



Energy Conversion

ELECTRICAL ENERGY SYSTEMS (EEEN20090)

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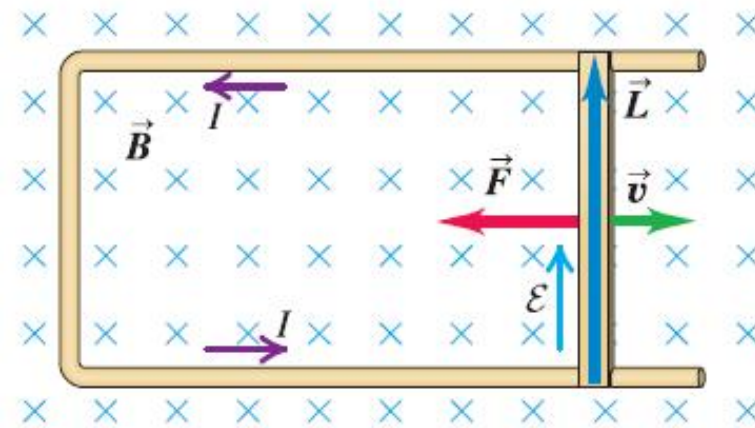


Energy Conversion in Magnetic Circuits

- Lorentz force
- Energy balance
- Energy and coenergy
- Examples:
 - Acyclic machine
 - Electromagnet
 - Reluctance motor

Linear Motor

- Consider a conductor (e.g. a copper rod) of length L resting on wires, carrying a current I , in the presence of a magnetic field whose flux density is in the direction shown, normal to the plane of the conductor.



- Current leads to moving charges in the conductor, which experience an electromagnetic force, which is transmitted to the conductor itself.
- The conductor thus experiences a force (Lorentz force), given by the vector product

$$\vec{F} = i\vec{\ell} \times \vec{B}$$

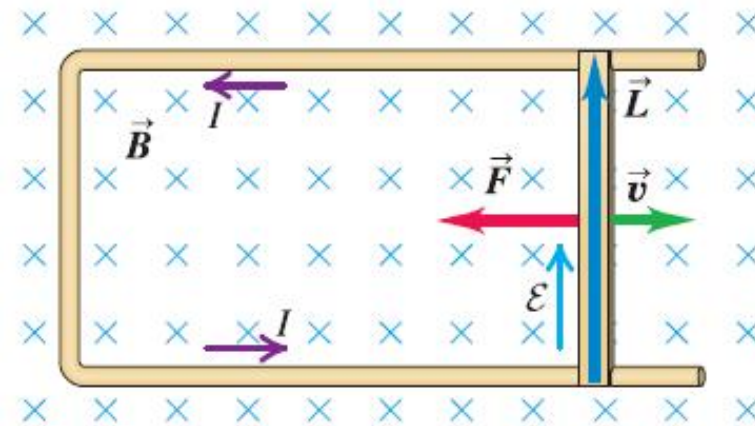
- In this example F has magnitude BLI

Linear Generator

- The system considered in the previous slide can also be used as a generator. If the bar moves with constant speed \vec{v} , then the area spanned by the coil varies, which leads to a variation of magnetic flux:

$$d\phi = B dA = BL dx$$

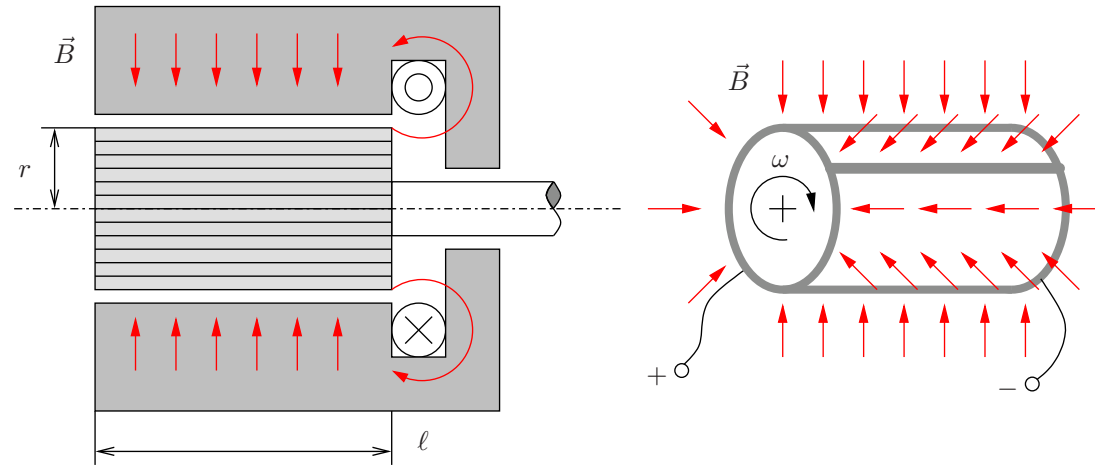
$$\Rightarrow \frac{d\phi}{dt} = BL \frac{dx}{dt} = BLv$$



- This is a very simple example of electric machine which can work either as a motor or as a generator.
- The linear structure is impractical: how can such an uniform \vec{B} be generated?
- That's why most electrical machines are rotating machines.

Acyclic Machine

- It is possible though to implement the machine seen previously using an *acyclic* structure
- Lets consider the machine in the figure



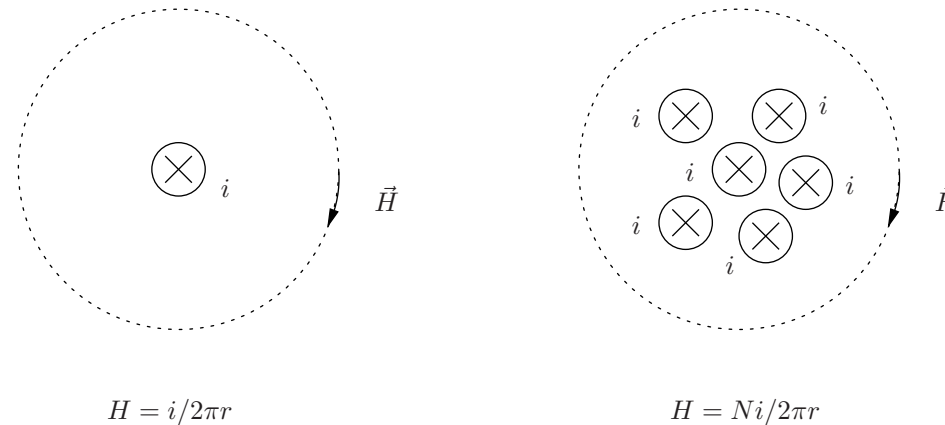
- The magnetic induction \vec{B} is radial and time-invariant. While rotating, the coil spanned area changes its relative position with respect to the magnetic induction:

$$e = -\frac{d\phi}{dt} = Bl\omega r = Blv$$

- This machine is characterized by a continuous (low) voltage and extremely high currents (order of kA).

