Electrical Energy Systems

ELECTRICAL ENERGY SYSTEMS (EEEN20090)

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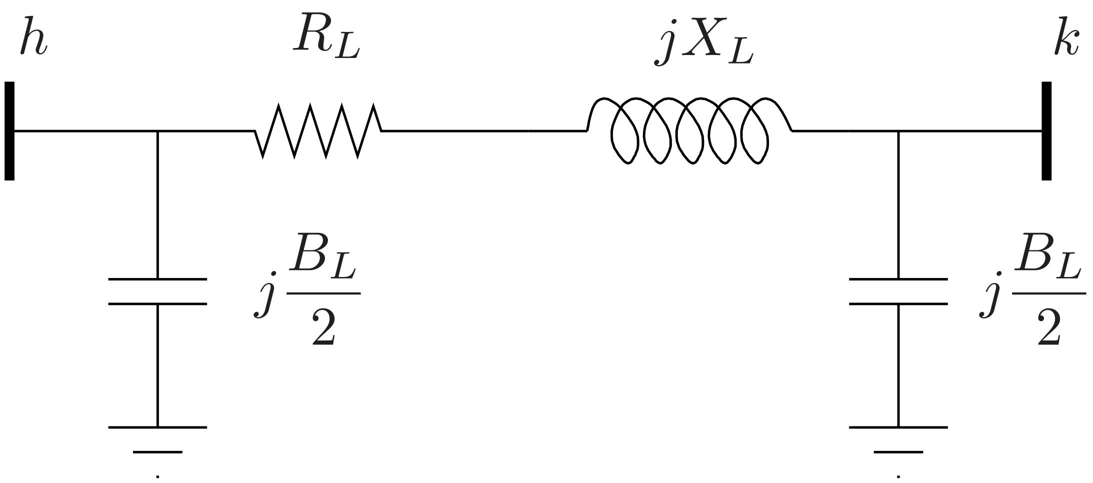
Transmission Line Modelling (I)

- Complex field: a rigorous analysis should be based on partial differential equations (Maxwell equations).
- However, for power system steady-state analysis as well as for “slow” dynamic analysis (e.g. electro-mechanical transient), we can use a “lumped” model:

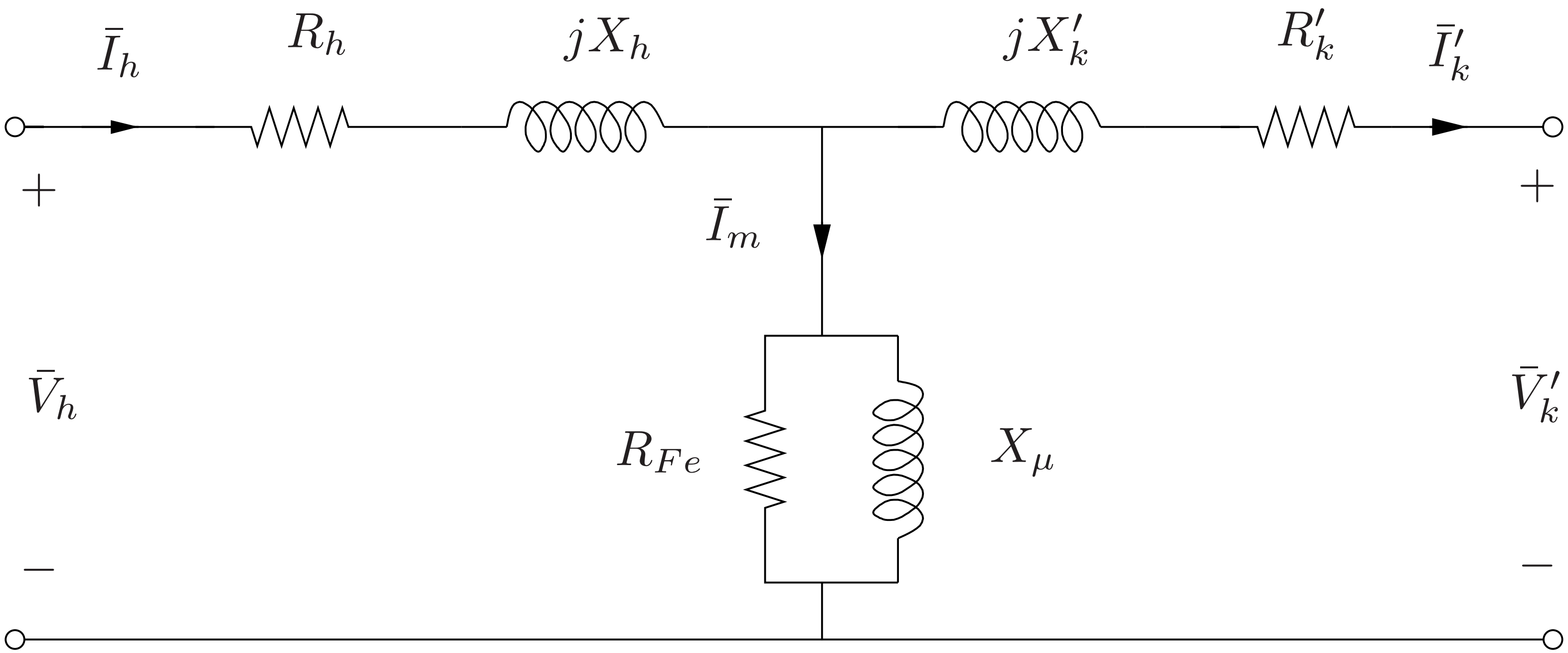


Transmission Line Modelling (II)

- The π model is accurate for "short" overhead transmission lines ($d < 250$ km).
- For overhead transmission lines, if $d < 100$ km, then $B_L \approx 0$.
- Overhead transmission lines are less capacitive than cables (installed underground).
- Note that underground cables can be up to 40 km long (otherwise Ferranti's Effect).
- For high voltage transmission lines $R_L \ll X_L$.
- For low voltage distribution lines $R_L \geq X_L$.

