Worked Problem on Per Unit System

EEEN20090 – Electrical Energy Systems

Problem 1

Consider the three-phases system shown in Figure 1:

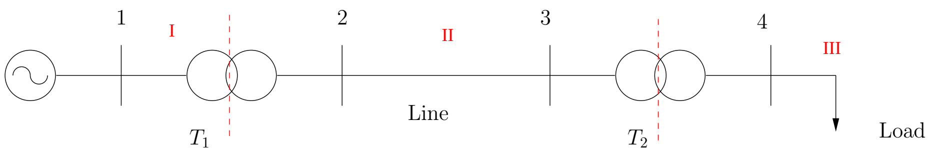

Figure 1

The data of the system are as follows:

T_1: 50 MVA, 11 kV/ 132 kV, $V_{sc\%} = 10$ %

Line: $X_L = 100\ \Omega$

T_2: 50 MVA, 132 kV/ 33 kV, $V_{sc\%} = 12$ %

Load: 50 MW, $\cos\varphi = 0.8$

a. Determine the voltage at bus 1 assuming that the voltage at bus 4 is $V_4 = 30$ kV. Use $S_b = 50$ MVA.

b. Determine per-unit values of the equivalent circuit using as power base $S_b = 100$ MVA.

c. Determine per-unit values of the equivalent circuit using as voltage base for region III $V_{b_{\text{III}}} = 30$ kV.

d. Check that V_1 in kV is the same for the three cases above.

Solution of Problem 1

a. Let's determine the equivalent circuit using the per-unit system.

Choose the bases first:

$$S_b = 50 \text{ MVA}$$
$$V_{b_{\mathrm{I}}} = 11 \text{ kV}$$
$$V_{b_{\mathrm{II}}} = 132 \text{ kV}$$
$$V_{b_{\mathrm{III}}} = 33 \text{ kV}$$
$$Z_{b_{\mathrm{II}}} = \tfrac{V_{b_{\mathrm{III}}}^2}{S_b} = \tfrac{132^2}{50} = 348\ \Omega$$
$$I_{b_{\mathrm{III}}} = \tfrac{S_b}{\sqrt{3}V_{b_{\mathrm{III}}}} = 0.875 \text{ kA}$$

Per-unit quantities:

$$x_{T_1} = 0.1 \text{ pu}$$
$$x_{T_2} = 0.12 \text{ pu}$$
$$x_L = \tfrac{100}{Z_{b_{\mathrm{II}}}} = \tfrac{100}{348} = 0.287 \text{ pu}$$
$$v_4 = \tfrac{V_4}{V_{b_{\mathrm{III}}}} = \tfrac{30}{33} = 0.91 \text{ pu}$$

Let $\bar{v}_4$ be the phase reference: $\bar{v}_4 = 0.91\angle 0$.

The current in the circuit is:

$$I_L = \frac{P_L}{\sqrt{3}V_4 \cos\varphi} = \frac{50 \cdot 10^6}{\sqrt{3} \cdot 30 \cdot 10^3 \cdot 0.8} = 1.2 \text{ kA}$$

which, in per-unit, is:

$$i_L = \frac{I_L}{I_{b_{\mathrm{III}}}} = \frac{1.2}{0.875} = 1.37 \text{ pu}$$

Since $\bar{v}_4$ is the phase reference, we have:

$$\bar{i}_L = c = i_c(0.8 - j0.6) = (1.096 - j0.822) \text{ pu}$$

Finally:

$$\begin{aligned}
\bar{v}_1 &= \bar{v}_4 + j(x_{T_1} + x_{T_2} + x_L)\bar{i}_c \\
&= 0.91 + j(0.1 + 0.12 + 0.287)(1.096 - j0.822) \\
&= (1.33 + j0.555) \text{ pu} = (1.44\angle 0.395 \text{ rad}) \text{ pu} \\
\Rightarrow \quad V_1 &= |\bar{v}_1| \cdot V_{b_{\mathrm{I}}} = 1.44 \cdot 11 = 15.84 \text{ kV}
\end{aligned}$$

Note that the voltage drop is 44%!

b. Bases:

$$S_b = 100 \text{ MVA}$$
$$V_{b_{\text{I}}} = 11 \text{ kV}$$
$$V_{b_{\text{II}}} = 132 \text{ kV}$$
$$V_{b_{\text{III}}} = 33 \text{ kV}$$
$$Z_{b_{\text{II}}} = \frac{V_{b_{\text{III}}}^2}{S_b} = \frac{132^2}{100} = 174\ \Omega$$
$$I_{b_{\text{III}}} = \frac{S_b}{\sqrt{3}V_{b_{\text{III}}}} = 1.75 \text{ kA}$$

System parameters, current and voltage on the load:

$$x_{T_1} = 0.1\frac{S_b}{S_{N_1}} = 0.1\frac{100}{50} = 0.2 \text{ pu}$$
$$x_{T_2} = 0.12\frac{S_b}{S_{N_2}} = 0.12\frac{100}{50} = 0.24 \text{ pu}$$
$$x_L = \frac{100}{174} = 0.575 \text{ pu}$$
$$i_L = \frac{1.2}{1.75} = 0.685 \text{ pu}$$
$$v_4 = \frac{30}{33} = 0.91 \text{ pu}$$

c. Bases:

$$S_b = 50 \text{ MVA}$$
$$V_{b_{\text{I}}} = 11\frac{132}{33}\frac{30}{132} = 10 \text{ kV}$$
$$V_{b_{\text{II}}} = 132\frac{30}{33} = 120 \text{ kV}$$
$$V_{b_{\text{III}}} = 30 \text{ kV}$$
$$Z_{b_{\text{II}}} = \frac{V_{b_{\text{III}}}^2}{S_b} = 288\ \Omega$$
$$I_{b_{\text{III}}} = \frac{S_b}{\sqrt{3}V_{b_{\text{III}}}} = 0.96 \text{ kA}$$

System parameters, current and voltage on the load:

$$x_{T_1} = 0.1\frac{V_{N_1}^2}{S_N}\frac{S_b}{V_{b_{\text{I}}}^2} = 0.1\frac{11^2}{50}\frac{50}{10^2} = 0.121 \text{ pu}$$
$$x_{T_2} = 0.12\frac{V_{N_1}^2}{S_N}\frac{S_b}{V_{b_{\text{II}}}^2} = 0.12\frac{132^2}{50}\frac{50}{120^2} = 0.145 \text{ pu}$$
$$x_L = \frac{100}{Z_{b_{\text{II}}}} = \frac{100}{288} = 0.347 \text{ pu}$$
$$v_4 = \frac{30}{V_{b_{\text{III}}}} = \frac{30}{30} = 1 \text{ pu}$$
$$i_L = \frac{1.2}{I_{b_{\text{III}}}} = \frac{1.2}{0.96} = 1.25 \text{ pu}$$