Motivation of Coils with High N

- The acyclic machine has not practical applications.
- This is mainly because the coil on the rotor has just one turn \Rightarrow inefficient!



- The advantage of using N turns is an easy way to *increase* the magnetic field \vec{H} while keeping relatively small the current of the winding.

Force in Magnetic Field - I

- The power balance of a coil:

$$p = vi = ri^2 + i \frac{d\lambda}{dt}$$

- Neglecting losses for simplicity, but without loss of generality:

$$pdt = id\lambda$$

- The total energy absorbed when the flux linkage changes from 0 to λ is:

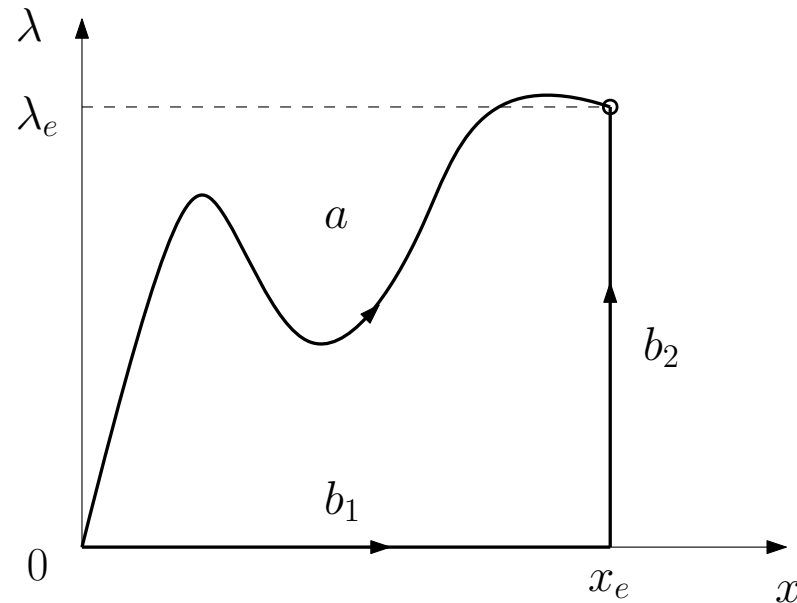
$$W_e = \int pdt = \int_0^\lambda id\lambda$$

Motivation

- Let's consider the variation of energy from $(0, 0)$ to (x_e, λ_e) .
- If there are no losses, any path can be used:

$$\Delta W_e = \Delta W_a = \Delta W_{b_1} + \Delta W_{b_2}$$

- However, path b_1 - b_2 is much simpler.



- Along b_1 , $\lambda = 0$ and $d\lambda = 0$, hence $i = 0$ and $f = 0$, hence: $\Delta W_{b_1} = 0$
- Along b_2 , $x = x_e = \text{const.}$, $dx = 0$, hence:

$$\Delta W_e = \Delta W_{b_2} = \int_0^{\lambda_e} i(x_e, \lambda) d\lambda$$

Force in Magnetic Field – II

- In the case of movable parts, the current is a function of λ and the position x :

$$i = i(\lambda, x)$$

- Then also the magnetic field energy W_f depends on λ and x :

$$W_f = W_f(\lambda, x)$$

- The incremental change of magnetic field energy gives:

$$dW_f(\lambda, x) = \frac{\partial W_f}{\partial \lambda} d\lambda + \frac{\partial W_f}{\partial x} dx$$



Force in Magnetic Field – III

- On the other hand, neglecting losses, the variation of electric energy has to be equal to the variation of magnetic energy plus the variation of mechanical energy:

$$dW_e = dW_f + f dx = i d\lambda \quad dW_f = i d\lambda - f dx$$

- Hence, we have:

$$i = \left. \frac{\partial W_f}{\partial \lambda} \right|_{dx=0} \quad f = - \left. \frac{\partial W_f}{\partial x} \right|_{d\lambda=0}$$

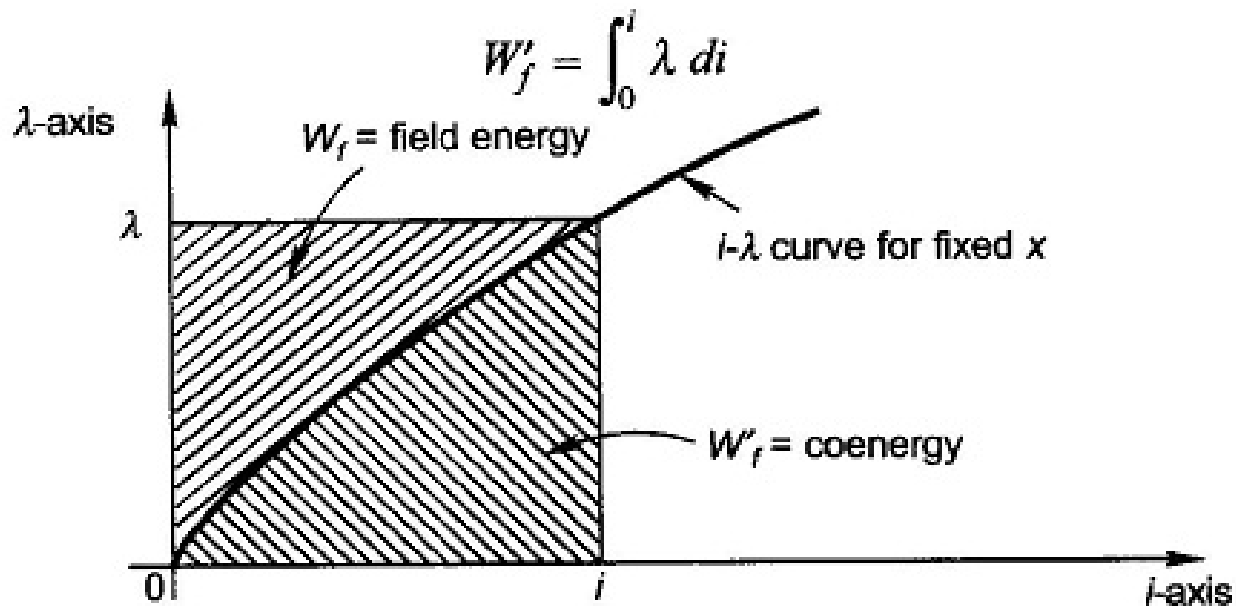
Coenergy

- For fixed x , the field energy is the area of the upper side of the λ - i curve:

$$dW_f = \int_0^\lambda i d\lambda$$

- Alternatively, for fixed x , one can define **coenergy** as the area of the lower side:

$$dW'_f = \int_0^i \lambda di$$



Mechanical Force in terms of Coenergy

- Let remember that:

$$dW_e = dW_f + dW_m = id\lambda$$

- Then, by definition of coenergy:

$$W_f + W'_f = i\lambda \quad dW_f + dW'_f = id\lambda + \lambda di$$

- Substituting into the expression of the energy balance:

$$\begin{aligned} id\lambda + \lambda di - dW'_f + dW_m &= id\lambda \\ \Rightarrow dW'_f &= \lambda di + dW_m = \lambda di + f dx \end{aligned}$$

- Which leads to:

$$f = \left. \frac{\partial W'_f}{\partial x} \right|_{di=0}$$

Linear Case with Time-invariant Inductance

- Let's assume the linear case (constant inductance):

$$\lambda = Li$$

- Then, the field energy is:

$$W_f = \int_0^\lambda id\lambda = \int_0^\lambda \frac{\lambda}{L} d\lambda = \frac{1}{2} \frac{\lambda^2}{L} = \frac{1}{2} Li^2$$

- The coenergy is:

$$W'_f = \int_0^i \lambda di = \int_0^i Lidi = \frac{1}{2} Li^2 = \frac{1}{2} \frac{\lambda^2}{L}$$

- Hence, $W_f = W'_f$, as it can be deduced straightforwardly from the curve on the λ - i plane.

Alternative Expressions

- The inductance can be expressed as:

$$L = N \frac{\phi}{i}$$

- Then, taking into account the Hopkinson law ($\mathcal{F} = \mathcal{R}\phi$), the inductance can be rewritten as:

$$L = N^2 \frac{\phi}{Ni} = N^2 \frac{\phi}{\mathcal{R}} = \frac{N^2}{\mathcal{R}} = N^2 \mathcal{P}$$

where \mathcal{P} is the permeance, $\mathcal{P} = 1/\mathcal{R}$

- So, in the linear case, one has:

$$W_m = W'_m = \frac{1}{2} \mathcal{F} \phi = \frac{1}{2} \mathcal{R} \phi^2 = \frac{1}{2} \frac{\phi^2}{\mathcal{P}} = \frac{1}{2} \frac{\mathcal{F}^2}{\mathcal{R}} = \frac{1}{2} \mathcal{P} \mathcal{F}^2$$

Special Case 1: Constant Flux

- The assumption is that the movement of the movable iron core parts is sufficiently **fast** to prevent the magnetic flux to vary.
- In this case, the energy absorbed from the feeder is zero:

$$dW_e = id\lambda = 0 \quad \Rightarrow \quad 0 = dW_f + dW_m$$

- Hence:

$$dW_m = -dW_f \quad \Rightarrow \quad f = -\left. \frac{dW_f}{dx} \right|_{d\lambda=0}$$

i.e., the mechanical work is obtained by varying the stored magnetic energy.

Special Case 2: Constant Current

- The assumption is that the movement of the movable iron core parts is sufficiently **slow** to prevent the current in the coil to vary.
- In this case, we have:

$$dW_e = id\lambda = dW_f + dW_m = id\lambda + \lambda di - dW'_f + dW_m$$

- Hence:

$$dW_m = dW'_f \quad \Rightarrow \quad f = \left. \frac{dW'_f}{dx} \right|_{di=0}$$

i.e., *the mechanical work is obtained by varying the coenergy of the magnetic circuit.*

Summary

- **ENERGY** (W) is useful if we know the function $i(x, \lambda)$:

$$W = \int_{\lambda} i(x, \lambda) d\lambda$$

- **COENERGY** (W') is useful if we know the function $\lambda(x, i)$:

$$W' = \int_i \lambda(x, i) di$$

Expressions of Mechanical Force (or Torque)

- From the differentiation of the energy, we obtain:

$$dW = id\lambda - f dx = \left. \frac{\partial W}{\partial \lambda} \right|_{x=\text{const.}} \cdot d\lambda + \left. \frac{\partial W}{\partial x} \right|_{\lambda=\text{const.}} \cdot dx$$

- From the differentiation of the coenergy, we obtain:

$$dW' = \lambda di + f dx = \left. \frac{\partial W'}{\partial i} \right|_{x=\text{const.}} \cdot di + \left. \frac{\partial W'}{\partial x} \right|_{i=\text{const.}} \cdot dx$$

- Hence, there two expressions of the force (torque):

$$f = - \left. \frac{\partial W}{\partial x} \right|_{\lambda=\text{const.}} = \left. \frac{\partial W'}{\partial x} \right|_{i=\text{const.}}$$

Example

- An electromechanical system has the following relation between the total magnetic flux and the current:

$$\lambda = \frac{4 \cdot 10^{-4}}{x^2} \cdot i^{1/3}$$

- Determine the mechanical force.