Transformer Modelling (I)

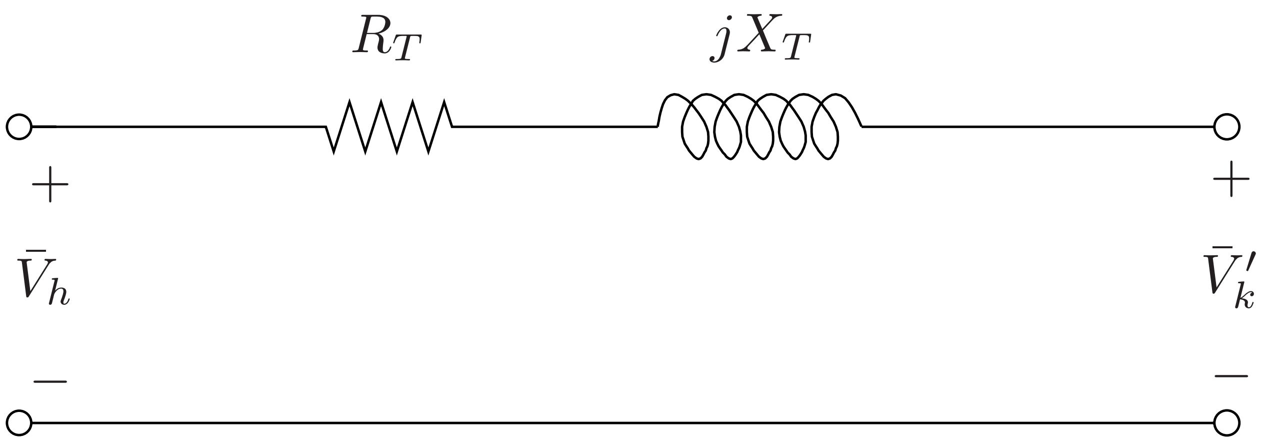
- We have already seen the transformer model:



- For transmission system analysis, the model above is often too detailed.

Transformer Modelling (II)

- In most applications, the following model is adequate:



where

  - $X_T = X_h + X_k'$
  - $R_T = R_h + R_k'$

- Note that the transformer modifies the voltage/current ratios.

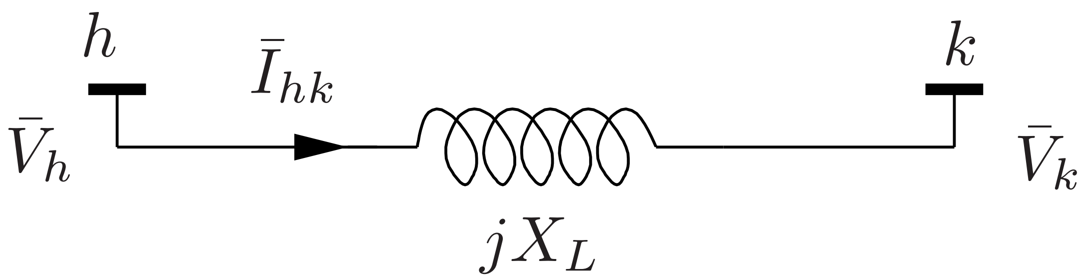
- The **per unit system** solves this issue.

Properties of High Voltage Transmission Systems

- Let's prove the following assertions:
  - Transmitted power is $P_{hk} \propto V^2$
  - Transmission losses are $P_{\text{loss}} \propto \frac{1}{V^2}$
  - Voltage drop $\Delta V \propto \frac{1}{V}$

Transmitted Power - $P_{hk} \propto V^2$ (I)

- Let's consider a single-phase equivalent of a three-phase system.



$$\bar{V}_h = V_h \angle \theta_h \qquad \bar{S}_{hk} = 3\, \bar{V}_h\, \bar{I}_{hk}^*$$

$$\bar{V}_k = V_k \angle \theta_k \qquad \bar{I}_{hk} = \frac{\bar{V}_h - \bar{V}_k}{jX_L}$$

$$\Rightarrow \qquad \bar{I}_{hk}^* = \frac{\bar{V}_h^* - \bar{V}_k^*}{-jX_L}$$

Transmitted Power - $P_{hk} \propto V^2$ (II)

- Hence:

$$
\begin{aligned}
P_{hk} &= \Re\left\{\bar{S_{hk}}\right\} \\
&= 3\,\frac{V_h V_k}{X_L}\sin(\theta_h - \theta_k) \\
&\propto V_h V_k
\end{aligned}
$$

and

$$
\begin{aligned}
Q_{hk} &= \Im\left\{\bar{S_{hk}}\right\} \\
&= 3\,\frac{V_h^2}{X_L} - 3\,\frac{V_h V_k}{X_L}\cos(\theta_h - \theta_k) \\
&\propto V_h^2
\end{aligned}
$$

