Worked Problems on Induction Machines

EEEN20090 – Electrical Energy Systems

Problem 1

A 4-pole, Y-connected, 380 V, 50 Hz, wound-rotor induction machine has the following equivalent circuit parameters:

$$\bar{Z}_1 = (0.6 + j1.8)\ \Omega/\text{phase} = R_1 + jX_1 \text{ (stator)}$$
$$\bar{Z}_2 = (0.2 + j0.5)\ \Omega/\text{phase} = R_2 + jX_2 \text{ (rotor)}$$

The stator/rotor tap ratio is $k_T = 2$.

a. Determine the slip factor (σ^*) for which the torque is maximum and the maximum torque (T_m^{max}).

b. Determine the starting torque (T_m^{su}).

c. Determine the external resistance (R_{ext}) to connect to the rotor to have $\tilde{T}_m^{\text{su}} = T_m^{\text{max}}$.

d. How to calculate (c) if the induction machine is squirrel-cage type?

Problem 2

A 4-pole, Y-connected, 400 V, 53 Hz induction machine has the following data:

$$\bar{Z}_1 = (0.125 + j0.3)\ \Omega$$
$$\bar{Z}_2' = (0.125 + j0.2)\ \Omega$$
$$I_\mu = 18\ \text{A}$$
$$P_{\text{Fe}} = 1350\ \text{W}$$
$$P_{m,\text{losses}} = 900\ \text{W}$$
$$\omega_N = 1500\ \text{rpm}.$$

At full load, determine:

a. Torque (T_{mN}).

b. Mechanical power (P_m) and torque (T_m) output.

c. Rotor current (I_N) and power factor ($\cos(\phi)$).

d. Efficiency (η).

Problem 3

A 60 Hz, $\triangle$ connected induction machine has the following results.

DC Test: $V_{\text{dc}} = 21$ V, $I_{\text{dc}} = 72$ A.

Open Circuit: $\omega_m = 715$ rpm.

Full Load: $\omega_{mN} = 679$ rpm.

Determine:

a. R_1 using the results of the DC test.

b. Number of pairs of poles (p).

c. Slip factor (σ) at full load.

d. Mechanical angular frequency (ω_m) for a load 25% of full load.

e. Electrical frequency in the rotor (ω_2) for a load 25% of full load.

Solution of Problem 1

a. The slip factor which for which the torque is maximum is:

$$\sigma_M = \frac{R_2^{'}}{\sqrt{R_1^2 + X_{sc}^{'2}}}$$

where $X'_{sc} = X_1 + X'_2$ and $X_1 = 1.8$. Now, lets find out X'_2.

$$\bar{Z}'_2 = R'_2 + jX'_2 = k_T^2 \bar{Z}_2 = (0.8 + j2)\ \Omega$$

So, $X'_{sc} = 1.8 + 2 = 3.8$. Then, we obtain:

$$\sigma^* = 0.21 \quad (\mathbf{21\%})$$

The torque equation is

$$T_m = \frac{3p}{\omega_1} \frac{E_1^2}{(R_1 + \frac{R'_2}{\sigma})^2 + {X'_{sc}}^2} \frac{R'_2}{\sigma}$$

where $E_1 = 380/\sqrt{3}$. The nominal angular speed of the stator is $\omega_1 = 2\pi f = 2 \cdot \pi \cdot 50 = 314.16$ rad/s, or $\omega_1 = 3000$ rpm. As, $p = 4/2 = 2$, $\omega_{s1} = 1500$ rpm. Moreover, $\omega_m^* = (1-\sigma^*)1500 = 1185$ rpm. At maximum torque, the slip is known ($\sigma^* = 0.21$). Then, one has:

