

# Worked Problems on Induction Machines

EEEN20090 – Electrical Energy Systems

## Problem 1

A 4-pole, Y-connected, 380 V, 50 Hz, wound-rotor induction machine has the following equivalent circuit parameters:

$$\bar{Z}_1 = (0.6 + j1.8) \Omega/\text{phase} = R_1 + jX_1 \text{ (stator)}$$

$$\bar{Z}_2 = (0.2 + j0.5) \Omega/\text{phase} = R_2 + jX_2 \text{ (rotor)}$$

The stator/rotor tap ratio is  $k_T = 2$ .

- Determine the slip factor ( $\sigma^*$ ) for which the torque is maximum and the maximum torque ( $T_m^{\max}$ ).
- Determine the starting torque ( $T_m^{\text{su}}$ ).
- Determine the external resistance ( $R_{\text{ext}}$ ) to connect to the rotor to have  $\tilde{T}_m^{\text{su}} = T_m^{\max}$ .
- How to calculate (c) if the induction machine is squirrel-cage type?

## Problem 2

A 4-pole, Y-connected, 400 V, 53 Hz induction machine has the following data:

$$\bar{Z}_1 = (0.125 + j0.3) \Omega$$

$$\bar{Z}'_2 = (0.125 + j0.2) \Omega$$

$$I_\mu = 18 \text{ A}$$

$$P_{\text{Fe}} = 1350 \text{ W}$$

$$P_{m,\text{losses}} = 900 \text{ W}$$

$$\omega_N = 1500 \text{ rpm.}$$

At full load, determine:

- Torque ( $T_{mN}$ ).
- Mechanical power ( $P_m$ ) and torque ( $T_m$ ) output.
- Rotor current ( $I_N$ ) and power factor ( $\cos(\phi)$ ).
- Efficiency ( $\eta$ ).

### Problem 3

A 60 Hz,  $\Delta$  connected induction machine has the following results.

**DC Test:**  $V_{dc} = 21$  V,  $I_{dc} = 72$  A.

**Open Circuit:**  $\omega_m = 715$  rpm.

**Full Load:**  $\omega_{mN} = 679$  rpm.

Determine:

- a.  $R_1$  using the results of the DC test.
- b. Number of pairs of poles ( $p$ ).
- c. Slip factor ( $\sigma$ ) at full load.
- d. Mechanical angular frequency ( $\omega_m$ ) for a load 25% of full load.
- e. Electrical frequency in the rotor ( $\omega_2$ ) for a load 25% of full load.

## Solution of Problem 1

- a. The slip factor which for which the torque is maximum is:

$$\sigma_M = \frac{R'_2}{\sqrt{R_1^2 + X'_{sc}{}^2}}$$

where  $X'_{sc} = X_1 + X'_2$  and  $X_1 = 1.8$ . Now, lets find out  $X'_2$ .

$$\bar{Z}'_2 = R'_2 + jX'_2 = k_T^2 \bar{Z}_2 = (0.8 + j2) \Omega$$

So,  $X'_{sc} = 1.8 + 2 = 3.8$ . Then, we obtain:

$$\sigma^* = 0.21 \quad (21\%)$$

The torque equation is

$$T_m = \frac{3p}{\omega_1} \frac{E_1^2}{(R_1 + \frac{R'_2}{\sigma})^2 + X'_{sc}{}^2} \frac{R'_2}{\sigma}$$

where  $E_1 = 380/\sqrt{3}$ . The nominal angular speed of the stator is  $\omega_1 = 2\pi f = 2 \cdot \pi \cdot 50 = 314.16$  rad/s, or  $\omega_1 = 3000$  rpm. As,  $p = 4/2 = 2$ ,  $\omega_{s1} = 1500$  rpm. Moreover,  $\omega_m^* = (1 - \sigma^*)1500 = 1185$  rpm. At maximum torque, the slip is known ( $\sigma^* = 0.21$ ). Then, one has:

$$\begin{aligned} T_m^{\max} &= \frac{3 \cdot 2}{314.16} \frac{E_1^2}{(R_1 + \frac{R'_2}{\sigma^*})^2 + X'_{sc}{}^2} \frac{R'_2}{\sigma^*} \\ &= \frac{3 \cdot 2}{314.16} \frac{(380/\sqrt{3})^2}{(0.6 + \frac{0.8}{0.21})^2 + (3.8)^2} \frac{0.8}{0.21} \\ &= \mathbf{103.4 \text{ Nm}} \end{aligned}$$