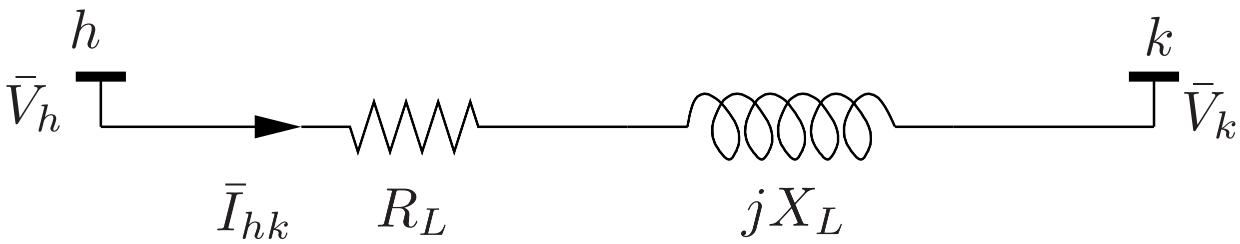
- Note also that $P_{hk} \propto \sin(\theta_h - \theta_k)$

Transmission Losses - $P_{\text{loss}} \propto \frac{1}{V^2}$ (I)

- Losses are defined as:

$$P_{\text{loss}} = 3\,R_L\;I_{hk}^2 = 3\,R_L\;\bar{I}_{hk}\;\bar{I}_{hk}^*$$

$$Q_{\text{loss}} = 3\,X_L\;I_{hk}^2 = 3\,X_L\;\bar{I}_{hk}\;\bar{I}_{hk}^*$$



Transmission Losses - $P_{\text{loss}} \propto \frac{1}{V^2}$ (II)

- The complex power flowing from h to k is:

$$\bar{S}_{hk} = 3\,\bar{V}_h\ \bar{I}_{hk}^*$$

$$\Rightarrow \quad \bar{I}_{hk} = \frac{\bar{S}_{hk}^*}{3\,\bar{V}_h^*}$$

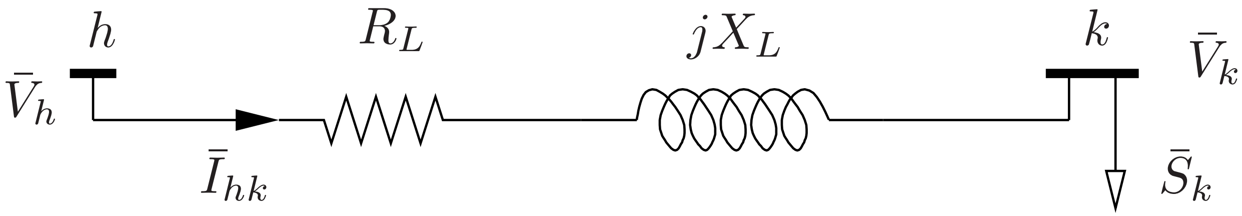
- Assuming $\bar{V}_h = V_h \angle 0$, we have:

$$\bar{I}_{hk}\ \bar{I}_{hk}^* = \frac{\bar{S}_{hk}^2}{9\,\bar{V}_h^2} = \frac{\bar{P}_{hk}^2 + Q_{hk}^2}{9\,\bar{V}_h^2}$$

- Hence $P_{\text{loss}} \propto \dfrac{1}{V_h^2}$
- We can obtain a similar conclusion for Q_{loss}

Voltage Drop - $\Delta V \propto \frac{1}{V}$ (I)

- Let's consider again a radial system:



- We have:

$$\bar{V}_h = \bar{V}_k + (R_L + jX_L)\bar{I}_{hk}$$

Voltage Drop - $\Delta V \propto \frac{1}{V}$ (II)

- The current can be written as

$$\bar{I}_{hk} = \frac{\bar{S}^*_{hk}}{3\,\bar{V}^*_k} \quad \text{(note that} \quad \bar{S}_{hk} \neq \bar{S}_k)$$

- Then

$$\begin{aligned} \Rightarrow \quad \bar{V}_h &= \bar{V}_k + (R_L + jX_L)\frac{P_h - Q_h}{3\,\bar{V}^*_h} \\ &= \bar{V}_k + \frac{R\;P_k + X_L\;Q_k}{3\,\bar{V}^*_k} + j\,\frac{X_L\;P_k - R_L\;Q_k}{3\,\bar{V}^*_k} \end{aligned}$$

Voltage Drop - $\Delta V \propto \frac{1}{V}$ (III)

- Let's assume $\bar{V}_k = V_k \angle 0$ (phase reference), then

$$\bar{V}_h = V_k + \frac{R_L\, P_k + X_L\, Q_k}{3\, V_k} + j\,\frac{X_L\, P_k - R_L\, Q_k}{3\, V_k}$$

$$\Rightarrow \quad \Delta\bar{V} = (\bar{V}_h - V_k) = \Delta V_R + j\Delta V_I$$

