2\theta)$$

- If $i(t) = \sqrt{2} I \sin(\omega t)$, then:

$$\begin{aligned} T &= -\frac{1}{2} \cdot 2I^2 \cdot \sin^2(\omega t) \cdot \frac{L_{\max} - L_{\min}}{2} \cdot 2 \cdot \cos(2\theta) \\ &= -\frac{1}{2} \cdot 2I^2 \cdot \frac{1}{2}(1 - \cos(2\omega t)) \cdot \Delta L \cdot \cos(2\theta) \\ &= \underbrace{-\frac{1}{2} \cdot I^2 \cdot \Delta L \cdot \cos(2\theta)}_{\mathrm{A}} + \underbrace{\frac{1}{2} \cdot I^2 \cdot \cos(2\omega t) \cdot \Delta L \cdot \cos(2\theta)}_{\mathrm{B}} \end{aligned}$$

where $\Delta L = L_{\max} - L_{\min}$.

Rotating Reluctance Machine - V

- The term A has null average as θ varies.
- The term B has non-null average only if $\theta = \omega_m t = \omega t$.
- In other words, this machine has non-null average torque only if it rotates synchronously with the electrical ac system.
- If $\omega_m = \omega$:

$$T = \frac{1}{2} I^2 \Delta L \cos^2(2\omega t)$$