Worked Problems on Magnetic Circuits

EEEN20090 – Electric Energy Systems

Problem 1

Figure 1 shows an iron core composed of an external magnetic circuit with cross section S_1 and a N_1-turn coil, and a shunt branch with cross section S_2 and a N_2-turn coil. The external magnetic circuit has an air gap of length ℓ_4. The data of the magnetic circuit are as follows.

$\ell_{11} + \ell_{12} + \ell_{13} = 45$ cm	$\ell_4 = 0.1$ cm	$S_1 = 150$ cm^2
$\ell_2 = 15$ cm	$\ell_{51} + \ell_{52} = 22.45$ cm	$S_2 = 250$ cm^2
$\ell_{31} + \ell_{32} = 22.45$ cm	$N_1 = 200$	$N_2 = 150$

Determine:

a. The current I_2 that leads to a 0.5 T magnetic induction in the air gap, assuming $I_1 = 2$ A and a relative permeability of the iron core $\mu_r = 5000$.

b. The current I_1 that leads to a 0.15 T magnetic induction in the air gap, assuming $I_2 = 0.8$ A and the magnetization curve shown in Figure 2.

Problem 2

Figure 3 shows the magnetic circuit of a rotaing machine with 2 stator poles. The rotor is a 200 mm long cylinder. The stator has two poles whose width and length are 40 mm and 200 mm, respectively. A coil with 360 turns is wound on each pole. The same current flows in the two coils. Both air gaps are 1.5 mm long.

a. Determine the reluctance of each air gap.

b. Neglecting the reluctance of the iron core, determine the current required in the coils to obtain in the air gaps a magnetic induction equal to 0.8 T.

c. Repeat the previous question, considering the reluctances of the air gaps and of the stator, but neglecting the reluctances of the poles and the rotor. The average radius of the stator is 200 mm and its cross section is $4,000$ mm^2 and the relative permeability of the iron core is $4,000$.

