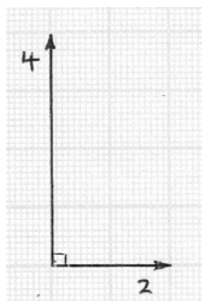
EXERCISE 212 Page 579

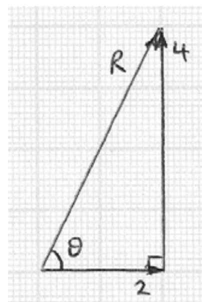
1. Determine a sinusoidal expression for $2 \sin \theta + 4 \cos \theta$ by drawing phasors.

The relative positions of $2 \sin \theta$ and $4 \cos \theta$ are shown as phasors in diagram (a)

The phasor diagram in diagram (b) is drawn to scale with a ruler and protractor



(a)



(b)

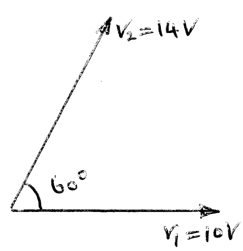
The resultant *R* is shown and is measured as 4.5 and angle ϕ as 63.5°

Hence, by drawing and measuring: $2 \sin \theta + 4 \cos \theta = 4.5 \sin(\theta + 63.5°)$

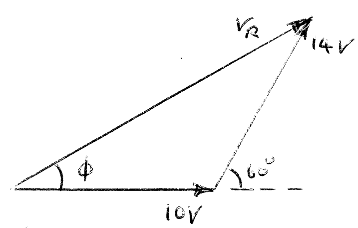
2. If $v_1 = 10 \sin \omega t$ volts and $v_2 = 14 \sin(\omega t + \pi/3)$ volts, determine by drawing phasor sinusoidal expressions for (a) $v_1 + v_2$ (b) $v_1 - v_2$

(a) The relative positions of v_1 and v_2 at time $t = 0$ are shown as phasors in diagram (a), where $\frac{\pi}{3}$ rad = 60°

The phasor diagram in diagram (b) is drawn to scale with a ruler and protractor



(a)



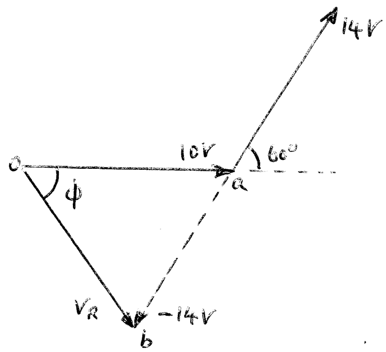
(b)

The resultant v_R is shown and is measured as 20.9 V

and angle ϕ as 35.5° or $35.5 \times \frac{\pi}{180}$ = 0.62 rad leading v_1.

Hence, by drawing and measuring: $v_R = v_1 + v_2 = 20.9 \sin(\omega t + 0.62)$ V

(b) At time $t = 0$, voltage v_1 is drawn 10 units long horizontally as shown by $0a$ in the diagram below. Voltage v_2 is shown, drawn 14 units long in a broken line and leading by 60°. The current $-v_2$ is drawn in the opposite direction to the broken line of v_2, shown as ab in the diagram. The resultant v_R is given by $0b$ lagging by angle ϕ



By measurement, $v_R = 12.5$ V and $\phi = 76°$ or 1.33 rad

Hence, by drawing phasors: $v_R = v_1 + v_2 = 12.5\sin(\omega t - 1.33)$ V

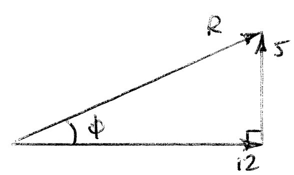
---

3. Express 12 sin ωt + 5 cos ωt in the form A sin($\omega t \pm \alpha$) by drawing phasors.

The relative positions of the two phasors at time $t = 0$ are shown in diagram (a)

The phasor diagram in diagram (b) is drawn to scale with a ruler and protractor

(a)



(b)