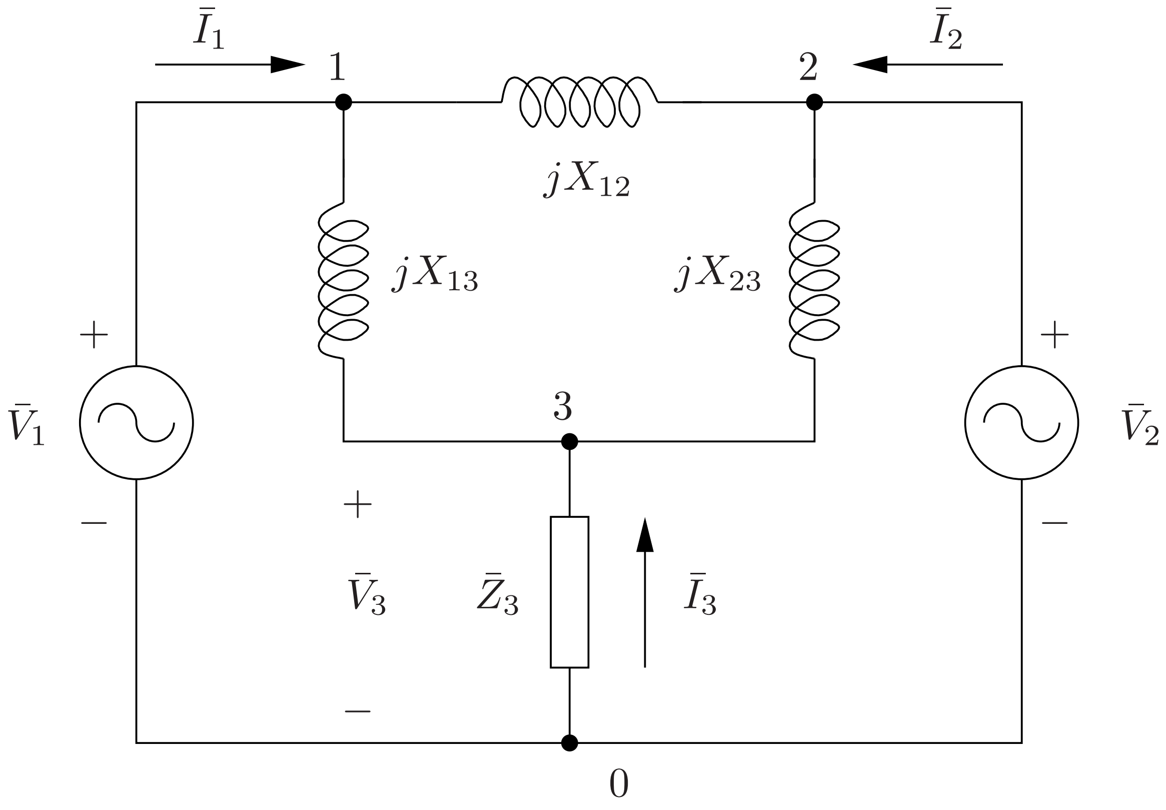
where

$$\Delta V_R = \frac{R_L\, P_k + X_L\, Q_k}{3\, V_k} \propto \frac{1}{V_k}$$

$$\Delta V_I = \frac{X_L\, P_k - R_L\, Q_k}{3\, V_k} \propto \frac{1}{V_k}$$

Power Flow Analysis

- In circuit analysis, we generally solve the following problem: given circuit parameters and voltages at the sources, find the current in the branches of the circuit.



Example – Power Flow Analysis (II)

- Using the branch current method (nodal), one has:

$$
\begin{aligned}
0 &= \frac{\bar{V}_1 - \bar{V}_2}{jX_{12}} + \frac{\bar{V}_1 - \bar{V}_3}{jX_{13}} - \bar{I}_1 \\
0 &= \frac{\bar{V}_2 - \bar{V}_1}{jX_{12}} + \frac{\bar{V}_2 - \bar{V}_3}{jX_{23}} - \bar{I}_2 \\
0 &= \frac{\bar{V}_3 - \bar{V}_1}{jX_{13}} + \frac{\bar{V}_3 - \bar{V}_2}{jX_{23}} - \bar{I}_3
\end{aligned}
$$

- In matrix form:

$$
\begin{bmatrix} \bar{I}_1 \\ \bar{I}_2 \\ \bar{I}_3 \end{bmatrix} = \begin{bmatrix} 1/jX_{12} + 1/jX_{13} & -1/jX_{12} & -1/jX_{13} \\ -1/jX_{12} & 1/jX_{12} + 1/jX_{23} & -1/jX_{23} \\ -1/jX_{13} & -1/jX_{23} & 1/jX_{13} + 1/jX_{23} \end{bmatrix} \begin{bmatrix} \bar{V}_1 \\ \bar{V}_2 \\ \bar{V}_3 \end{bmatrix}
$$

- In compact notation:

$$
\bar{\boldsymbol{I}} = [\bar{\mathbf{Y}}]\bar{\boldsymbol{V}}
$$

Example – Power Flow Analysis (III)

- $[\bar{\boldsymbol{Y}}]$ is the **admittance matrix** of the circuit.
- $[\bar{\boldsymbol{Y}}]$ is often called $[\mathrm{Y}_{\mathrm{bus}}]$.
- In this example, the load is linear:

$$\bar{V}_3 = -\bar{Z}_3\,\bar{I}_3$$

hence one can write the circuit as:

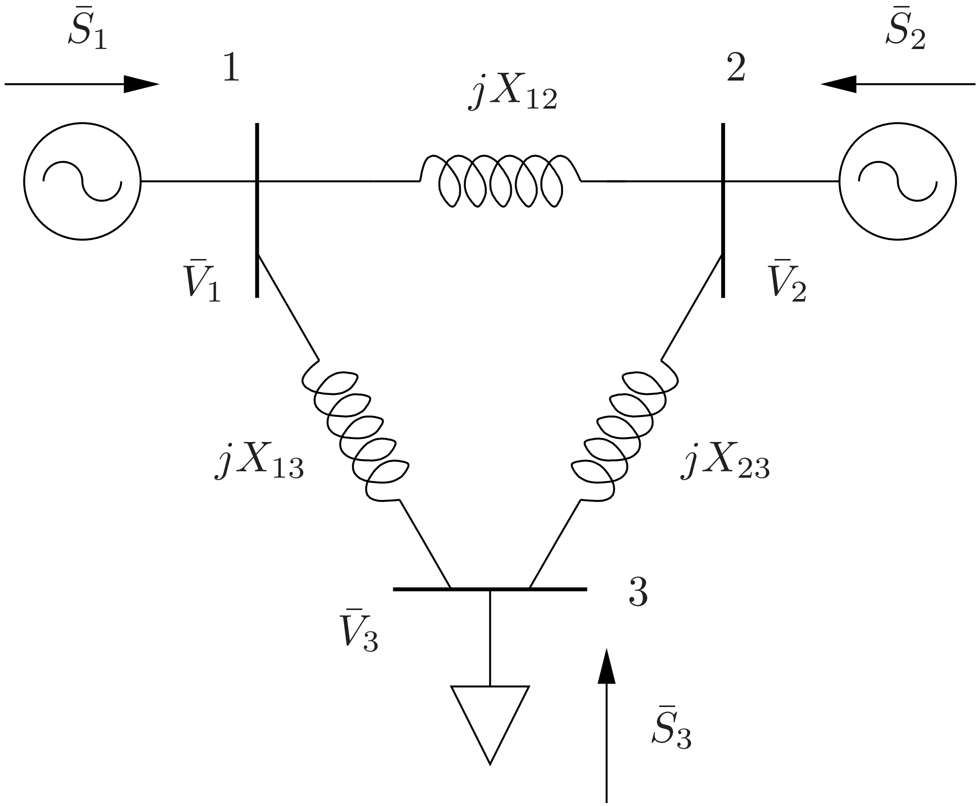
$$\begin{bmatrix} \bar{I}_1 \\ \bar{I}_2 \\ 0 \end{bmatrix} = \left([\bar{\boldsymbol{Y}}] + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1/\bar{Z}_3 \end{bmatrix} \right) \begin{bmatrix} \bar{V}_1 \\ \bar{V}_2 \\ \bar{V}_3 \end{bmatrix} = [\bar{\boldsymbol{Y}}_{\mathrm{tot}}]\bar{\boldsymbol{V}}$$

Power Flow Analysis (IV)

- In power system analysis, we have the following data:
  - Loads are typically known as power consumptions, i.e., (P, Q).
  - Generators are typically known as an active power injection and a voltage magnitude, i.e., (P, V).
- Hence, the problem to be solved is: determine the voltage profile at every bus of the systems given load power consumption and generator active powers and voltage magnitudes.
- This problem is known as **Power Flow Analysis**.

Power Flow Analysis (V)

- The previous circuit becomes:



Power Flow Analysis (VI)

- We know the following quantities:
  - Bus 1 : $P_1 \quad , \quad V_1$
  - Bus 2 : $P_2 \quad , \quad V_2$
  - Bus 3 : $P_3 \quad , \quad Q_3$
- Actually we need to impose a **phase reference**.
- Generally one bus is as a reference.
- So, we impose V and $\theta = \angle\bar{V}$ at a bus.

Power Flow Analysis (VII)

- Hence the data becomes:
  - Bus 1 : $V_1 \quad , \quad \theta_1 = 0$ (the reference is arbitrary)
  - Bus 2 : $P_2 \quad , \quad V_2$
  - Bus 3 : $P_3 \quad , \quad Q_3$
- Note that for lossless systems, $P_1 = P_3 - P_2$, but, in general $P_1 = P_3 - P_2 + P_{\text{loss}}$ where P_{loss} is unknown a priori.
- The reference bus is, for this reason, also called **SLACK BUS**.

Formulation of the Power Flow Problem (I)

- The vector of currents injected at each node is:

$$\bar{\boldsymbol{I}} = [\bar{\boldsymbol{Y}}]\bar{\boldsymbol{V}}$$

which leads to write the power flow problem as the complex power injections at buses:

$$\bar{\boldsymbol{S}} = 3\,\bar{\boldsymbol{V}} \circ \bar{\boldsymbol{I}}^* = 3\,\bar{\boldsymbol{V}} \circ \left([\bar{\boldsymbol{Y}}]^* \bar{\boldsymbol{V}}^*\right)$$

where $\circ$ is the element-by-element product of two vectors and $*$ indicates the conjugate of a complex number.

Formulation of the Power Flow Problem (II)

- For every bus we can write:

$$P_h = 3\,V_h \sum_{k=1}^{n} V_k (G_{hk} \cos\theta_{hk} + B_{hk} \sin\theta_{hk})$$

$$Q_h = 3\,V_h \sum_{k=1}^{n} V_k (G_{hk} \sin\theta_{hk} - B_{hk} \cos\theta_{hk})$$

where

$$\bar{Y}_{hk} = G_{hk} + j B_{hk} = \frac{1}{R_{hk} + j X_{hk}}$$

$$\theta_{hk} = \theta_h - \theta_k$$

and

$$\bar{V}_h = V_h \angle \theta_h \,, \qquad \bar{V}_k = V_k \angle \theta_k$$

Formulation of the Power Flow Problem (III)

- Then we can write :
  - for every load bus (PQ bus) the equation of P_h and
  - for every generator bus (PV bus) the equation of P_h and impose the voltage magnitude V_h.
  - for the slack bus (VQ bus), we impose V_h and θ_h.