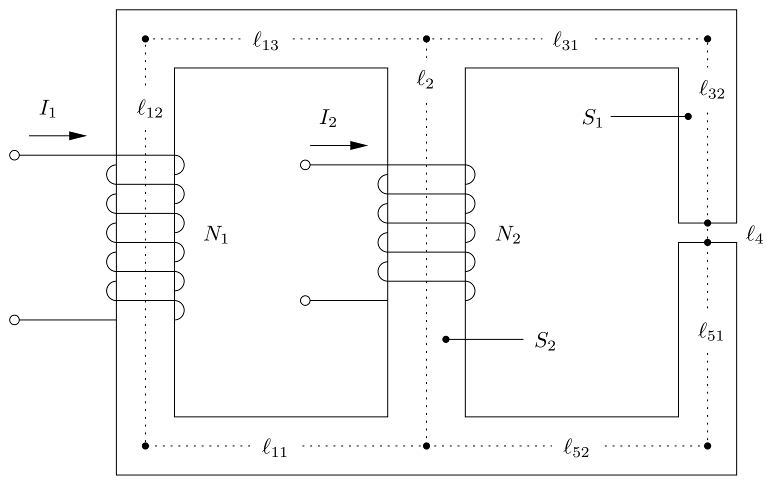

Figure 1

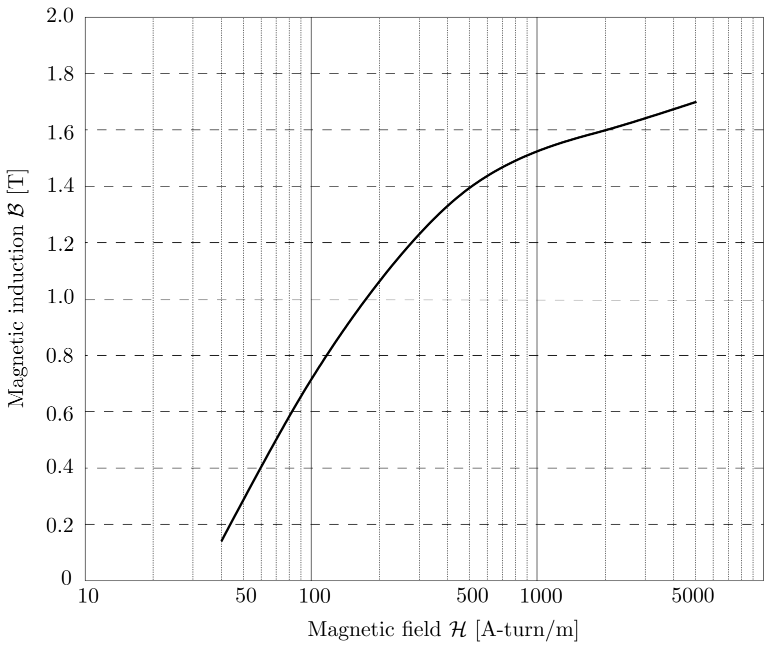

Figure 2

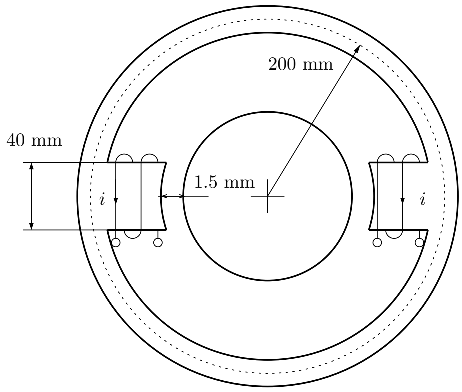

Figure 3

Problem 3

The magnetic circuit shown in Figure 4 has a constant cross section $A = 16$ cm^2, average length $\ell = 70$ cm, and $\alpha = 45°$. The magnetic induction and magnetic field in the iron core are linked by the following expression:

$$B = \frac{2.2H}{215 + H} \tag{1}$$

where B is in T and H in A-turn/m. The coil has $N = 600$ turns and its current is 2.5 A. Determine the length of the air gap g that leads to a magnetic flux equal to $\phi = 0.8$ mWb. Determine also the density of magnetic energy in the iron core and in the air gap.

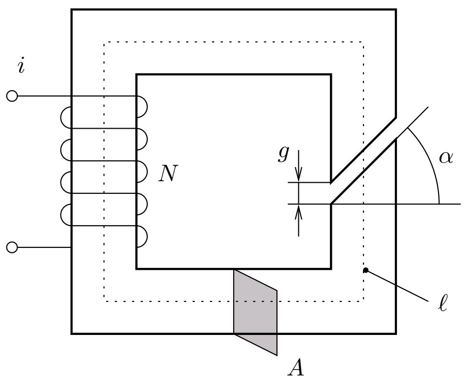

Figure 4

Problem 4

Figure 5 shows an iron core whose relative permeability is 2,000. The dimensions of the core are shown in the figure. The width of the core is 7 cm. The lengths of the air gaps are 0.05 cm and 0.07 cm, respectively. The cross section in the air gaps is 5% bigger than that of the iron core. The coil has 300 turns and its current is 1.0 A. Determine the magnetic flux in each column of the iron core and the magnetic induction in the air gaps.

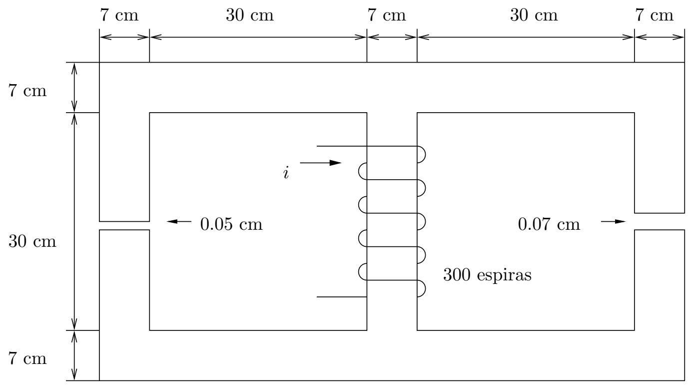

Figure 5

Solution of Problem 1

a. The reluctances of the magnetic circuit are (see Figure 6):