- b. At start up, the slip,  $\sigma = 1$ . Using the torque equation,

$$\begin{aligned} T_m^{\text{su}} &= \frac{3p}{\omega_1} \frac{E_1^2}{(R_1 + R'_2)^2 + X'_{sc}{}^2} R'_2 \\ &= \mathbf{44.84 \text{ Nm}} \end{aligned}$$

- c. We have to impose  $R'_2 + R'_{\text{ext}} = \sqrt{R_1^2 + X'_{sc}{}^2}$ . Hence,

$$\begin{aligned} R'_{\text{ext}} &= \sqrt{R_1^2 + X'_{sc}{}^2} - R'_2 \\ &= \mathbf{3.05 \Omega} \end{aligned}$$

The actual resistance to be connected is:

$$\begin{aligned} R_{\text{ext}} &= \frac{1}{k_T^2} R'_{\text{ext}} \\ &= \mathbf{0.76 \Omega} \end{aligned}$$

- d. In case of squirrel-cage machine, one cannot connect any resistance, so it is impossible to calculate.

## Solution of Problem 2

- a. We need to compute the slip factor at full load. At full load the primary (stator) synchronous speed:

$$\begin{aligned}\omega_{s1} &= \frac{60 \cdot f}{p} = \frac{53 \text{ Hz} \cdot 60}{2} \\ &= \frac{3,180}{2} = 1,590 \text{ rpm}\end{aligned}$$

Slip factor:

$$\begin{aligned}\sigma_N &= \frac{1,590 - 1,500}{1,590} \\ &= 0.0566 \text{ (5.66\%)}\end{aligned}$$

Torque:

$$\begin{aligned}T_{mN} &= \frac{3 \cdot 2}{2 \cdot \pi \cdot f} \frac{E_1^2}{(R_1 + \frac{R_2'}{\sigma_N})^2 + X_{sc}'^2} \frac{R_2'}{\sigma_N} \\ &= \frac{3.2}{333.0} \frac{(400/\sqrt{3})^2}{(0.125 + \frac{0.125}{0.0566})^2 + (0.5)^2} \frac{0.125}{0.0566} \\ &= \mathbf{372.7 \text{ Nm}}\end{aligned}$$

- b. Mechanical power output:

$$\begin{aligned}P_{m,\text{net}} &= P_m - P_{m,\text{losses}} = T_m \cdot \omega_{mN} - P_{m,\text{losses}} \\ &= (372.7) \cdot (157.07) - 900 = 58,536.6 - 900 = \mathbf{57636.6 \text{ W}}\end{aligned}$$

Mechanical torque output:

$$\begin{aligned}T_{m,\text{net}} &= \frac{P_{m,\text{net}}}{\omega_{mN}} \\ &= \frac{57,636.6}{157.07} = \mathbf{366.9 \text{ Nm}}\end{aligned}$$

- c. Rotor current magnitude:

$$I_{2N}' = \frac{E_1^2}{\sqrt{(R_1 + \frac{R_2'}{\sigma_N})^2 + X_{sc}'^2}}$$

Stator current:

$$\begin{aligned}\bar{I}_{1N} &= \bar{I}_{2N}' + \bar{I}_{10} \\ &= \frac{\bar{E}_1}{jX_\mu} + \frac{\bar{E}_1}{R_{Fe}} + \frac{\bar{E}_1}{(R_1 + \frac{R_2'}{\sigma_N}) + jX_{sc}'}\end{aligned}$$

where,  $\bar{E}_1 = \frac{400}{\sqrt{3}} \angle 0$  V.