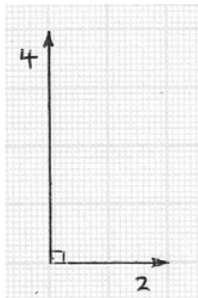
The resultant R is shown and is measured as 13 and angle α as 23° or $23 \times \frac{\pi}{180}$ = 0.40 rad

Hence, by drawing and measuring: 12 sin ωt + 5 cos ωt = 13 sin(ωt + 0.40)

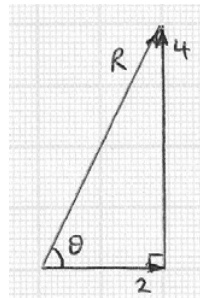
EXERCISE 213 Page 580

1. Determine, using the cosine and sine rules, a sinusoidal expression for: $y = 2 \sin A + 4 \cos A$.

The space diagram is shown in (a) below and the phasor diagram is shown in (b)



(a)



(b)

Using the cosine rule: $R^2 = 2^2 + 4^2 - 2(2)(4)\cos 90° = 20$

from which, $R = \sqrt{20} = 4.472$

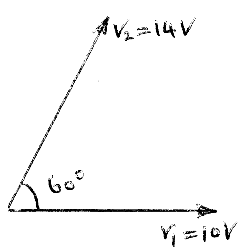
Using the sine rule: $\frac{4}{\sin\theta} = \frac{4.472}{\sin 90°}$ from which, $\sin\theta = \frac{4\sin 90°}{4.472} = 0.894454...$

and $\theta = \sin^{-1} 0.894454... = 63.44°$

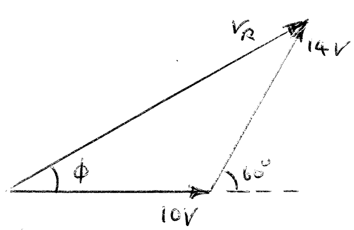
Hence, in sinusoidal form, resultant = $4.472\sin(\theta + 63.44°)$

2. Given $v_1 = 10 \sin \omega t$ volts and $v_2 = 14 \sin(\omega t + \pi/3)$ volts, use the cosine and sine rules to determine sinusoidal expressions for (a) $v_1 + v_2$ (b) $v_1 - v_2$

(a) The space diagram is shown in (a) below and the phasor diagram is shown in (b)



(a)



(b)

Using the cosine rule: $v_R^2 = 10^2 + 14^2 - 2(10)(14)\cos 120° = 436$

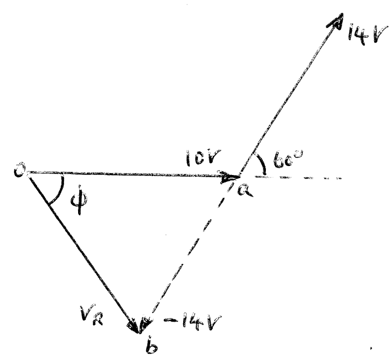
from which, $v_R = \sqrt{436} = 20.88$

Using the sine rule: $\dfrac{14}{\sin\theta} = \dfrac{20.88}{\sin 120°}$ from which, $\sin\theta = \dfrac{14\sin 120°}{20.88} = 0.580668...$

and $\theta = \sin^{-1} 0.580668... = 35.50°$ or 0.62 rad

Hence, in sinusoidal form, $v_1 + v_2 = 20.88\sin(\omega t + 0.62)$ V

(b) $v_1 - v_2$ is given by length 0*b* in the diagram below.



Using the cosine rule: $v_R{}^2 = 10^2 + 14^2 - 2(10)(14)\cos 60° = 156$

from which, $v_R = \sqrt{156} = 12.50$

Using the sine rule: $\dfrac{14}{\sin\theta} = \dfrac{12.50}{\sin 60°}$ from which, $\sin\theta = \dfrac{14\sin 60°}{12.50} = 0.969948...$

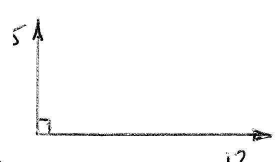
and $\theta = \sin^{-1} 0.969948... = 75.92°$ or 1.33 rad

Hence, in sinusoidal form, $v_1 - v_2 = 12.50\sin(\omega t - 1.33)$ V

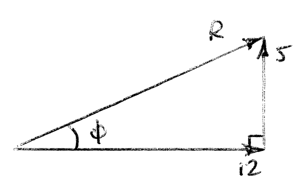
3. Express 12 sin ωt + 5 cos ωt in the form A sin($\omega t \pm \alpha$) by using the cosine and sine rules.