$$\mathcal{R}_1 = \frac{\ell_{11}+\ell_{12}+\ell_{13}}{\mu_0\mu S_1} = 4775 \text{ H}^{-1} \tag{2}$$

$$\mathcal{R}_2 = \frac{\ell_2}{\mu_0\mu S_2} = 955 \text{ H}^{-1} \tag{3}$$

$$\mathcal{R}_4 = \frac{\ell_4}{\mu_0 S_1} = 53052 \text{ H}^{-1} \tag{4}$$

$$\mathcal{R}_3 = \mathcal{R}_5 = \frac{\ell_{31}+\ell_{32}}{\mu_0\mu S_1} = 2382 \text{ H}^{-1} \tag{5}$$

Then:

$$\phi_3 = \mathcal{B}_3 S_1 = 0.5 \cdot 0.015 = 0.0075 \text{ Wb} \tag{6}$$

$$\Delta U = (\mathcal{R}_3+\mathcal{R}_4+\mathcal{R}_5)\phi_3 = 433.62 \text{ A-turn} \tag{7}$$

$$\Delta U = N_1 I_1 - \mathcal{R}_1\phi_1 = N_2 I_2 - \mathcal{R}_2\phi_2 \tag{8}$$

$$\phi_3 = \phi_1 + \phi_2 \tag{9}$$

$$\phi_1 = \frac{N_1 I_1 - \Delta U}{\mathcal{R}_1} = -0.007 \text{ Wb} \tag{10}$$

$$I_2 = \frac{\Delta U + \mathcal{R}_2(\phi_3-\phi_1)}{N_2} = 2.98 \text{ A} \tag{11}$$

b. Due to nonliearity, one cannot define the reluctances.

$$\mathcal{B}_3 = 0.15 \text{ T} \quad \Rightarrow \quad \mathcal{H}_3 = 40 \text{ A-turn/m} \tag{12}$$

$$\mathcal{H}_2 = \frac{N_2 I_2 - \mathcal{H}_3(\ell_{31}+\ell_{32}+\ell_{51}+\ell_{52}) - \mathcal{H}_4\ell_4}{\ell_2} \tag{13}$$

$$= -115.5 \text{ A-turn/m} \tag{14}$$

$$\mathcal{H}_2 = -115.5 \text{ A-turn/m} \quad \Rightarrow \quad \mathcal{B}_2 = -0.8 \text{ T} \tag{15}$$

$$\mathcal{B}_1 = \frac{\mathcal{B}_3 S_1 - \mathcal{B}_2 S_2}{S_1} = 1.48 \text{ T} \tag{16}$$

$$\mathcal{F}_1 = \mathcal{H}_1(\ell_{11}+\ell_{12}+\ell_{13}) + N_2 I_2 - \mathcal{H}_2\ell_2 = 542 \text{ A-turn} \tag{17}$$

$$I_1 = \frac{\mathcal{F}_1}{N_1} = 2.71 \text{ A} \tag{18}$$

Solution of Problem 2

a. The cross section of the air gap is:

$$A_g = 0.04 \cdot 0.2 = 8 \cdot 10^{-3} \text{ m}^2 \tag{19}$$

The reluctance of each air gap is:

$$\mathcal{R}_g = \frac{g}{\mu_0 A_g} = \frac{0.0015}{(4\pi 10^{-7})(8\cdot 10^{-3)}} = 149207.8 \text{ A-turn/Wb} \tag{20}$$

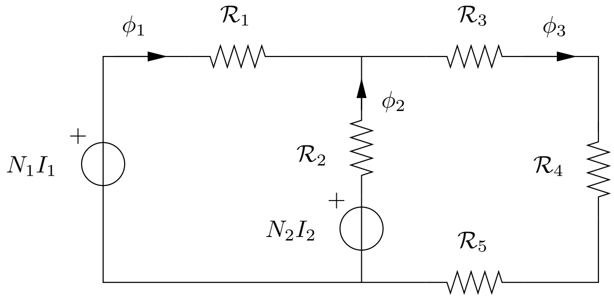

Figure 6

b. The magnetic flux that flows in the air gap is:

$$\phi = BA_g = (0.8)(8 \cdot 10^{-3}) = 6.4 \cdot 10^{-3} \text{ Wb} \tag{21}$$

Neglecting the reluctances on the iron core, the equivalent electrical circuit consists of two mmfs in series with the reluctances of the two air gaps (see Figure 7).