$$\begin{aligned} T_m^{\text{max}} &= \frac{3 \cdot 2}{314.16} \frac{E_1^2}{(R_1 + \frac{R'_2}{\sigma^*})^2 + {X'_{sc}}^2} \frac{R'_2}{\sigma^*} \\ &= \frac{3 \cdot 2}{314.16} \frac{(380/\sqrt{3})^2}{(0.6 + \frac{0.8}{0.21})^2 + (3.8)^2} \frac{0.8}{0.21} \\ &= \mathbf{103.4}\ \text{Nm} \end{aligned}$$

b. At start up, the slip, $\sigma = 1$. Using the torque equation,

$$\begin{aligned} T_m^{\text{su}} &= \frac{3p}{\omega_1} \frac{E_1^2}{(R_1 + R'_2)^2 + {X'_{sc}}^2} R'_2 \\ &= \mathbf{44.84}\ \text{Nm} \end{aligned}$$

c. We have to impose $R'_2 + R'_{\text{ext}} = \sqrt{R_1^2 + {X'_{sc}}^2}$. Hence,

$$\begin{aligned} R'_{\text{ext}} &= \sqrt{R_1^2 + {X'_{sc}}^2} - R'_2 \\ &= 3.05\ \Omega \end{aligned}$$

The actual resistance to be connected is:

$$\begin{aligned} R_{\text{ext}} &= \frac{1}{k_T^2} R'_{\text{ext}} \\ &= \mathbf{0.76}\ \Omega \end{aligned}$$

d. In case of squirrel-cage machine, one cannot connect any resistance, so it is impossible to calculate.

Solution of Problem 2

a. We need to compute the slip factor at full load. At full load the primary (stator) synchronous speed:

$$\omega_{s1} = \frac{60 \cdot f}{p} = \frac{53 \text{ Hz} \cdot 60}{2}$$
$$= \frac{3,180}{2} = 1,590 \text{ rpm}$$

Slip factor:

$$\sigma_N = \frac{1,590 - 1,500}{1,590}$$
$$= 0.0566 \ (\mathbf{5.66\%})$$

Torque:

$$T_{mN} = \frac{3 \cdot 2}{2 \cdot \pi \cdot f} \frac{E_1^2}{(R_1 + \frac{R_2'}{\sigma_N})^2 + {X_{sc}'}^2} \frac{R_2'}{\sigma_N}$$
$$= \frac{3.2}{333.0} \frac{(400/\sqrt{3})^2}{(0.125 + \frac{0.125}{0.0566})^2 + (0.5)^2} \frac{0.125}{0.0566}$$
$$= \mathbf{372.7} \text{ Nm}$$

b. Mechanical power output:

$$P_{m,\text{net}} = P_m - P_{m,\text{losses}} = T_m \cdot \omega_{mN} - P_{m,\text{losses}}$$
$$= (372.7) \cdot (157.07) - 900 = 58,536.6 - 900 = \mathbf{57636.6} \text{ W}$$

Mechanical torque output:

$$T_{m,\text{net}} = \frac{P_{m,\text{net}}}{\omega_{mN}}$$
$$= \frac{57,636.6}{157.07} = \mathbf{366.9} \text{ Nm}$$

c. Rotor current magnitude:

$$I_{2N}' = \frac{E_1}{\sqrt{(R_1 + \frac{R_2'}{\sigma_N})^2 + {X_{sc}'}^2}}$$

Stator current:

$$\bar{I}_{1N} = \bar{I}_{2N}' + \bar{I}_{10}$$
$$= \frac{\bar{E}_1}{jX_\mu} + \frac{\bar{E}_1}{R_{\text{Fe}}} + \frac{\bar{E}_1}{(R_1 + \frac{R_2'}{\sigma_N}) + jX_{sc}'}$$

where, $\bar{E}_1 = \frac{400}{\sqrt{3}}\angle 0$ V, $R_{\text{Fe}} = E_1^2/P_{\text{Fe}}$, and $X_\mu = E_1/I_\mu$.