The relative positions of the two phasors at time $t = 0$ are shown in diagram (a)

The phasor diagram is shown in diagram (b)



(a)



(b)

Using the cosine rule: $R^2 = 12^2 + 5^2 - 2(12)(5)\cos 90° = 169$

from which, $R = \sqrt{169} = 13$

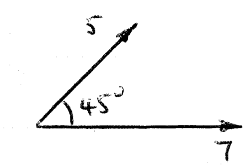
Using the sine rule: $\dfrac{5}{\sin\phi} = \dfrac{13}{\sin 90°}$ from which, $\sin\phi = \dfrac{5\sin 90°}{13} = 0.384615...$

and $\theta = \sin^{-1} 0.384615... = 22.62°$ or 0.395 rad

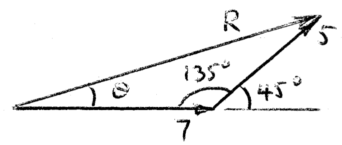
Hence, in sinusoidal form, resultant = $13\sin(\omega t + 0.395)$

4. Express $7 \sin \omega t + 5 \sin\left(\omega t + \dfrac{\pi}{4}\right)$ in the form $A \sin(\omega t \pm \alpha)$ by using the cosine and sine rules.

The space diagram is shown in (a) below and the phasor diagram is shown in (b)



(a)



(b)

Using the cosine rule: $R^2 = 7^2 + 5^2 - 2(7)(5)\cos 135° = 123.497$

from which, $R = \sqrt{123.497} = 11.11$

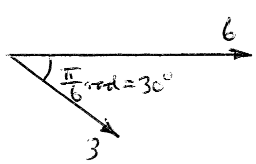
Using the sine rule: $\dfrac{5}{\sin\theta} = \dfrac{11.11}{\sin 135°}$ from which, $\sin\theta = \dfrac{5\sin 135°}{11.11} = 0.31823$

and $\theta = \sin^{-1} 0.31823 = 18.56°$ or 0.324 rad

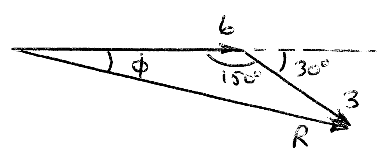
Hence, in sinusoidal form, $7\sin\omega t + 5\sin\left(\omega t + \dfrac{\pi}{4}\right) = 11.11\sin(\omega t + 0.324)$

5. Express $6 \sin \omega t + 3 \sin\left(\omega t - \dfrac{\pi}{6}\right)$ in the form $A \sin(\omega t \pm \alpha)$ by using the cosine and sine rules.

The space diagram is shown in (a) below and the phasor diagram is shown in (b)



(a)



(b)

Using the cosine rule: $R^2 = 6^2 + 3^2 - 2(6)(3)\cos 150° = 76.177$

from which, $R = \sqrt{76.177} = 8.73$

Using the sine rule: $\dfrac{3}{\sin\theta} = \dfrac{8.73}{\sin 150°}$ from which, $\sin\theta = \dfrac{3\sin 150°}{8.73} = 0.171821...$

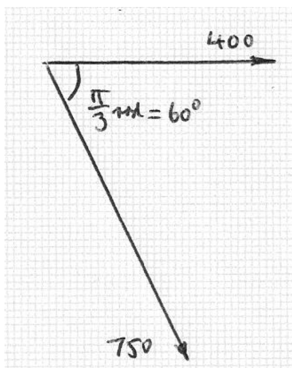
and $\theta = \sin^{-1} 0.171821... = 9.89°$ or 0.173 rad

Hence, in sinusoidal form, $6 \sin \omega t + 3 \sin\left(\omega t - \frac{\pi}{6}\right) = 8.73\sin(\omega t - 0.173)$