Example - Coenergy-based Solution

- We determine W' first and then we calculate $f = \frac{\partial W'}{\partial x}$.
- Since the function $\lambda(x, i)$ is given, we have:

$$\begin{aligned}W' &= \int \lambda(x, i) di = \int \frac{4 \cdot 10^{-4}}{x^2} \cdot i^{1/3} di \\ &= \frac{3 \cdot 10^{-4}}{x^2} i^{4/3}\end{aligned}$$

- Then, the force is obtained as:

$$\begin{aligned}f &= \left. \frac{\partial W'}{\partial x} \right|_{i=\text{const.}} \\ &= -\frac{2 \cdot 3 \cdot 10^{-4}}{x^3} \cdot i^{4/3} = -\frac{6 \cdot 10^{-4}}{x^3} \cdot i^{4/3}\end{aligned}$$

Example - Energy-based Solution

- We determine W first and then we calculate $f = -\frac{\partial W}{\partial x}$.
- We first compute $i(x, \lambda)$, then we have:

$$\begin{aligned} W &= \int i(x, \lambda) d\lambda = \int \frac{x^6}{(4 \cdot 10^{-4})^3} \cdot \lambda^3 d\lambda \\ &= \frac{x^6}{(4 \cdot 10^{-4})^3} \cdot \frac{1}{4} \cdot \lambda^4 \end{aligned}$$

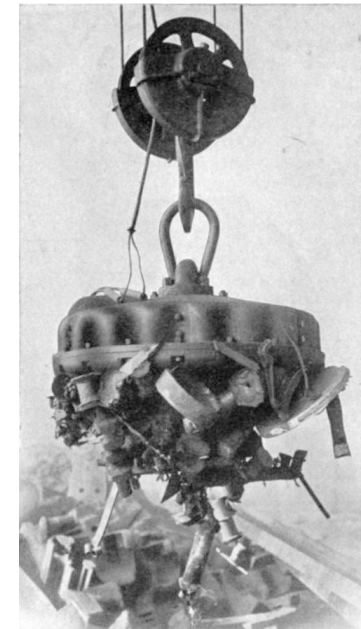
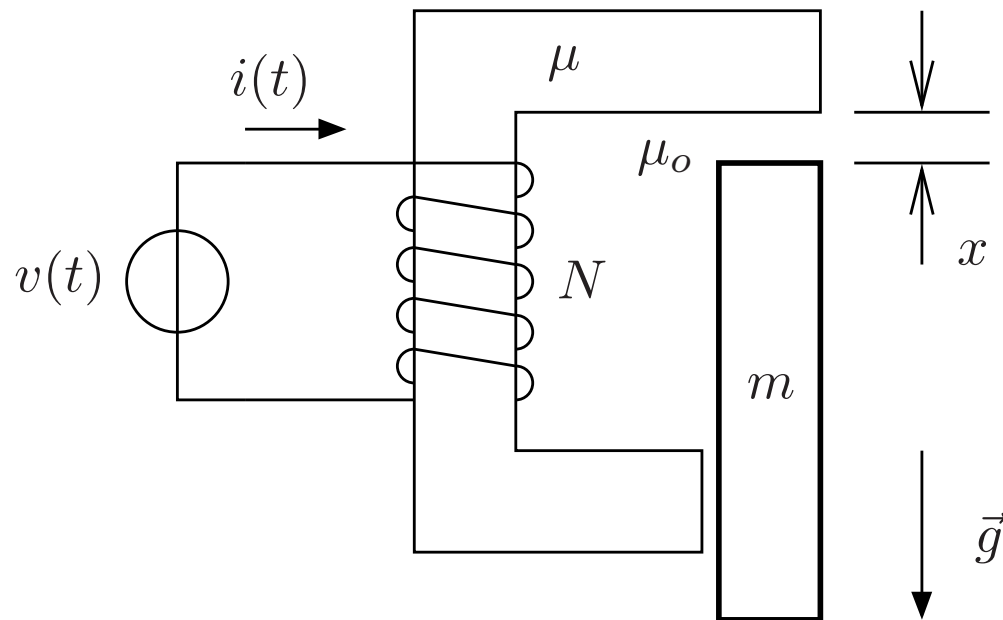
- Then, the force is obtained as:

$$f = - \left. \frac{\partial W}{\partial x} \right|_{\lambda=\text{const.}} = -6 \cdot \frac{x^5}{(4 \cdot 10^{-4})^3} \cdot \frac{1}{4} \cdot \lambda^4 ,$$

- which, substituting back the original expression of $\lambda(x, i)$, gives the same solution obtained with the coenergy approach.

Electromagnet - I

- Let consider a simple electro-mechanical system: the electromagnet (on the right, industrial electromagnet lifting scrap iron, 1914).



- What is the force acting on the armature?

Electromagnet – II

- The electrical equation is:

$$v = Ri + e$$

where R is the resistance of the coil and e is the mmf induced by the magnetic circuit:

$$e = \frac{d\phi}{dt}$$

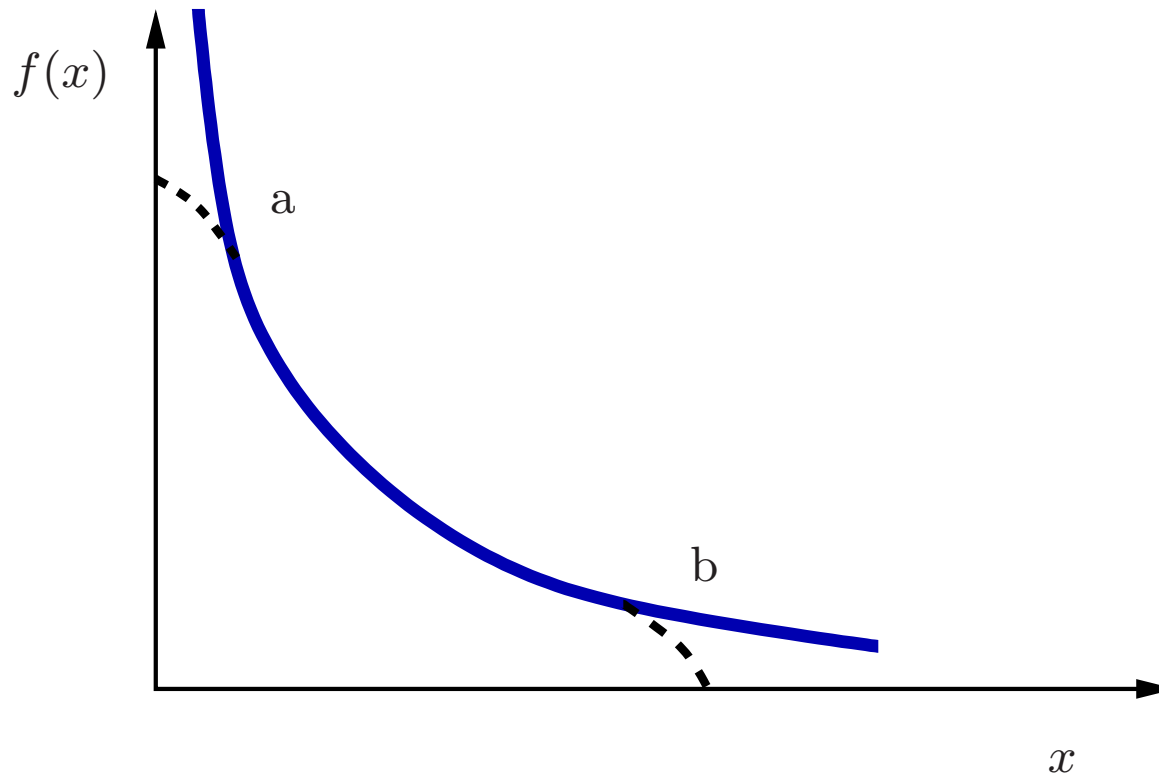
and ϕ is the magnetic flux in the iron core, which depends on the current and on the position x :