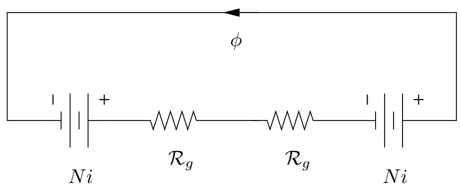

Figure 7

Applying the KVL the the circuit shown in Figure 7, one obtains:

$$2Ni = 2\phi\mathcal{R}_g \tag{22}$$

$$i = \frac{2(6.4 \cdot 10^{-3})(149207.8)}{2 \cdot 360} = 2.65 \text{ A} \tag{23}$$

c. The reluctance of each half of the stator is:

$$\mathcal{R}_e = \frac{l_e}{\mu_0 \mu_r A_e} = \frac{\pi 0.2}{(4\pi 10^{-7})(4000)(4000 \cdot 10^{-6)}} = 31250 \text{ A-turn/Wb} \tag{24}$$

Applying the KVL to the upper mesh of the circuit of Figure 8, one obtains:

$$2Ni = 2\phi\mathcal{R}_g + \mathcal{R}_e\frac{\phi}{2} \tag{25}$$

$$i = \frac{2(6.4 \cdot 10^{-3})(149207.8)}{2 \cdot 360} + \frac{(3.2 \cdot 10^{-3})(31250)}{2 \cdot 360} = 2.79 \text{ A} \tag{26}$$

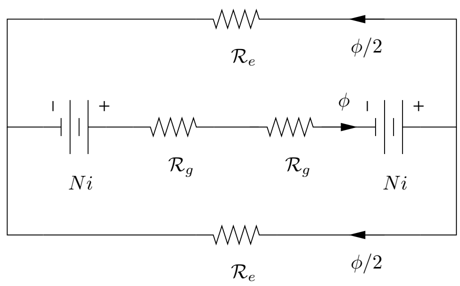

Figure 8

Solution of Problem 3

From Ampere's law, one obtains:

$$Ni = H_\ell(\ell - g) + H_g g \tag{27}$$

where H_ℓ is the magnetic field in the iron core and H_g is the magnetic field in the air gap. In the air gap:

$$H_g = \frac{B}{\mu_0} \tag{28}$$

where B is the magnetic induction of the magnetic circuit. Considering that $\phi = \int \vec{B} \cdot d\vec{S} = BA$, the magnetic induction does not depend on the angle α. Hence:

$$B = \frac{\phi}{A} = \frac{0.8 \cdot 10^{-3}}{16 \cdot 10^{-4}} = 0.5 \text{ T} \tag{29}$$

The magnetic field in the iron core is:

$$H_\ell = \frac{215B}{2.2 - B} = 63.235 \text{ A-turn/m} \tag{30}$$

Hence, the remaining unknwon in equation (27) is g:

$$g = \frac{Ni - H_\ell \ell}{H_g - H_\ell} = \frac{1\,500 - 7.35 \cdot 0.7}{397\,887 - 7.35} = 3.76 \text{ mm} \tag{31}$$

Finally, the density of the magnetic energy w_{mag} as a function of the flux density field B can be obtained as follows:

- In the airgap:

$$w_{\text{mag}}(B) = \int_0^B \frac{B}{\mu_0} dB = \frac{1}{2} \frac{B^2}{\mu_0} \tag{32}$$

Hence, $w_{\text{mag}}(0.5) = 9.95 \cdot 10^4 \text{ J/m}^3$.