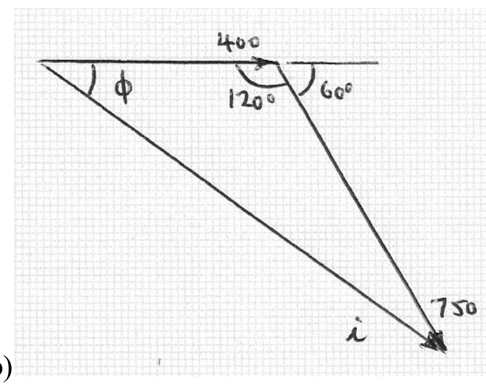
6. The sinusoidal currents in two parallel branches of an electrical network are 400 sin ωt and 750 sin($\omega t - \pi/3$), both measured in milliamperes. Determine the total current flowing into the parallel arrangement. Give the answer in sinusoidal form and in amperes.

Total current, $i = 400 \sin \omega t + 750 \sin(\omega t - \pi/3)$ mA

The space diagram is shown in (a) below and the phasor diagram is shown in (b)



(a)



(b)

Using the cosine rule: $R^2 = 400^2 + 750^2 - 2(400)(750)\cos 120° = 1\,022\,500$

from which, $R = \sqrt{1\,022\,500} = 1011$ mA = 1.011 A

Using the sine rule: $\frac{750}{\sin\phi} = \frac{1011}{\sin 120°}$ from which, $\sin\phi = \frac{750\sin 120°}{1011} = 0.642452...$

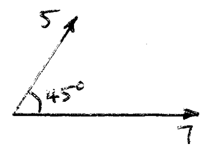
and $\theta = \sin^{-1} 0.642452... = 39.97°$ or 0.698 rad

Hence, in sinusoidal form, $400 \sin \omega t + 750 \sin(\omega t - \pi/3) = 1.01\sin(\omega t - 0.698)$ A

EXERCISE 214 Page 582

1. Express 7 sin ωt + 5 sin$\left(\omega t+\frac{\pi}{4}\right)$ in the form A sin(ωt ± α) by horizontal and vertical components.

From the phasors shown:



Total horizontal component, H = 7 cos 0° + 5 cos 45° = 10.536 (since $\frac{\pi}{4}$ rad = 45°)

Total vertical component, V = 7 sin 0° + 5 sin 45° = 3.536

By Pythagoras, the resultant, $i_R=\sqrt{\left[10.536^2+3.536^2\right]}$ = 11.11 A

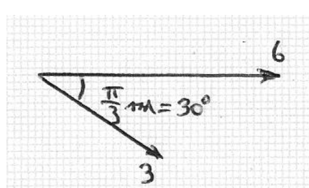
Phase angle, $\phi=\tan^{-1}\left(\frac{3.536}{10.536}\right)$ = 18.55° or 0.324 rad

Hence, by using horizontal and vertical components,

$$7\sin\omega t+5\sin\left(\omega t+\frac{\pi}{4}\right)=11.11\sin\left(\omega t+0.324\right)$$

2. Express 6 sin ωt + 3 sin$\left(\omega t-\frac{\pi}{6}\right)$ in the form A sin(ωt ± α) by horizontal and vertical components.

From the phasors shown:



Total horizontal component, H = 6 cos 0° + 3 cos(–30°) = 8.598 (since $\frac{\pi}{6}$ rad = 30°)

Total vertical component, V = 6 sin 0° + 3 sin(–30°) = –1.5

By Pythagoras, the resultant, $i_R=\sqrt{\left[8.598^2+1.5^2\right]}$ = 8.73

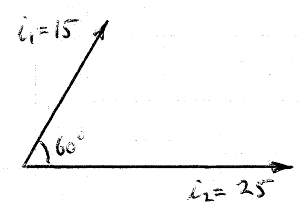
Phase angle, $\phi = \tan^{-1}\left(\frac{1.5}{8.598}\right) = 9.896°$ or 0.173 rad

Hence, by using horizontal and vertical components,

$$6 \sin \omega t + 3 \sin\left(\omega t - \frac{\pi}{6}\right) = 8.73 \sin(\omega t - 0.173)$$

3. Express $i = 25 \sin \omega t - 15 \sin\left(\omega t + \frac{\pi}{3}\right)$ in the form $A \sin(\omega t \pm \alpha)$ by horizontal and vertical components.