$$\phi = L(x)i = \frac{N^2 i}{\mathcal{R}_{\text{Fe}} + \mathcal{R}_0(x)} \approx \mu_0 \frac{N^2 A i}{x}$$

where L is the inductance of the coil, A is the iron core section area and we have assumed that the reluctance in the iron core is negligible with respect to the one in the air gap ($\mathcal{R}_{\text{Fe}} \ll \mathcal{R}_0$).

Electromagnet – III

- The function of the force is not valid for *all* values of x .



- In region a, the force does not become infinite due to the iron core reluctance.
- In region b, the force goes to zero as the magnetic flux field disperses in the air.

Electromagnet – IV

- The time derivative of the magnetic flux gives:

$$\frac{d\phi}{dt} = L(x) \frac{di}{dt} + \frac{\partial L}{\partial x} \frac{dx}{dt} i$$

- The mechanical equation is:

$$m \frac{d^2 x}{dt^2} = f(x) + b \frac{dx}{dt} + mg$$

where b is a viscous friction coefficient, $f(x)$ is the force generated by the magnetic circuit:

$$f(x) = \frac{1}{2} \frac{\partial L(x)}{\partial x} i^2 = -\frac{1}{2} \frac{N^2}{x^2} \mu_0 A i^2$$

Electromagnet – V

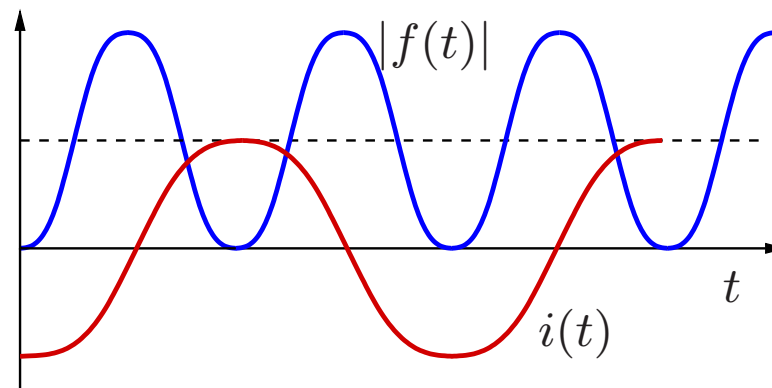
- If the current is ac, we have:

$$f = -\frac{1}{2} \frac{\partial L}{\partial x} i^2(t) = -\frac{1}{2} \frac{\partial L}{\partial x} I_M^2 \sin^2(\omega t)$$

- Since $\sin^2 \alpha = 0.5(1 - \cos(2\alpha))$, the previous expression can be rewritten as:

$$f = -\frac{1}{2} \frac{\partial L}{\partial x} I^2 + \frac{1}{2} \frac{\partial L}{\partial x} I^2 \cos(2\omega t)$$

where $I = I_M / \sqrt{2}$ is the rms value of the current.



Electromagnet – VI

- Note that the term that depends on the electrical pulsation ω is filtered by the inertia of the mobile iron core, i.e., **the mobile iron core does not oscillate!**
- Observe also that, assuming ac stationary conditions, i.e., no dynamic interactions between electromagnetic and mechanic dynamics, the electromagnetic force becomes:

$$f \approx -\frac{1}{2} \frac{\partial L}{\partial x} I^2 = -\frac{1}{2} \frac{AN^2}{x^2} I^2 \approx -\frac{1}{2} \frac{V^2}{\mu_0 \omega^2 N^2 A}$$

- The force is only a function of the voltage rms value, not of the position of the mobile iron core.

Electromagnet – VII

- For the particular case of ac voltage, stationary conditions can be described by static phasors:

$$I = \frac{V}{\sqrt{R^2 + X^2}}$$

where I and V are the rms values of the current and the voltage, respectively, and X is the system reactance:

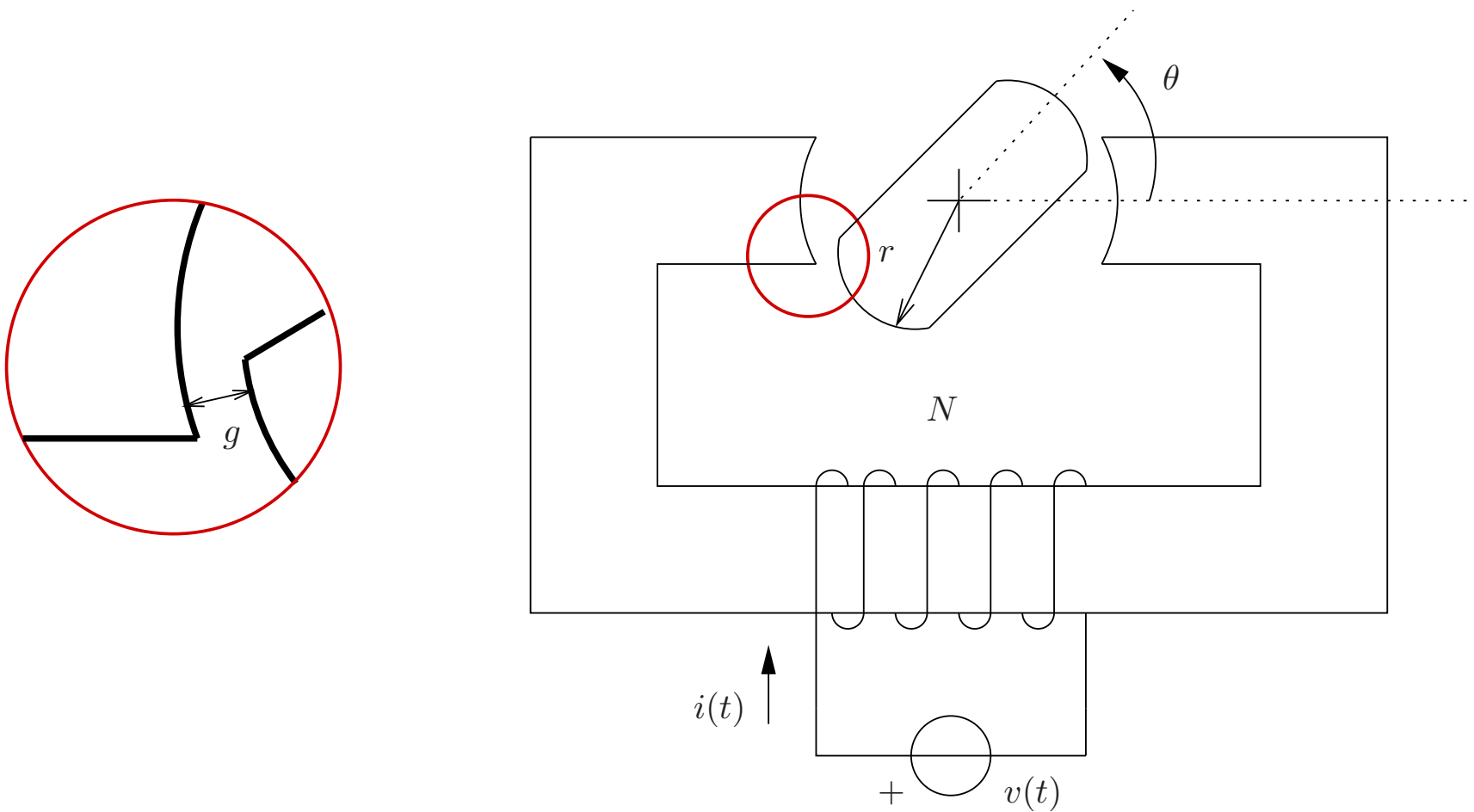
$$X = \omega L(x) = \omega \mu_0 \frac{N^2 A}{x}$$

- Assuming $R \ll X$:

$$I \approx \frac{V}{X} = \frac{V x}{\mu_0 \omega N^2 A}$$

Reluctance Machine

- Let's consider the following magnetic circuit:



Steady-State Reluctance Machine

- The reluctance of the two airgaps is:

$$\mathcal{R} = \frac{1}{\mu_0} \cdot \frac{2g}{r\theta\ell}$$

where ℓ is the width of the magnetic core and we have neglected the reluctances of the fixed and mobile iron cores.

- The resulting torque is:

$$T = \frac{1}{2} \frac{\partial L}{\partial \theta} i^2 = \frac{1}{2} \frac{\mu_0 r \ell}{2g} N^2 i^2$$

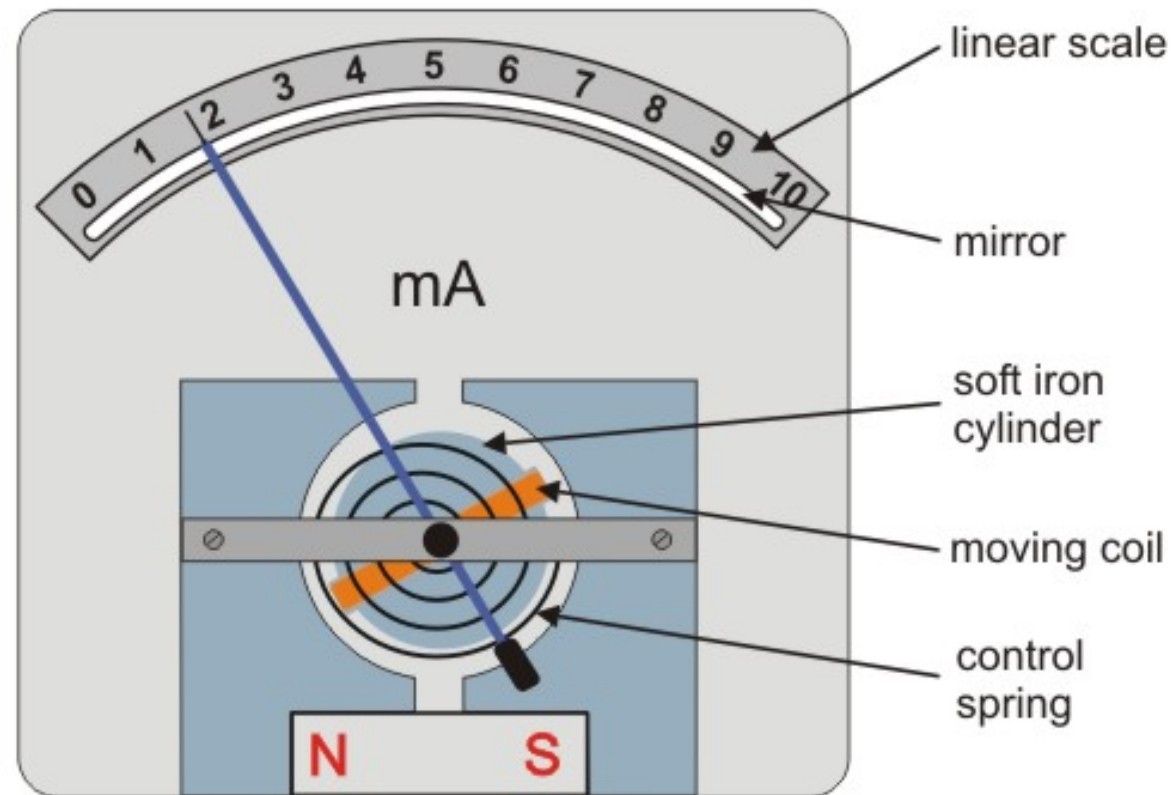
- If the circuit is AC and in steady state:

$$T \approx \frac{1}{2} \frac{\mu_0 r \ell}{2g} N^2 I^2 = \text{const.}$$

- The torque is thus constant for a given current.

