Worked Problems on Synchronous Machines

EEEN20090 – Electrical Energy Systems

Problem 1

A 3-phase synchronous generator has the following synchronous reactance $X_s = 5\ \Omega$ and $R_a = 0$. It is connected to a grid with $V = 6{,}600$ V. The e.m.f. of the machine is $E = 6{,}000$ V. Determine the maximum active power that the machine can deliver to the grid.

Problem 2

A 3-phase synchronous machine has the following nominal data:

$$S_N = 5{,}000 \text{ kVA}$$
$$V_N = 6{,}600 \text{ V}$$

The e.m.f. is a function of the excitation current I_c, as follows:

$$E = \frac{7{,}400 I_e}{85 + I_e}\ , \tag{1}$$

where E is expressed in V and I_e in A. E in (1) is the phase-to-neutral voltage. Moreover, the machine has: $R_a = 0.2\ \Omega$ and $X_d = 1\ \Omega$. The mmf of the armature reaction is equivalent to a current $I_c = 20$ A.

Determine:

a. Variation of the current to obtain the nominal voltage of $6{,}600$ V from open-circuit to full load with $\cos\varphi = 0.6$ lagging.

b. Efficiency of the machine at full load with $\cos\varphi = 0.6$ lagging.

Note: Iron losses and mechanical losses are 100 kW and $V_e = 200$ V is the exitation dc voltage.

Problem 3

A salient-pole generator has the following data:

$$
\begin{aligned}
S_N &= 1500 \text{ kVA} \\
V_N &= 3000 \text{ V} \\
f_N &= 50 \text{ Hz} \\
X_d &= 2\ \Omega \\
X_q &= 1.5\ \Omega
\end{aligned}
$$

Determine:

a. The emf at full load with $\cos\varphi = 0.6$ lagging.

b. Maximal active power that the machine can deliver to the grid.

Solution of Problem 1

The phase-to-neutral voltage and emf are:

$$V = \frac{6600}{\sqrt{3}} = 3810.5 \text{ V}$$
$$E = \frac{6000}{\sqrt{3}} = 3464.1 \text{ V}$$

The active power provided by the generator is:

$$P_e = \frac{3EV}{X_s} \sin\delta$$

This power is maximized for $\delta = \frac{\pi}{2}$, hence:

$$\begin{aligned} P_e^{\max} &= \frac{3EV}{X_s} = \frac{3 \cdot 6,000/\sqrt{3} \cdot 6,600/\sqrt{3}}{X_s} \\ &= \frac{6,000 \cdot 6,600}{5} = 7,920 \text{ W} \end{aligned}$$

One can solve the same problem by imposing:

$$\begin{aligned} \bar{E} &= \bar{V} + jX_s\bar{I} \\ \Rightarrow 3,464.1\angle 90^\circ &= 3,810.5\angle 0^\circ + j5I\angle\varphi \\ \Rightarrow I\sin\varphi &= 762.1 \text{ A and } I\cos\varphi = 692.8 \text{ A} \\ \Rightarrow \tan\varphi &= \frac{I\sin\varphi}{I\cos\varphi} = 1.1 \\ \Rightarrow \varphi &= 47.74^\circ \\ \Rightarrow \cos\varphi &= 0.673 \text{ (leading)} \\ \Rightarrow I \cdot 0.673 &= 692.8 \text{ A} \\ \Rightarrow I &= 1,029.95 \text{ A} \\ P_{\max} &= \sqrt{3}VI\cos\varphi = 7,920 \text{ W} \end{aligned}$$

Solution of Problem 2

a. The emf in open-circuit is:

$$E = \frac{6,600}{\sqrt{3}} = 3,810.5 \text{ V}$$

Then:

$$E = \frac{7,400I_e}{85 + I_e} \qquad \Rightarrow I_e = 90.23 \text{ A}$$

At full-load, the nominal current is:

$$I_N = \frac{S_N}{\sqrt{3}\,V_N} = \frac{5,000,000}{\sqrt{3} \cdot 6,600} = 437.39 \text{ A}$$

Then, one has:

$$\begin{aligned}
\bar{E}_r &= \bar{V} + (R_a + jX_d)\bar{I} \\
\bar{E}_r &= \frac{6,600}{\sqrt{3}}\angle 0^\circ + (0.2 + j1)437.39\angle - 53.13^\circ \\
&= 4217.3\angle 2.62^\circ \text{ V}
\end{aligned}$$

Hence the I_r required to produce E_r is:

$$\begin{aligned}
E_r &= \frac{7,400I_r}{85 + I_r} \\
\Rightarrow 4,217.3 &= \frac{7,400I_r}{85 + I_r} \\
\Rightarrow I_r &= 112.63 \text{ A}
\end{aligned}$$

The excitation current is given by:

$$\bar{I}_e = \bar{I}_r - \frac{\bar{I}}{K_P}$$

where

$$\begin{aligned}
\bar{I}_r &= I_r(\angle\bar{E}_r + 90^\circ) \\
&= 112.63\angle 2.62^\circ + 90^\circ \\
&= 112.63\angle 92.62^\circ \text{ A}
\end{aligned}$$

and

$$\begin{aligned}
\bar{I}_c = \frac{\bar{I}}{K_p} &= 20\angle - 53.13^\circ \\
\Rightarrow \cos\varphi &= 0.6 \text{ (lagging)}
\end{aligned}$$

