Per-unit System

ELECTRICAL ENERGY SYSTEMS (EEEN20090)

Prof. Federico Milano

Email: federico.milano@ucd.ie

Tel.: 01 716 1844

Room 157a – Engineering and Materials Science Centre

School of Electrical & Electronic Engineering

University College Dublin

Dublin, Ireland

Per-unit System - I

- The per-unit system is widely used in power system analysis and simulation.
- It consists of a set of **BASES** with respect to which actual quantities are referred:

$$\text{per-unit value} = \frac{\text{actual value}}{\text{base value}}$$

- We need, thus, to choose a meaningful and complete set of base values for the electrical system.

Per-unit System- II

- We just need 3 quantities:
  - $S_{\text{base}} = S_b$ [MVA]
  - $V_{\text{base}} = V_b$ [kV]
  - $f_{\text{base}} = f_b$ [kV]
- All other bases can be obtained from the quantities above.

Per-unit System - III

- Current base:
  - Single-phase system:

$$I_b = \frac{S_b}{V_b} \quad \text{[kA]}$$

  - Three-phase system:

$$I_b = \frac{S_b}{\sqrt{3}V_b} \quad \text{[kA]}$$

- Impedance base:
  - Both single and three-phase systems:

$$Z_b = \frac{V_b}{I_b^2} \quad [\Omega]$$

Example 1

- Let's consider the following data:
  - $S_b = 100$ MVA
  - $V_b = 10$ kV
  - $x = 0.8$ pu with respect to S_b and V_b.
- What is the absolute value of the reactance in Ω?

- Solution:

$$X = x \cdot X_b = x \cdot \frac{V_b^2}{S_b} = 0.8 \cdot \frac{10^2}{100} = 0.8\ \Omega$$

Example 2

- Let's consider a transformer with following parameters and nominal values:

$$S_N = 700 \text{ MVA}, \quad V_{N1}/V_{N2} = 10\text{kV}/100\text{KV} \ , \quad V_{sc\%} = 10\%$$

- Determine the short-circuit impedance reduced to the primary and the secondary windings:

$$Z_{sc(1)} = V_{sc\%} \cdot \frac{V_{N1}^2}{S_N} = 0.1 \cdot \frac{10^2}{700} = 0.0143 \ \Omega$$

$$Z_{sc(2)} = V_{sc\%} \cdot \frac{V_{N2}^2}{S_N} = 0.1 \cdot \frac{100^2}{700} = 1.43 \ \Omega$$

- Note that the per-unit value of the short-circuit impedance of the transformer is $\frac{1}{100} V_{sc\%}$ if we choose as bases the nominal quantities of the transformer.

Example 2

- Note also that the per-unit value of the impedance is the same independently from the side of the transformer.
- In fact, let:
  - $Z_{b1} = \frac{V_{b1}^2}{S_b} = \frac{V_{N1}^2}{S_N}$
  - $Z_{b2} = \frac{V_{b2}^2}{S_b} = \frac{V_{N2}^2}{S_N}$
  - $z_{pu(1)} =$ per unit value reduced to the primary winding
  - $z_{pu(2)} =$ per unit value reduced to the secondary winding