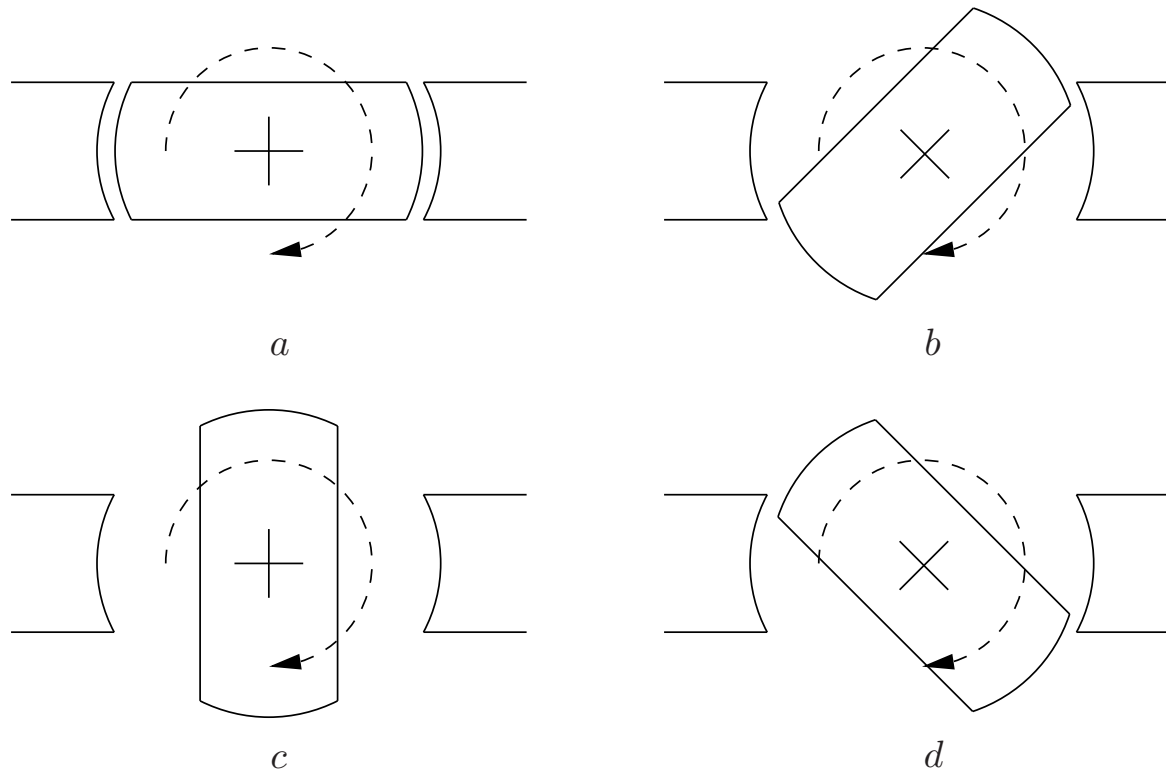
Galvanometer

- This electro-magnetic circuit is the base for instruments to measure the current, voltage and power. For example, the figure below shows a galvanometer.



Rotating Reluctance Machine - I

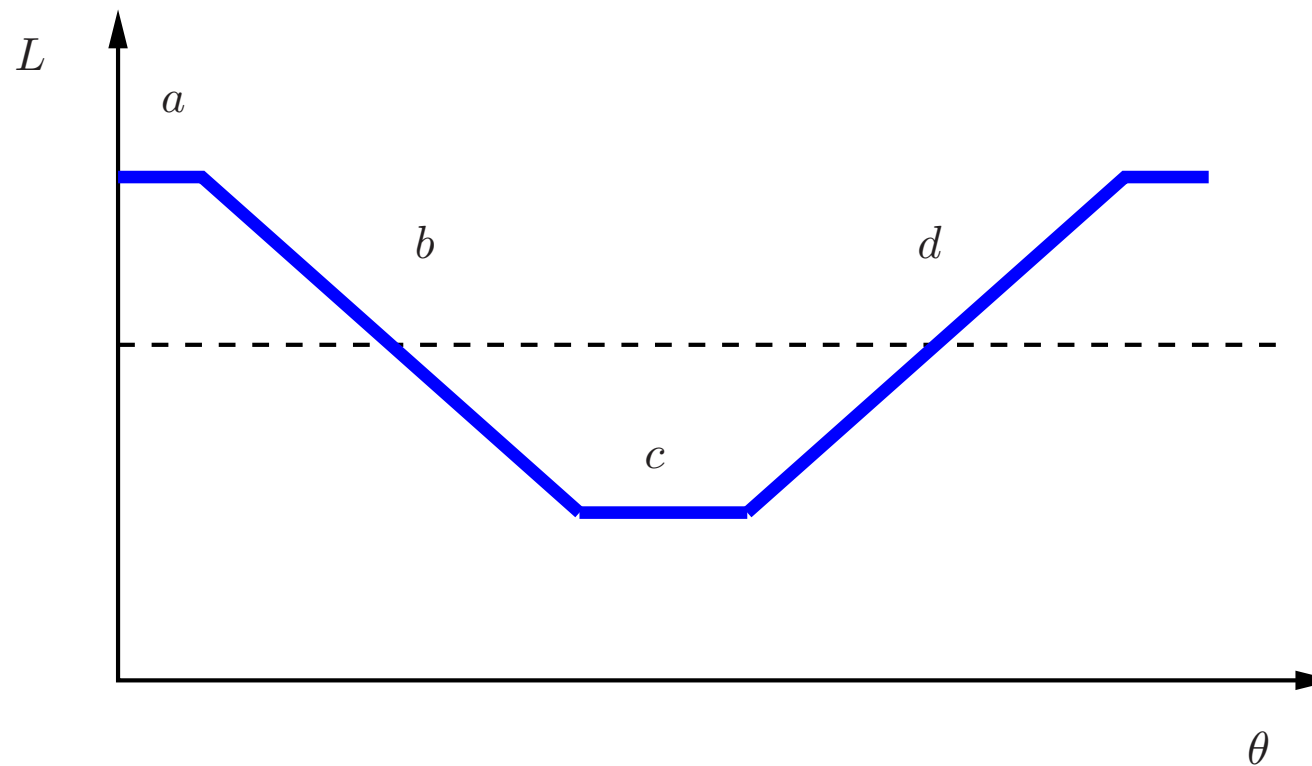
- The inductance of the machine changes as the rotor changes position.