Example

- Hence, the equations of the previous example becomes:

$$P_2 = 3\,\frac{V_2\,V_1}{X_{12}}\,\sin(\theta_{21}) + 3\,\frac{V_2\,V_3}{X_{23}}\,\sin(\theta_{23})$$

$$P_3 = 3\,\frac{V_3\,V_1}{X_{13}}\,\sin(\theta_{31}) + 3\,\frac{V_3\,V_2}{X_{23}}\,\sin(\theta_{32})$$

$$Q_3 = 3\,\frac{V_3^2}{X_{13}} + 3\,\frac{V_3^2}{X_{23}} - 3\,\frac{V_3\,V_1}{X_{13}}\,\cos(\theta_{31}) - 3\,\frac{V_3\,V_2}{X_{23}}\,\cos(\theta_{32})$$

  where

$$\theta_{21} = \theta_2 - \theta_1 = \theta_2, \quad \theta_{31} = \theta_3 - \theta_1 = \theta_3, \quad \theta_{23} = \theta_2 - \theta_3, \quad \theta_{32} = \theta_3 - \theta_2,$$

- Unknowns: V_3, θ_3, and θ_2.
- 3 unknowns and 3 equations $\Rightarrow$ solvable.
- **But**, the power flow problem is non-linear. We need to use iterative techniques.

Summary of Power Flow Problem (I)

- Knowns and unknowns:
  - **Load Buses**: we know P and Q, we find V and θ.
  - **Generator Buses**: we know P and V, we find θ (and Q).
  - **Slack Bus**: we know V and θ, (we find P and Q.
- Equations:
  - **Load Buses**: 1 eq. for P-injection and 1 eq. for Q-injection.
  - **Generator Buses**: 1 eq. for P-injection.
  - **Slack Bus**: no need of writing any equation.

Summary of Power Flow Problem (II)

- The determination of Q for generator buses and of P and Q for the slack bus can be done after the solution of the power flow problem.
- If at a bus we have both a load and a generator, the bus is a generator bus with voltage magnitude and power injection:

$$P_h = P_{Gh} - P_{Lh}$$

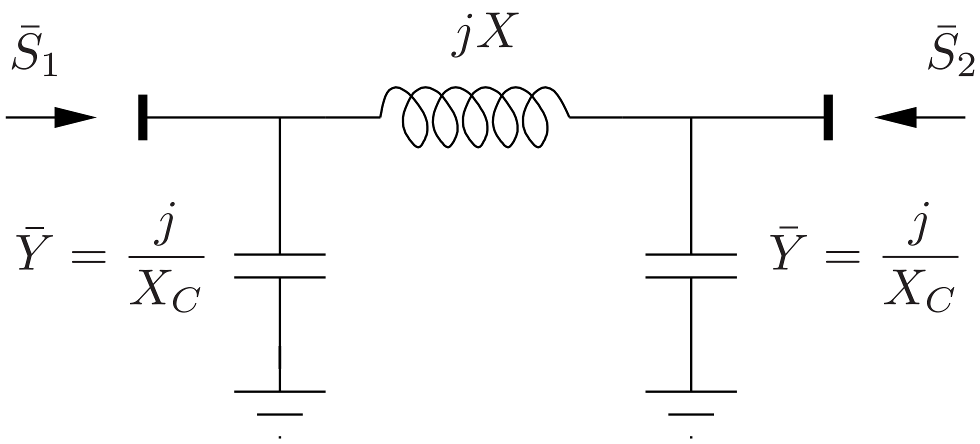
Once we have solved the power flow, we know Q_h and thus:

$$Q_{Gh} = Q_h - Q_{Lh}$$

- **Note**: the general power flow problem can be solved only with numerical methods.

Power Flow Equations of a Lossless Line (I)

- A lossless transmission line has following model:



$$\bar{I}_1 = \frac{\bar{S}_1^*}{3\,\bar{V}_1^*} = \bar{Y}\;\bar{V}_1 \;+\; \frac{\bar{V}_1 - \bar{V}_2}{jX}$$

$$\bar{I}_2 = \frac{\bar{S}_2^*}{3\,\bar{V}_2^*} = \bar{Y}\;\bar{V}_2 \;+\; \frac{\bar{V}_2 - \bar{V}_1}{jX}$$

Power Flow Equations of a Lossless Line (II)

- Let:

$$\bar{V}_1 = V_1 \, e^{j\theta_1}, \qquad \bar{V}_2 = V_2 \, e^{j\theta_2}$$

$$\bar{S}_1 = P_1 + jQ_1, \qquad \bar{S}_2 = P_2 + jQ_2$$

- Then:

$$\bar{S}_1^* = P_1 - jQ_1 = 3\,\bar{Y} V_1^2 + 3\,\frac{V_1^2}{jX} - 3\,\frac{V_1 \, V_2 \, e^{-j\theta_1} \, e^{j\theta_2}}{jX}$$

$$\bar{S}_2^* = P_2 - jQ_2 = 3\,\bar{Y} V_2^2 + 3\,\frac{V_2^2}{jX} - 3\,\frac{V_2 \, V_1 \, e^{-j\theta_2} \, e^{j\theta_1}}{jX}$$

Power Flow Equations of a Lossless Line - III

- Note that:

$$e^{j\theta_2}\ e^{-j\theta_1} = \cos(\theta_2 - \theta_1) + j\ \sin(\theta_2 - \theta_1)$$

$$e^{j\theta_1}\ e^{-j\theta_2} = \cos(\theta_1 - \theta_2) + j\ \sin(\theta_1 - \theta_2)$$