The relative positions of currents i_1 and i_2 are shown in the diagram below.



Total horizontal component, $H = 25 \cos 0° - 15 \cos 60° = 17.50$ (since $\frac{\pi}{3}$ rad = 60°)

Total vertical component, $V = 25 \sin 0° - 15 \sin 60° = -12.99$

By Pythagoras, the resultant, $i_R = \sqrt{[17.50^2 + 12.99^2]} = 21.79$

Phase angle, $\phi = \tan^{-1}\left(\frac{-12.99}{17.50}\right) = -36.59°$ or -0.639 rad

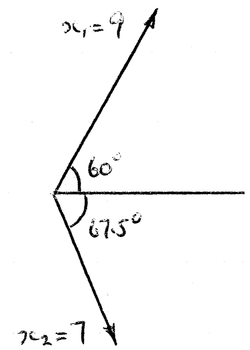
Hence, by using horizontal and vertical components

$$i = 25 \sin \omega t - 15 \sin\left(\omega t + \frac{\pi}{3}\right) = 21.79 \sin(\omega t - 0.639)$$

4. Express $x = 9 \sin\left(\omega t + \frac{\pi}{3}\right) - 7 \sin\left(\omega t - \frac{3\pi}{8}\right)$ in the form $A \sin(\omega t \pm \alpha)$ by horizontal and vertical components.

$\frac{\pi}{3}\text{rad} = \frac{\pi}{3} \times \frac{180°}{\pi} = 60°$ and $\frac{3\pi}{8}\text{rad} = \frac{3\pi}{8} \times \frac{180°}{\pi} = 67.5°$

The relative positions of currents x_1 and x_2 are shown in the diagram below.



Total horizontal component, $H = 9 \cos 60° - 7 \cos(-67.5°) = 1.821$

Total vertical component, $V = 9 \sin 60° - 7 \sin(-67.5°) = 14.261$

By Pythagoras, the resultant, $i_R = \sqrt{[1.821^2 + 14.261^2]} = 14.38$

Phase angle, $\phi = \tan^{-1}\left(\frac{14.261}{1.821}\right) = 82.72°$ or 1.444 rad

Hence, by using horizontal and vertical components,

$$x = 9\sin\left(\omega t + \frac{\pi}{3}\right) - 7\sin\left(\omega t - \frac{3\pi}{8}\right) = 14.38\sin(\omega t + 1.444)$$

5. The voltage drops across two components when connected in series across an a.c. supply are:

$v_1 = 200 \sin 314.2t$ and $v_2 = 120 \sin(314.2t - \pi/5)$ volts, respectively. Determine the

(a) voltage of the supply (given by $v_1 + v_2$) in the form $A \sin(\omega t \pm \alpha)$, and

(b) frequency of the supply.

(a) Total horizontal component, $H = 200 \cos 0° + 120 \cos(-36°) = 297.082$

(since $\frac{\pi}{5}$ rad $= \frac{180°}{5} = 36°$)

Total vertical component, $V = 200 \sin 0° + 120 \sin(-36°) = -70.534$

By Pythagoras, the resultant, $i_R = \sqrt{[297.082^2 + 70.534^2]} = 305.3$ V

Phase angle, $\phi = \tan^{-1}\left(\frac{-70.534}{297.082}\right) = -13.36°$ or -0.233 rad

Hence, by using horizontal and vertical components,

$v_1 + v_2 = 200 \sin 314.2t + 120 \sin (314.2t - \pi/5) = 305.3\sin(314.2t - 0.233)$ volts

(b) Angular velocity, $\omega = 314.2$ rad/s $= 2\pi f$

from which, frequency, $f = \dfrac{314.2}{2\pi} = 50$ Hz

> **6.** If the supply to a circuit is $v = 20 \sin 628.3t$ volts and the voltage drop across one of the components is $v_1 = 15 \sin (628.3t - 0.52)$ volts, calculate the:
>
> (a) voltage drop across the remainder of the circuit, given by $v - v_1$, in the form $A \sin(\omega t \pm \alpha)$
>
> (b) supply frequency
>
> (c) periodic time of the supply.