- Position a : L_{\max} ; Position c : L_{\min}

Rotating Reluctance Machine - II

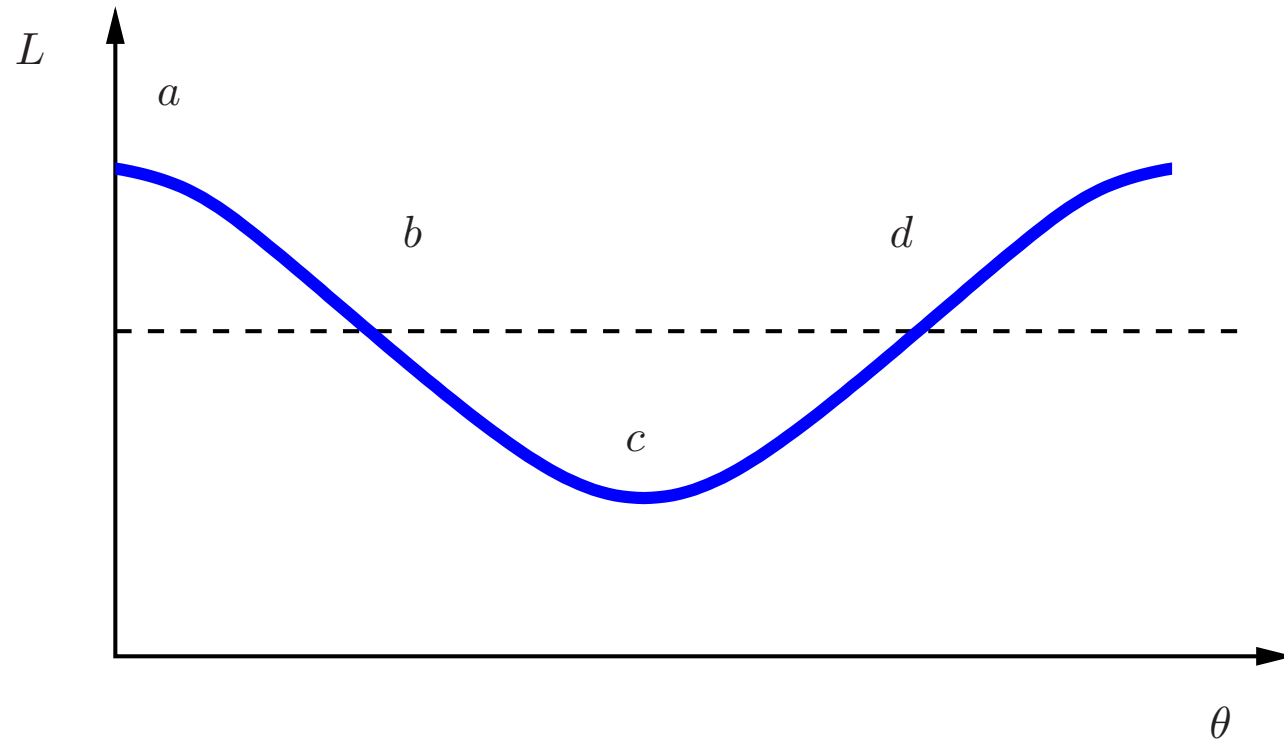
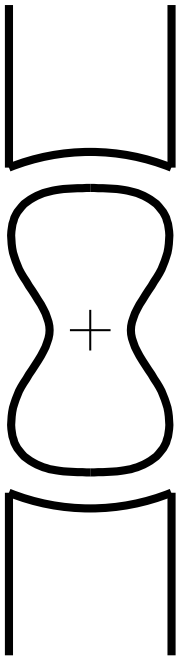
- The inductance L varies periodically as a function of the position θ .



Rotating Reluctance Machine - III

- It is possible to shape the rotor so that the inductance L varies sinusoidally:

$$L(\theta) = \frac{L_{\max} + L_{\min}}{2} + \frac{L_{\max} - L_{\min}}{2} \sin(2\theta)$$



Rotating Reluctance Machine - IV

- The resulting torque is:

$$T = -\frac{1}{2} \cdot i^2 \cdot \frac{\partial L}{\partial \theta} = -\frac{1}{2} \cdot i^2 \cdot \frac{L_{\max} - L_{\min}}{2} \cdot 2 \cdot \cos(2\theta)$$

- If $i(t) = \sqrt{2}I \sin(\omega t)$, then:

$$\begin{aligned} T &= -\frac{1}{2} \cdot 2I^2 \cdot \sin^2(\omega t) \cdot \frac{L_{\max} - L_{\min}}{2} \cdot 2 \cdot \cos(2\theta) \\ &= -\frac{1}{2} \cdot 2I^2 \cdot \frac{1}{2}(1 - \cos(\omega t)) \cdot \Delta L \cdot \cos(2\theta) \\ &= \underbrace{-\frac{1}{2} \cdot I^2 \cdot \Delta L \cdot \cos(2\theta)}_A + \underbrace{\frac{1}{2} \cdot I^2 \cdot \cos(\omega t) \cdot \Delta L \cdot \cos(2\theta)}_B \end{aligned}$$

where $\Delta L = L_{\max} - L_{\min}$.

Rotating Reluctance Machine - V

- The term A has null average as θ varies.
- The term B has non-null average only if $\theta = \omega_m t = \omega t$.
- In other words, this machine has non-null average torque only if it rotates synchronously with the electrical ac system.
- If $\omega_m = \omega$:

$$T = \frac{1}{2} I^2 \Delta L \cos^2(2\omega t)$$