- In the air gap:

$$w_{\text{mag}}(B) = \int_0^B \frac{215B}{2.2 - B} dB = -215B - 473 \log\left(\frac{B - 2.2}{-2.2}\right) \quad (33)$$

Hence, $w_{\text{mag}}(0.5) = 14.5$ J/m^3. Note that log in the equation above indicates the *natural* logarithm, i.e., the logarithm with base e.

Solution of Problem 4

Let's define the following reluctances:

$$\begin{aligned}
\mathcal{R}_1 &= \frac{\ell_1}{\mu_r \mu_0 A_1} = \frac{1.11}{2000 \cdot 4\pi \cdot 10^{-7} \cdot 0.07 \cdot 0.07} = 90.1 \text{ kA-turn/Wb} && (34)\\
\mathcal{R}_2 &= \frac{\ell_2}{\mu_0 A_2} = \frac{0.0005}{4\pi \cdot 10^{-7} \cdot 0.07 \cdot 0.07 \cdot 1.05} = 77.3 \text{ kA-turn/Wb} && (35)\\
\mathcal{R}_3 &= \frac{\ell_3}{\mu_r \mu_0 A_3} = \frac{1.11}{2000 \cdot 4\pi \cdot 10^{-7} \cdot 0.07 \cdot 0.07} = 90.1 \text{ kA-turn/Wb} && (36)\\
\mathcal{R}_4 &= \frac{\ell_4}{\mu_0 A_4} = \frac{0.0007}{4\pi \cdot 10^{-7} \cdot 0.07 \cdot 0.07 \cdot 1.05} = 108.3 \text{ kA-turn/Wb} && (37)\\
\mathcal{R}_5 &= \frac{\ell_5}{\mu_r \mu_0 A_5} = \frac{0.37}{2000 \cdot 4\pi \cdot 10^{-7} \cdot 0.07 \cdot 0.07} = 30.0 \text{ kA-turn/Wb} && (38)
\end{aligned}$$

The total reluctance is:

$$\begin{aligned}
\mathcal{R}_{\text{TOT}} &= \mathcal{R}_5 + \frac{(\mathcal{R}_1 + \mathcal{R}_2)(\mathcal{R}_3 + \mathcal{R}_4)}{\mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3 + \mathcal{R}_4} && (39)\\
&= 30.0 + \frac{(90.1 + 77.3)(90.1 + 108.3)}{90.1 + 77.3 + 90.1 + 108.3} = 120.8 \text{ kA-turn/Wb} && (40)
\end{aligned}$$

The total flux in the iron core is equal to the flux in the central column:

$$\phi_{\text{center}} = \phi_{\text{TOT}} = \frac{\mathcal{F}}{\mathcal{R}_{\text{TOT}}} = \frac{300 \cdot 1.0}{120.8} = 0.00248 \text{ Wb} \quad (41)$$

Using the current divider principle, one obtains the fluxes in the other columns:

$$\begin{aligned}
\phi_{\text{left}} &= \frac{\mathcal{R}_3 + \mathcal{R}_4}{\mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3 + \mathcal{R}_4} \phi_{\text{TOT}} = 0.00135 \text{ Wb} && (42)\\
\phi_{\text{right}} &= \frac{\mathcal{R}_1 + \mathcal{R}_2}{\mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3 + \mathcal{R}_4} \phi_{\text{TOT}} = 0.00113 \text{ Wb} && (43)
\end{aligned}$$

The magnitues of the density flux fields in the air gaps are:

$$\begin{aligned}
B_{\text{left}} &= \frac{\phi_{\text{left}}}{A_2} = \frac{0.00135}{0.07 \cdot 0.07 \cdot 1.05} = 0.262 \text{ T} && (44)\\
B_{\text{right}} &= \frac{\phi_{\text{right}}}{A_4} = \frac{0.00113}{0.07 \cdot 0.07 \cdot 1.05} = 0.220 \text{ T} && (45)
\end{aligned}$$