(a) $v - v_1 = 20 \sin 628.3t - 15 \sin (628.3t - 0.52)$

Total horizontal component, $H = 20 \cos 0 - 15 \cos(-0.52) = 6.9827$ (Remember – radians)

Total vertical component, $V = 20 \sin 0 - 15 \sin(-0.52) = 7.4532$

By Pythagoras, the resultant, $i_R = \sqrt{\left[6.9827^2 + 7.4532^2\right]} = 10.21$ V

Phase angle, $\phi = \tan^{-1}\left(\dfrac{7.4532}{6.9827}\right) = 0.818$ rad

Hence, by using horizontal and vertical components,

$v - v_1 = 20 \sin 628.3t - 15 \sin (628.3t - 0.52) = 10.21\sin(628.3t + 0.818)$ volts

(b) Angular velocity, $\omega = 628.3$ rad/s $= 2\pi f$

from which, frequency, $f = \dfrac{628.3}{2\pi} = 100$ Hz

(c) Periodic time, $T = \dfrac{1}{f} = \dfrac{1}{100} = 0.01$ s $= 10$ ms

> **7.** The voltages across three components in a series circuit when connected across an a.c. supply are: $v_1 = 25\sin\left(300\pi t + \dfrac{\pi}{6}\right)$ volts, $v_2 = 40\sin\left(300\pi t - \dfrac{\pi}{4}\right)$ volts and

$v_3 = 50\sin\left(300\pi t + \frac{\pi}{3}\right)$ volts.

Calculate the: (a) supply voltage, in sinusoidal form, in the form $A \sin(\omega t \pm \alpha)$

(b) frequency of the supply

(c) periodic time

(a) Total horizontal component, H = 25 cos 30° + 40 cos(–45°) + 50 cos 60° = 74.935

Total vertical component, V = 25 sin 30° + 40 sin(–45°) + 50 sin 60° = 27.517

By Pythagoras, the resultant, $v_1 + v_2 + v_3 = \sqrt{\left[74.935^2 + 27.517^2\right]}$ = 79.83 V

Phase angle, $\phi = \tan^{-1}\left(\frac{27.517}{74.935}\right)$ = 20.16° or 0.352 rad

Hence, by using horizontal and vertical components,

supply voltage, $v_1 + v_2 + v_3 = 79.83\sin(300\pi t + 0.352)$

(b) Angular velocity, $\omega = 300\pi$ rad/s = $2\pi f$

from which, frequency, $f = \frac{300\pi}{2\pi}$ = 150 Hz

(c) Periodic time, $T = \frac{1}{f} = \frac{1}{150}$ = 0.006667 s = 6.667 ms

8. In an electrical circuit, two components are connected in series. The voltage across the first component is given by 80 sin($\omega t + \pi/3$) volts, and the voltage across the second component is given by 150 sin($\omega t - \pi/4$) volts. Determine the total supply voltage to the two components. Give the answer in sinusoidal form.

Total horizontal component, H = 80 cos 60° + 150 cos(–45°) = 146.066

(since $\frac{\pi}{3}\text{rad} = \frac{180°}{3}$ = 60° and $\frac{\pi}{4}\text{rad} = \frac{180°}{4}$ = 45°)

Total vertical component, V = 80 sin 60° + 150 sin(–45°) = –36.784

By Pythagoras, the resultant, $i_R = \sqrt{\left[146.066^2 + 36.784^2\right]}$ = 150.6 V

Phase angle, $\phi = \tan^{-1}\left(\dfrac{-36.784}{146.066}\right) = -14.135°$ or -0.247 rad

Hence, by using horizontal and vertical components,

$$80\sin(\omega t + \pi/3) + 150\sin(\omega t - \pi/4) = 150.6\sin(\omega t - 0.247) \text{ volts}$$

EXERCISE 215 Page 584

1. Express $8 \sin \omega t + 5 \sin\left(\omega t + \frac{\pi}{4}\right)$ in the form $A \sin(\omega t \pm \alpha)$ by using complex numbers.