- Hence:

$$P_1 = 3\,\frac{V_1\ V_2}{X}\sin(\theta_1 - \theta_2)$$

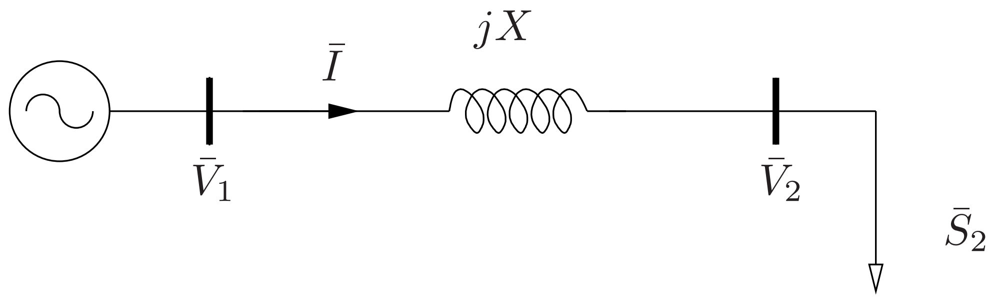
$$P_2 = 3\,\frac{V_1\ V_2}{X}\sin(\theta_2 - \theta_1)$$

$$Q_1 = 3\,\frac{V_1^2}{X} - 3\,\frac{V_1^2}{X_c} - 3\,\frac{V_1\ V_2}{X}\ \cos(\theta_1 - \theta_2)$$

$$Q_2 = 3\,\frac{V_2^2}{X} - 3\,\frac{V_2^2}{X_c} - 3\,\frac{V_1\ V_2}{X}\ \cos(\theta_2 - \theta_1)$$

Example A

- We know $\bar{S}_2$ and $\bar{V}_2$, then we can compute $\bar{V}_1$.



- In this case, no need of using power flow equations.
- If we choose as reference angle $\theta_2 = \angle\bar{V}_2 = 0$, then:

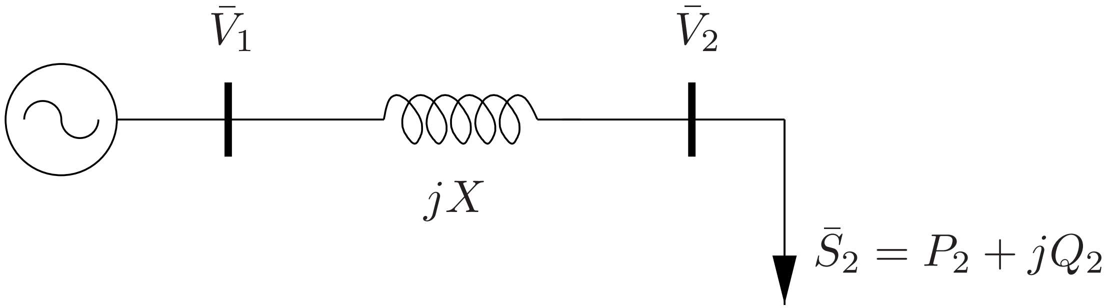
$$\bar{V}_1 = \bar{V}_2 + jX\ \bar{I} = V_2 + jX\bar{I}$$

$$\bar{S}_1 = P_1 + jQ_1 = 3\,\bar{V}_1\ \bar{I}^*$$

$$\bar{I} = \frac{\bar{S}_2^*}{3\,\bar{V}_2^*} = \frac{\bar{S}_2^*}{3\,V_2}$$

Example B (I)

- We know $\bar{V}_1 = V_1 \angle 0$ and $\bar{S}_2 = P_2 + jQ_2$.
- Determine $\bar{V}_2$. In this case, we need to use power flow equations.



- We have:

$$-P_2 = 3\,\frac{V_2\,V_1}{X}\sin\theta_2 \tag{1}$$

$$-Q_2 = 3\,\frac{V_2^2}{X} - 3\,\frac{V_2\,V_1}{X}\cos\theta_2 \tag{2}$$

- Note that we use $(-P_2)$ and $(-Q_2)$, because powers must be "injected" into the grid, hence load consumption is a negative power injection.

Example B (II)

- Let's solve (1) and (2). Unknowns are θ_2 and V_2. We can write:

$$-P_2 = 3\,\frac{V_2\,V_1}{X}\sin\theta_2 \tag{3}$$

$$-Q_2 - 3\,\frac{V_2^2}{X} = -3\,\frac{V_2\,V_1}{X}\cos\theta_2 \tag{4}$$

- Dividing by 3 and summing the squares of (3) and (4) leads to:

$$\Rightarrow \quad \frac{P_2^2}{9} + \left(\frac{Q_2}{3} + \frac{V_2^2}{X}\right)^2 = \frac{V_2^2\,V_1^2}{X^2}\left(\sin^2\theta_2 + \cos^2\theta_2\right) = \frac{V_2^2\,V_1^2}{X^2} \tag{5}$$

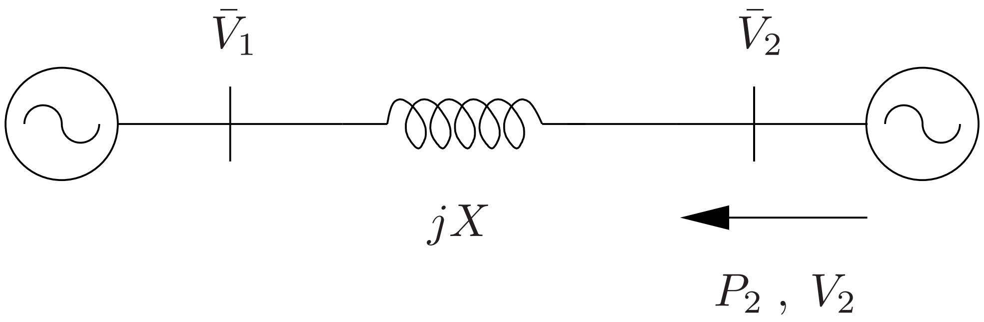
- Equation (5) is quadratic in V_2^2:

$$a\,V_2^4 + b\,V_2^2 + c = 0$$

- Solve for $V_2^2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, then find V_2 (only positive solutions close to the nominal voltage are meaningful) and finally substitute V_2 in (1) to determine θ_2.