Magnitude of the current and power factor,

$$\begin{aligned} I_1 &= \mathbf{103.9}\text{ A} \\ \cos(\phi) &= \cos(\angle\bar{I}_1) \\ &= \mathbf{0.9297}\text{ (lagging)} \end{aligned}$$

d. Electrical power input:

$$\begin{aligned} P_1 &= 3E_1I_1\cos(\phi_1) \\ &= 3\frac{400}{\sqrt{3}}(I_1)(0.9297) \\ &= 66,911.0 \end{aligned}$$

Efficiency:

$$\begin{aligned} \eta &= \frac{P_{m,\text{net}}}{P_1} \\ &= \frac{57,636.6}{66,911.0}\cdot 100\% \\ &= \mathbf{86.14}\ \% \end{aligned}$$

Solution of Problem 3

a. The equivalent circuit of the dc test is shown in Figure 1. This leads to:

$$\frac{V_{\text{dc}}}{I_{\text{dc}}} = R_{\text{eq}} = R_1 \| 2R_1 = \frac{2R_1^2}{3R_1} = \frac{2}{3}R_1$$

Then

$$R_1 = \frac{3}{2}\frac{V_{dc}}{I_{dc}} = \frac{3}{2}\cdot\frac{21}{72} = \mathbf{0.438}\ \Omega$$

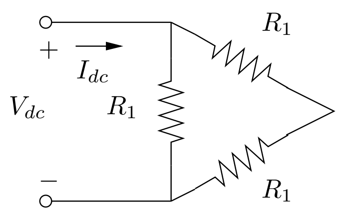

Figure 1

b. Let's consider different values of p:

$$\begin{aligned}
&p = 1, \omega_{s1} = 3600 \text{ rpm } \rightarrow \text{No} \\
&p = 2, \omega_{s1} = 1800 \text{ rpm } \rightarrow \text{No} \\
&p = 3, \omega_{s1} = 1200 \text{ rpm } \rightarrow \text{No} \\
&p = 4, \omega_{s1} = 900 \text{ rpm } \rightarrow \text{No} \\
&p = 5, \omega_{s1} = \mathbf{720} \text{ rpm } \rightarrow \text{Ok} \\
&p = 6, \omega_{s1} = 600 \text{ rpm } < \omega_{\text{mN}} \rightarrow \text{No}
\end{aligned}$$

c. Slip factor at full load:

$$\begin{aligned}
\sigma_N &= \frac{\omega_{s1} - \omega_{mN}}{\omega_{s1}} \\
&= \frac{720 - 679}{720} = \mathbf{0.0569}
\end{aligned}$$

d. At 25% full load, the torque/speed characteristic is almost linear (see Figure 2).

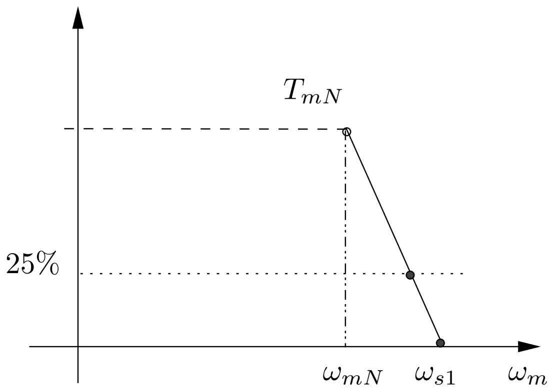

Figure 2

hence:

$$\begin{aligned}
\sigma(25\%) &\cong \frac{1}{4} 0.0569 \\
&= \mathbf{0.0142}
\end{aligned}$$

and the mechanical angular frequency is:

$$\omega_m = (1 - \sigma(25\%)) \cdot \omega_{s1} \tag{1}$$

e. The electric frequency corresponding to a 25% load is:

$$\omega_2 = p \cdot \sigma(25\%) \cdot \omega_{s1}$$
$$f_2 = \sigma f_1 = 0.0142 \cdot 60 = \mathbf{0.852}\ \text{Hz}$$