Using complex numbers, $8 \sin \omega t + 5 \sin\left(\omega t + \frac{\pi}{4}\right) \equiv 8\angle 0° + 5\angle 45°$ in polar form

$= (8 + j0) + (3.536 + j3.536)$

$= 11.536 + j3.536$

$= 12.07\angle 17.04° = 12.07\angle 0.297$ rad

Hence, in sinusoidal form, $8 \sin \omega t + 5 \sin\left(\omega t + \frac{\pi}{4}\right) = 12.07 \sin(\omega t + 0.297)$

2. Express $6 \sin \omega t + 9 \sin\left(\omega t - \frac{\pi}{6}\right)$ in the form $A \sin(\omega t \pm \alpha)$ by using complex numbers.

Using complex numbers, $6 \sin \omega t + 9 \sin\left(\omega t - \frac{\pi}{6}\right) \equiv 6\angle 0° + 9\angle -30°$ in polar form

(since $\frac{\pi}{6}$ rad $= \frac{180°}{6}$)

$= (6 + j0) + (7.794 - j4.500)$

$= 13.794 - j4.500$

$= 14.51\angle -18.068° = 14.51\angle -0.315$ rad

Hence, in sinusoidal form, $6 \sin \omega t + 9 \sin\left(\omega t - \frac{\pi}{6}\right) = 14.51 \sin(\omega t - 0.315)$

3. Express $v = 12 \sin \omega t - 5 \sin\left(\omega t - \frac{\pi}{4}\right)$ in the form $A \sin(\omega t \pm \alpha)$ by using complex numbers.

Using complex numbers, $12 \sin \omega t - 5 \sin\left(\omega t + \frac{\pi}{4}\right) \equiv 12\angle 0° - 5\angle -45°$ in polar form

$= (12 + j0) - (3.536 - j3.536)$

$= 8.464 - j3.536$

$= 9.173\angle{-22.67°} = 9.173\angle{-0.396}$ rad

Hence, in sinusoidal form, $12 \sin \omega t - 5 \sin\left(\omega t - \frac{\pi}{4}\right) = 9.173 \sin(\omega t - 0.396)$

4. Express $x = 10 \sin\left(\omega t + \frac{\pi}{3}\right) - 8 \sin\left(\omega t - \frac{3\pi}{8}\right)$ in the form $A \sin(\omega t \pm \alpha)$ by using complex numbers.

$\frac{\pi}{3}\text{rad} = \frac{\pi}{3} \times \frac{180°}{\pi} = 60°$ and $\frac{3\pi}{8}\text{rad} = \frac{3\pi}{8} \times \frac{180°}{\pi} = 67.5°$

Using complex numbers, $10 \sin\left(\omega t + \frac{\pi}{3}\right) - 8 \sin\left(\omega t - \frac{3\pi}{8}\right) \equiv 10\angle 60° - 8\angle{-67.5°}$ in polar form

$= (5 + j8.660) - (3.061 - j7.391)$

$= 1.939 + j16.051$

$= 16.168\angle 83.11° = 16.168\angle 1.451$ rad

Hence, in sinusoidal form, $10 \sin\left(\omega t + \frac{\pi}{3}\right) - 8 \sin\left(\omega t - \frac{3\pi}{8}\right) = 16.168 \sin(\omega t + 1.451)$

5. The voltage drops across two components when connected in series across an a.c. supply are:

$v_1 = 240 \sin 314.2t$ and $v_2 = 150 \sin(314.2t - \pi/5)$ volts, respectively. Determine the

(a) voltage of the supply (given by $v_1 + v_2$) in the form $A \sin(\omega t \pm \alpha)$

(b) frequency of the supply.