Magnitude of the current and power factor,

$$\begin{aligned} I_1 &= \mathbf{103.9} \text{ A} \\ \cos(\phi) &= \cos(\angle \bar{I}_1) \\ &= \mathbf{0.9297} \text{ (lagging)} \end{aligned}$$

d. Electrical power input:

$$\begin{aligned} P_1 &= 3E_1 I_1 \cos(\phi_1) \\ &= 3 \frac{400}{\sqrt{3}} (I_1) (0.9297) \\ &= 66,911.0 \end{aligned}$$

Efficiency:

$$\begin{aligned} \eta &= \frac{P_{m,\text{net}}}{P_1} \\ &= \frac{57,636.6}{66,911.0} \cdot 100\% \\ &= \mathbf{86.14} \% \end{aligned}$$

### Solution of Problem 3

a. The equivalent circuit of the dc test is shown in Figure 1. This leads to:

$$\frac{V_{dc}}{I_{dc}} = R_{\text{eq}} = R_1 \parallel 2R_1 = \frac{2R_1^2}{3R_1} = \frac{2}{3} R_1$$

Then

$$R_1 = \frac{3 V_{dc}}{2 I_{dc}} = \frac{3}{2} \cdot \frac{21}{72} = \mathbf{0.438} \ \Omega$$

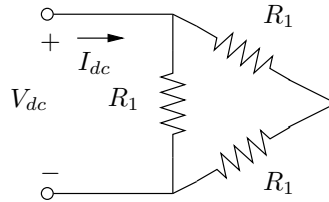


Figure 1

b. Let's consider different values of  $p$ :

$$\begin{aligned}
 p = 1, \omega_{s1} = 3600 \text{ rpm} &\rightarrow \text{No} \\
 p = 2, \omega_{s1} = 1800 \text{ rpm} &\rightarrow \text{No} \\
 p = 3, \omega_{s1} = 1200 \text{ rpm} &\rightarrow \text{No} \\
 p = 4, \omega_{s1} = 900 \text{ rpm} &\rightarrow \text{No} \\
 p = 5, \omega_{s1} = \mathbf{720} \text{ rpm} &\rightarrow \text{Ok} \\
 p = 6, \omega_{s1} = 600 \text{ rpm} &< \omega_{mN} \rightarrow \text{No}
 \end{aligned}$$

c. Slip factor at full load:

$$\begin{aligned}
 \sigma_N &= \frac{\omega_{s1} - \omega_{mN}}{\omega_{s1}} \\
 &= \frac{720 - 679}{720} = \mathbf{0.0569}
 \end{aligned}$$

d. At 25% full load, the torque/speed characteristic is almost linear (see Figure 2).

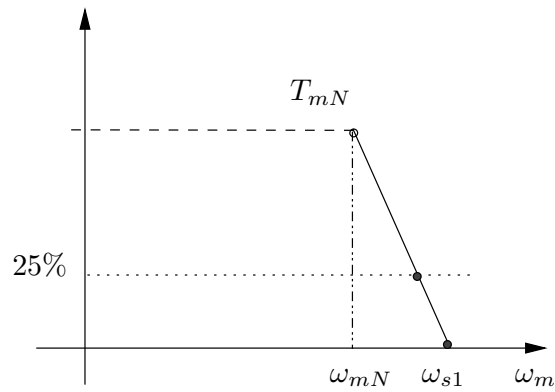


Figure 2

hence:

$$\begin{aligned}
 \sigma(25\%) &\cong \frac{1}{4} 0.0569 \\
 &= \mathbf{0.0142}
 \end{aligned}$$

e. The electric frequency corresponding to a 25% load is:

$$\begin{aligned}
 \omega_2 &= p \cdot \sigma(25\%) \cdot \omega_{s1} \\
 f_2 &= \sigma f_1 = 0.0142 \cdot 60 = \mathbf{0.852} \text{ Hz}
 \end{aligned}$$