Example 2

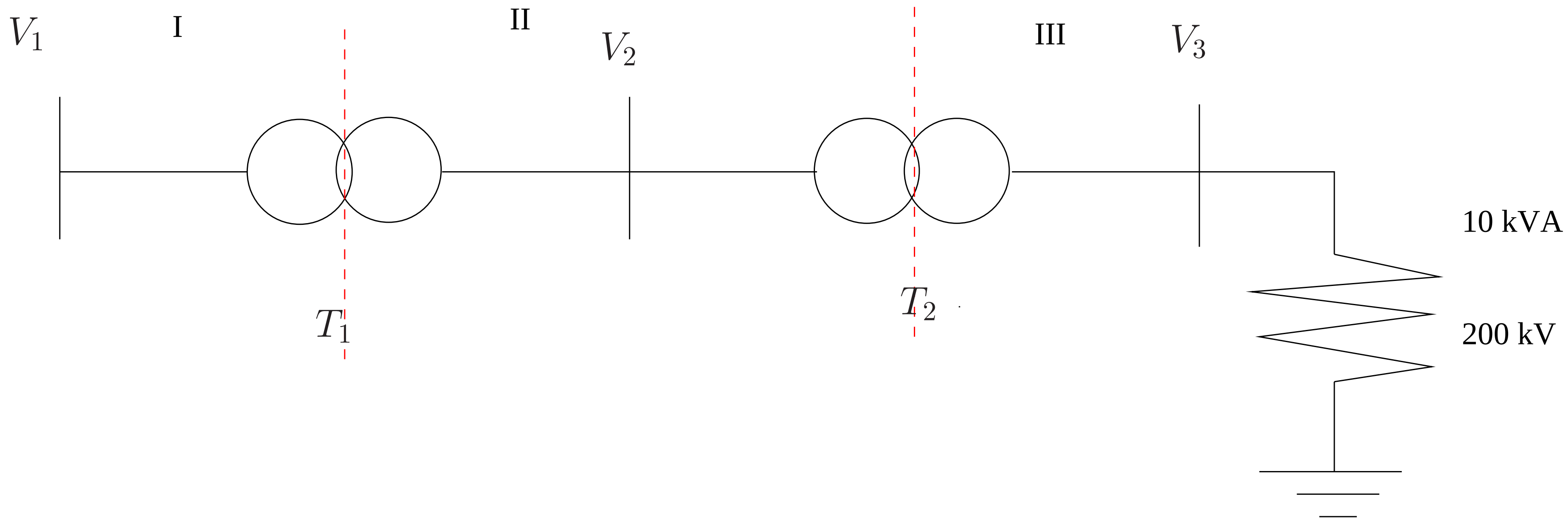
- Then:

$$
\begin{aligned}
Z_{sc(1)} &= Z_{sc(2)} \frac{N_1^2}{N_2^2} \\
\Rightarrow \quad z_{pu(1)} &= Z_{sc(1)} \frac{1}{Z_{b1}} = Z_{sc(1)} \frac{S_b}{V_{b1}^2} \\
&= Z_{sc(2)} \frac{N_1^2}{N_2^2} \frac{S_b}{V_{b1}^2} = Z_{sc(2)} \frac{S_b}{V_{b1}^2 \frac{N_1^2}{N_2^2}} \\
&= Z_{sc(2)} \frac{S_b}{V_{b2}^2} = Z_{sc(2)} \frac{1}{Z_{b2}} = z_{pu(2)}
\end{aligned}
$$

- Hence, if we choose as voltage bases the nominal voltages of the transformers, transformers can be modelled simply as impedances when using the per-unit system.

Example 3

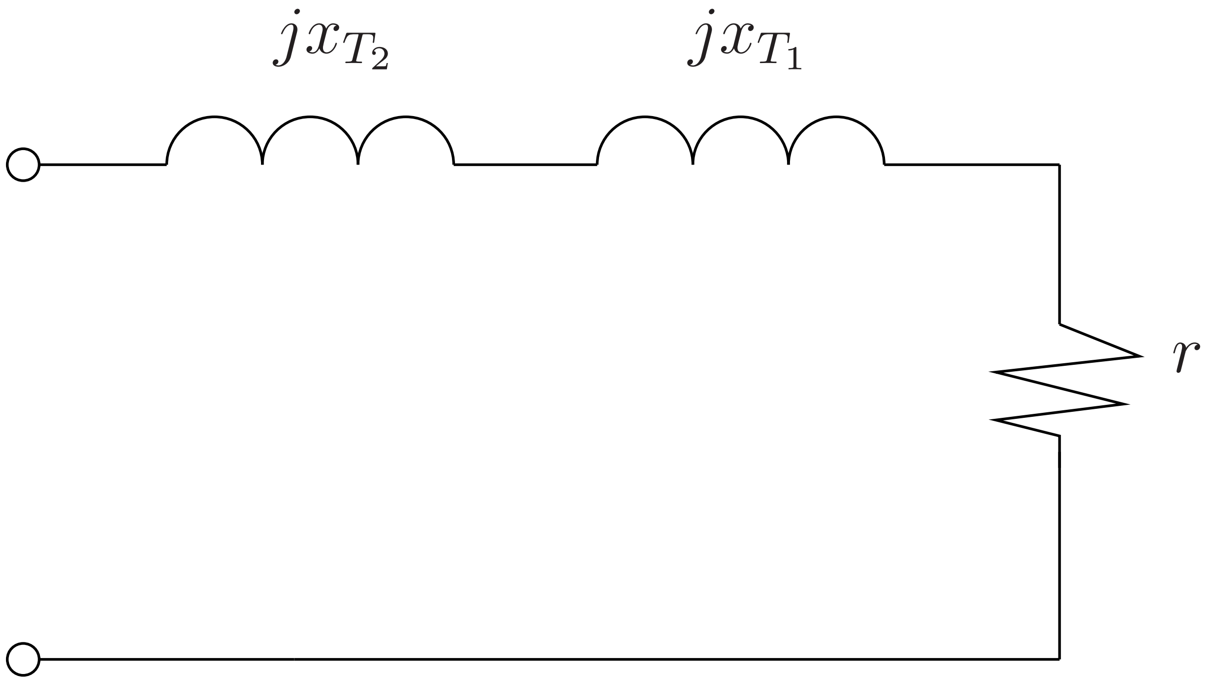
- Consider the following system:



- The data of the transformers are:
  - T_1 : 10 kVA, 100V / 400V , $V_{sc\%} = 10\%$
  - T_2 : 10 kVA, 400V / 200V , $V_{sc\%} = 15\%$

Example 3

- Determine V_1 such that $V_3 = 200$ V on the resistance.
- Equivalent circuit in pu:



Example 3

- First step: choose the bases.
  - S_b = 10 kVA $\longrightarrow$ unique base for all regions.
- Regions are defined by transformers:
  - V_{b1} = 100 V $\longrightarrow$ region I
  - V_{b2} = 400 V $\longrightarrow$ region II
  - V_{b3} = 200 V $\longrightarrow$ region III
- Equivalent resistance of the load:

$$R = \frac{V_3^2}{P} = \frac{200^2}{10^4} = 4\ \Omega$$

Example 3

- The impedance base for region III is:

$$Z_{b_{III}} = \frac{V_{b_{III}}^2}{S_b} = \frac{200^2}{10^4} = 4\,\Omega$$

- Hence:

$$r(pu) = \frac{R}{Z_{b_{III}}} = \frac{4}{4} = 1 \text{ pu}$$

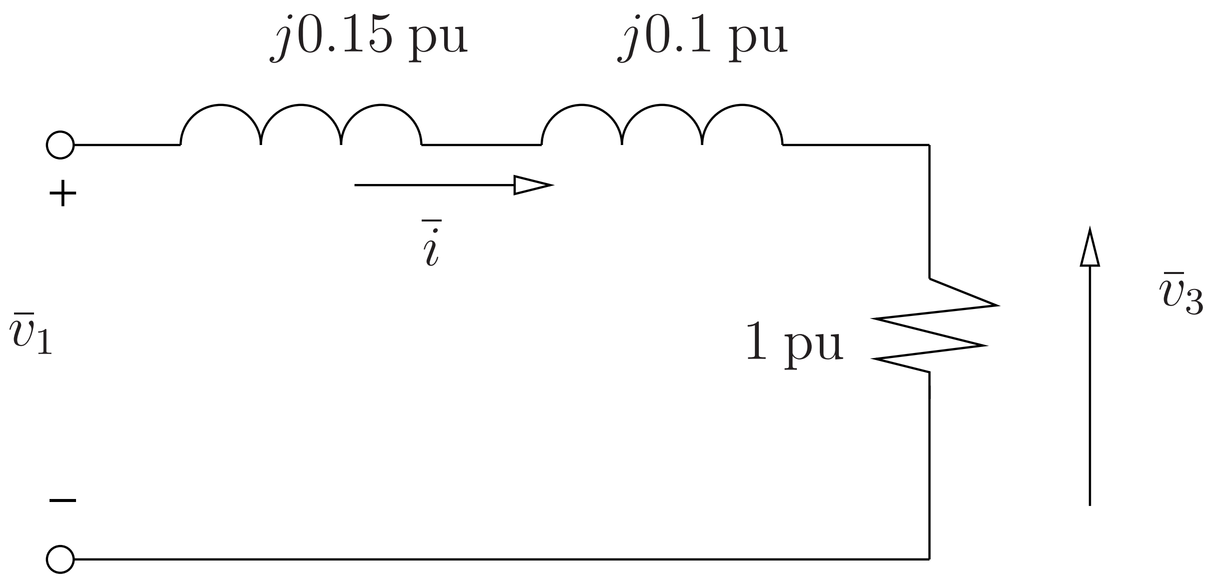
- Transformer reactances:

$$x_{T_2} = 0.15 \text{ pu}$$
$$x_{T_1} = 0.1 \text{ pu}$$

- The values of x_{T_1} and x_{T_2} are the same as short-circuit voltage as the bases of the transformers nominal quantities.

Example 3

- The resulting circuit is:



Example 3

- We assume $\angle \bar{v}_3 = 0$, i.e, phase reference.

$$\bar{v}_3 = \frac{200}{v_{b_{III}}} = \frac{200}{200} = 1 \text{ pu} \bar{v}_1 \qquad = \bar{v}_3 + j(x_{T_1} + x_{T_2})\overline{\bar{i}} = \frac{\bar{v}_3}{r} = 1 \text{ pu}$$

- Hence,

$$\bar{v}_1 = (1 + j0.25) \text{ pu}$$

Example 4 - I

- A 21 MW load at 4 kV and 60 Hz is made of:
  - An inductive impedance load with $G = 0.06047$, $B = -0.03530$.
  - An aggregated induction motor model with $r_1 = 0.07825$, $x_1 = 0.8320$, $r'_2 = 0.1055$, $x'_2 = 0.8320$, $x_\mu = 16.48$.
  - This data is all in pu on a 100 MVA, 4 kV base.

Example 4 - II

- The Z load model is then:

$$
\begin{aligned}
P_L &= \frac{21 \text{ MW}}{100 \text{ MVA}} = 0.21 \\
&= P_Z + P_{IM} \\
P_Z &= V_L^2 G = 0.06 \\
\Rightarrow \quad P_{IM} &= 0.15
\end{aligned}
$$

Example 4 - III

$$
\begin{aligned}
Z_{IM} &= r_1 + jx_1 + \frac{jx_\mu(r_2'/\sigma + jx_2')}{r_2'/\sigma + j(x_\mu + x_2')} \\
&= 0.07825 + j0.832 + \frac{-13.7114 + j1.7386/\sigma}{0.1055/\sigma + j17.312} \\
&= \left(0.07825 + \frac{28.652/\sigma}{0.01113\sigma^2 + 299.71}\right) \\
&\quad +j\left(0.832 + \frac{0.18342/\sigma^2 + 237.37}{0.01113/\sigma^2 + 299.71}\right)
\end{aligned}
$$

$$
= \frac{(0.00087/\sigma^2 + 28.652/\sigma + 23.452) + j(0.19628/\sigma^2 + 486.73)}{0.01113/\sigma^2 + 299.71}
$$

Example 4 - IV

$$
\begin{aligned}
P_{IM} &= V_L^2 G_{IM} = G_{IM} \\
0.15 &= \frac{(0.01113/\sigma^2 + 299.71)(0.00087/\sigma^2 + 28.652/\sigma + 23.452)}{(0.00087/\sigma^2 + 28.652/\sigma + 23.452)^2 + (0.19628/\sigma^2 + 486.73)^2} \\
\Rightarrow \sigma &= 0.0191 \quad \text{(by trial-and-error)} \\
\Rightarrow Z_{IM} &= 4.6221 + j3.0742 \\
Y_L &= (G + jB) + \frac{1}{Z_{IM}} \\
Z_L &= \frac{1}{Y_L} = 3.3654 + j2.1597
\end{aligned}
$$