(a) Using complex numbers, $v_1 + v_2 = 240 \sin 314.2t + 150 \sin(314.2t - \pi/5)$

$\equiv 240\angle 0° + 150\angle{-36°}$ in polar form (since $\frac{\pi}{5}\text{rad} = \frac{180°}{5} = 36°$)

$= (240 + j0) + (121.353 - j88.168)$

$= 361.353 - j88.168$

$= 371.95\angle{-13.71°} = 371.95\angle{-0.239}$ rad

Hence, in sinusoidal form, supply voltage, $v_1 + v_2 = 371.95 \sin(314.2t - 0.239)$ V

(b) Angular velocity, $\omega = 314.2$ rad/s $= 2\pi f$

from which, frequency, $f = \frac{314.2}{2\pi} = 50$ Hz

6. If the supply to a circuit is $v = 25 \sin 200\pi t$ volts and the voltage drop across one of the components is $v_1 = 18 \sin(200\pi t - 0.43)$ volts, calculate the:

(a) voltage drop across the remainder of the circuit, given by $v - v_1$, in the form $A \sin(\omega t \pm \alpha)$

(b) supply frequency

(c) periodic time of the supply.

(a) Using complex numbers, $v - v_2 = 25 \sin 200\pi t - 18 \sin(200\pi t - 0.43)$

$$\equiv 25\angle 0° - 18\angle -0.43 \text{ rad in polar form}$$

$$= (25 + j0) - (16.361 - j7.504)$$

$$= 8.639 + j7.504$$

$$= 11.44\angle 0.715 \text{ rad}$$

Hence, in sinusoidal form, voltage across remainder of circuit,

$$v - v_2 = 11.44 \sin(200\pi t + 0.715) \text{ V}$$

(b) Angular velocity, $\omega = 200\pi$ rad/s $= 2\pi f$

from which, frequency, $f = \frac{200\pi}{2\pi} = 100$ Hz

(c) Periodic time, $T = \frac{1}{f} = \frac{1}{100} = 0.010$ s = 10 ms

7. The voltages across three components in a series circuit when connected across an a.c. supply are: $v_1 = 20\sin\left(300\pi t - \frac{\pi}{6}\right)$ volts, $v_2 = 30\sin\left(300\pi t + \frac{\pi}{4}\right)$ volts and $v_3 = 60\sin\left(300\pi t - \frac{\pi}{3}\right)$ volts. Calculate the: (a) supply voltage, in sinusoidal form, in the form $A \sin(\omega t \pm \alpha)$

(b) frequency of the supply

(c) periodic time

(d) r.m.s. value of the supply voltage.

(a) Using complex numbers, supply voltage = $v_1 + v_2 + v_3$

$\equiv 20\angle{-30°} + 30\angle 45° + 60\angle{-60°}$ in polar form

$= (17.321 - j10) + (21.213 + j21.213) + (30 - j51.962)$

$= 68.534 - j40.749$

$= 79.73\angle{-30.73°} = 79.73\angle{-0.536}$ rad

Hence, by using complex numbers,

supply voltage, $v_1 + v_2 + v_3 = 79.73\sin\left(300\pi t - 0.536\right)$

(b) Angular velocity, $\omega = 300\pi$ rad/s $= 2\pi f$

from which, frequency, $f = \dfrac{300\pi}{2\pi} = 150$ Hz

(c) Periodic time, $T = \dfrac{1}{f} = \dfrac{1}{150} = 0.006667\ s = 6.667$ ms

(d) R.m.s. value of the supply voltage = 0.707 × 79.73 = 56.37 V

8. Measurements made at a substation at peak demand of the current in the red, yellow and blue phases of a transmission system are: $I_{\text{red}} = 1248\angle -15°$ A, $I_{\text{yellow}} = 1120\angle -135°$ A and $I_{\text{blue}} = 1310\angle 95°$ A. Determine the current in the neutral cable if the sum of the currents flows through it.

Current in neutral cable = $I_{\text{red}} + I_{\text{yellow}} + I_{\text{blue}}$

$= 1248\angle -15° + 1120\angle -135° + 1310\angle 95°$

$= (1205.475 - j323.006) + (-791.960 - j791.960) + (-114.174 + j1305.015)$

$= 299.341 + j190.049$

$= 354.6\angle 32.41°$

Hence, by using complex numbers,

current in neutral cable = $354.6\angle 32.41°$A