Hence:

$$\begin{aligned}
\bar{I}_e &= \bar{I}_r - \frac{\bar{I}}{K_p} \\
&= 112.63\angle 92.62^\circ - 20\angle - 53.13^\circ \\
&= 129.64\angle 97.6^\circ \text{ A}
\end{aligned}$$

Finally:

$$\begin{aligned}
& I_e(\text{open-circuit}) = 90.23 \text{ A} \\
& I_e(\text{full-load}, \cos\varphi = 0.6 \text{ lagging}) = 129.64 \text{ A} \\
\Rightarrow \quad & \Delta I \approx 40 \text{ A}
\end{aligned}$$

b. Let's compute the efficiency:

$$\begin{aligned}
P_e &= S_N \cos\varphi = 5000 \cdot 0.6 = 3000 \text{ kW} \\
P_{\text{Fe}} + P_{m,\text{losses}} &= 100 \text{ kW} \\
P_{\text{ex}} &= V_e I_e = 200 \cdot 129.6 = 25.92 \text{ kW} \\
P_{j,a} &= 3 R_a I^2 = 3 \cdot 0.2 \cdot 437.39^2 = 114.8 \text{ kW} \\
P_m &= P_e + P_{\text{Fe}} + P_{j,a} + P_{m,\text{losses}} \\
P_{\text{tot}} &= P_m + P_{\text{ex}} = 3,240.72 \text{ kW}
\end{aligned}$$

Finally:

$$\eta = \frac{P_e}{P_{\text{tot}}} = \frac{3,000}{3,240.72} 100 = 92.6\%$$

Solution of Problem 3

a. Blondell Model $\Rightarrow \bar{E} = \bar{V} + jX_d\bar{I}_d + jX_q\bar{I}_q$

Let's use $\bar{E}' = \bar{E} - j(X_d - X_q)\bar{I}_d$

Then we can write:

$$\bar{E}' = \bar{V} + jX_q\bar{I}$$

At full load:

$$\begin{aligned}
I &= \frac{S_N}{\sqrt{3}V} = \frac{1,500,000}{\sqrt{3} \cdot 3,000} = 288.7 \text{ A} \\
\bar{V} &= \frac{3,000}{\sqrt{3}} \angle 0^\circ = 1,732.1 \angle 0^\circ \text{ V (phase reference)} \\
\bar{I} &= 288.7 \angle -36.87^\circ \text{ A} \quad (\cos\varphi = 0.8 \text{ lagging})
\end{aligned}$$

Then

$$\begin{aligned}
\bar{E}' &= \bar{V} + jX_q\bar{I} \\
&= 1732.1\angle 0^\circ + j1.5 \cdot 288.7 \angle -35.87^\circ \\
&= 2022 \angle 9.9^\circ \\
\Rightarrow \delta &= 9.9^\circ \\
\psi &= \delta + \varphi = 9.9^\circ + 36.87^\circ = 46.77^\circ
\end{aligned}$$

$$\begin{cases} I_d = I \sin\psi = 288.7 \sin 46.77^\circ = 198 \text{ A} \\ I_q = I \cos\psi = 288.7 \cos 46.77^\circ = 210 \text{ A} \end{cases}$$

Then

$$\begin{aligned}
&(X_d - X_q) I_d = (2 - 1.5) \cdot 198 = 99 \text{ V} \\
&E = E' + (X_d + X_q) I_d = 2,022 + 99 = 2,121 \text{ V} \\
&\bar{E} = 2,121 \angle 9.9^\circ \text{ V} \\
&\rightarrow E(\text{phase-to-phase}) = \sqrt{3} \cdot 2,121 = 3,674 \text{ V}
\end{aligned}$$

b. To determine the maximum active power, we know that:

$$P_{em} = \frac{3EV}{X_d}\sin\delta + 3\frac{V^2}{2}(\frac{1}{X_q} - \frac{1}{X_d})\sin 2\delta$$
$$P_{em}^{\max}(\delta_M) \quad \Rightarrow \quad \frac{\partial P_{em}}{\partial \delta}|_{\delta_M = 0}$$
$$\frac{\partial P_{em}}{\partial \delta} = 0 = \frac{3EV}{X_d}\cos\delta + V^2(\frac{1}{X_q} - \frac{1}{X_d})\cos 2\delta$$

Substituting the values of E, V, X_d and X_q:

$$0 = 5.51\cos\delta + 1.5\cos 2\delta \tag{2}$$

Since

$$\cos 2\delta = \cos^2\delta - \sin^2\delta = 2\cos^2\delta - 1$$

Equation (2) becomes:

$$3\cos^2\delta + 5.51\cos\delta - 1.5 = 0$$

$\Rightarrow$ 2 solutions with known $\cos\delta = y$:

$$3y^2 + 5.51y - 1.5 = 0$$
$$\Rightarrow \quad \delta = 76^\circ$$

Hence:

$$P_{em}^{\max} = 35.31\sin 76^\circ + 0.75\sin 152^\circ = 5.7 \text{ MW}$$