Example C

- We know V_1, $\theta_1 = 0$, V_2 and P_2.



- Determine Q_1, Q_2 and θ_2.
- In this case, we need to use power flow equations.
- We have:

$$P_2 = 3\,\frac{V_2\,V_1}{X}\sin\theta_2 \quad \Rightarrow \quad \theta_2 = \arcsin\left(\frac{X\,P_2}{3\,V_2\,V_1}\right)$$

- Then we compute directly Q_1 and Q_2.

Rationale behind Load Models (I)

- Loads at the high voltage level are never a single device.
- For the transmission system each “load” is generally an entire distribution system, which connects thousands of small loads.
- While the behaviour of each single appliance is unknown to the transmission system, the aggregated model of the distribution network and its expected power consumption can be predicted with a very good approximation.
- Even so, the combination of small loads and the distribution network does not behave exactly as a constant PQ load.

Rationale behind Load Models (II)

- In general, the equivalent model, in steady-state, depends on the voltage, as follows:

$$P = P_0 \, V^{\alpha}$$

$$Q = Q_0 \, V^{\beta}$$

where coefficients α and β range from 0 to 2.

- Note that:

$\alpha = \beta = 0$	constant PQ
$\alpha = \beta = 1$	constant current
$\alpha = \beta = 2$	constant impedence

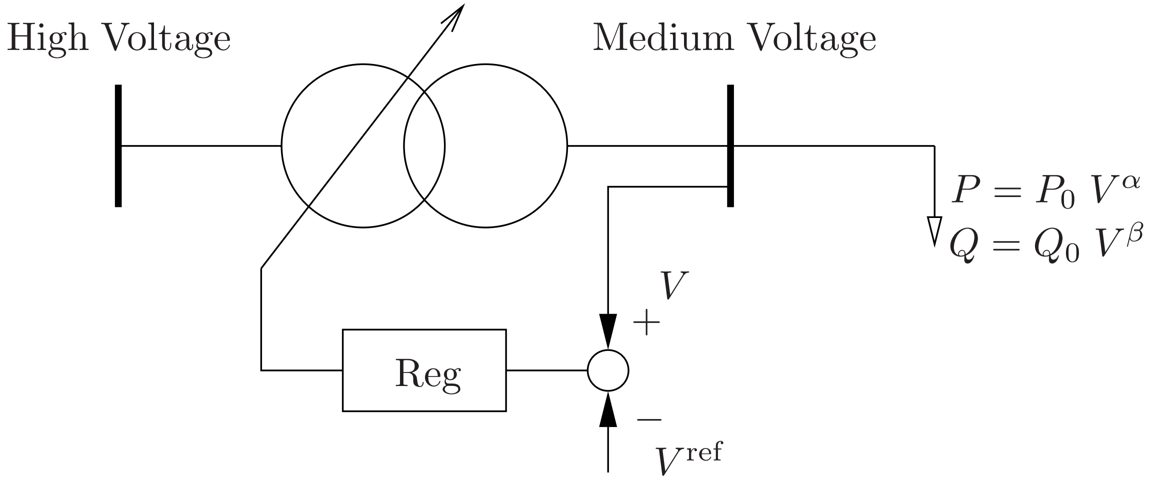
- So, why loads (even aggregated loads) are modelled as constant PQ for the power flow analysis?

Rationale behind Load Models (III)

- There are two main reasons:
  - **Practical**: from the analysis of historical data, weather conditions, etc., the transmission system can estimate the active and reactive power consumption of each load for each hour of the day. This estimation is then used to schedule generator power productions. From the transmission system operator point of view, it is better to use constant PQ load models.
  - **Technical**: each distribution system is connected to HV transmission system through an under-load tap changer transformer. Such transformers regulate and keep constant the voltage on the secondary winding, e.g., MV level.

Rationale behind Load Models (IV)

- In conclusion:



- If V on the MV side is constant, so is the power consumption.
- Note that the voltage is constant only in steady-state.

Rationale behind Generator Models (I)

- Generators have two roles in power systems:
  - Provide active power to feed the loads.
  - Regulate the voltage to compensate reactive power losses of the network and loads.
- Synchronous machines can do both regulations as they have two inputs ($P_m \Rightarrow$ mechanical power and $I_e \Rightarrow$ excitation current).
- In steady-state we can thus assume that a synchronous machine is able to generate a given amount of active power (P) and keep a given voltage (V) at the generator terminal bus.

Rationale behind Generator Models (II)

- The votlage magnitude is kept constant only if the reactive power generated by the machine is within capability limits.
- If not, the machine generates P and $Q^{\max}$ or $Q^{\min}$. It thus becomes a PQ generator.
- Renewables, so far, have not been designed to do much regulation.
- Their capability to provide reactive power is limited and often operate as PQ generators, where $Q = 0$.