Worked Problems on Power Systems

EEEN20090 – Electrical Energy Systems

Problem 1

Consider the 3-phase system shown in Figure 1. Determine (P_G, Q_G) and $\cos\phi_G$ at the generator bus.

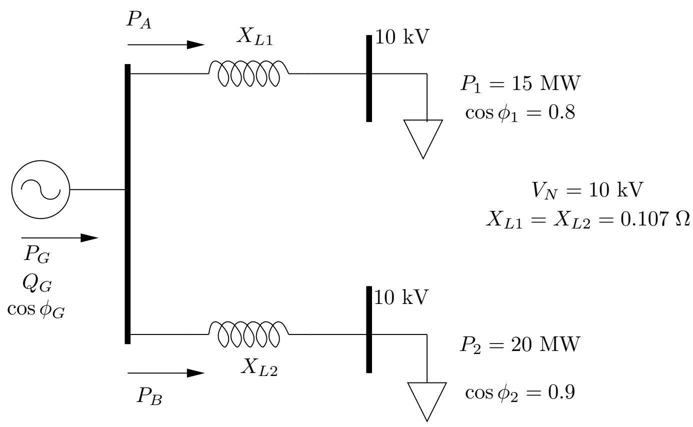

Figure 1

Problem 2

Consider the system shown in Figure 2.

a. Determine P_A, Q_A and $\cos\phi_A$ at the feeder connection A.

b. Determine the voltage $\bar{V}_A$.

Problem 3

Consider the non-balanced system shown in Figure 3.
Data:

$\bar{Z}_A = 10\ \Omega$

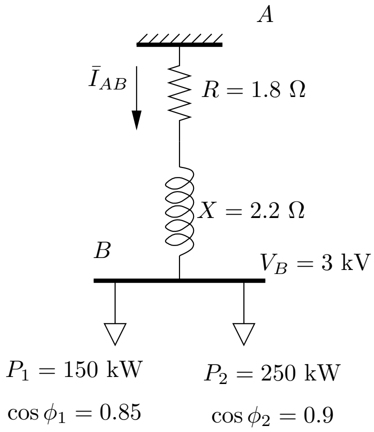

Figure 2

$\bar{Z}_B = 5 - j5\ \Omega$

$\bar{Z}_C = 10\ \Omega$

Motor M: $P_M = 2500W$, $\cos\phi_M = 0.8$ lagging

Nominal Voltage: $V_N = 380$ V.

Determine:

- $\bar{I}_A$, $\bar{I}_B$ and $\bar{I}_C$.
- The total power produced by the three-phase generator (P_G, Q_G) and $\cos\phi_G$ to feed the load and the motor.

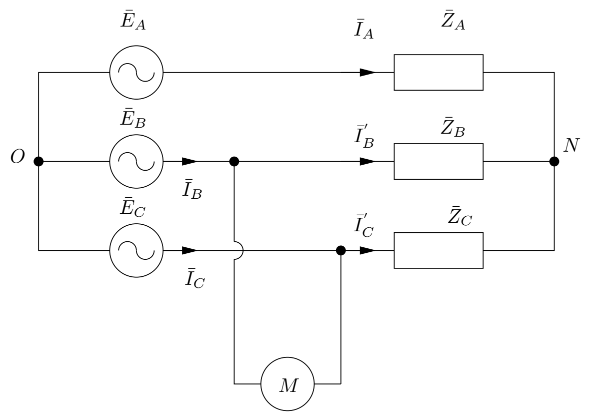

Figure 3

Solution of Problem 1

The feeder power is:

$$P_G = P_A + P_B$$
$$Q_G = Q_A + Q_B$$

where:

$$P_A = P_1 = 15 \text{ MW} \quad \text{(no resistive losses)}$$
$$P_B = P_2 = 20 \text{ MW}$$
$$Q_A = Q_1 + Q_{L1}$$
$$Q_B = Q_2 + Q_{L2}$$

where:

$$Q_1 = P_1 \tan\phi_1 = 11.25 \text{ MVAr}$$
$$Q_2 = P_2 \tan\phi_2 = 9.68 \text{ MVAr}$$
$$Q_{L1} = 3X_{L1}I_1^2 = 0.35 \text{ MVAr}$$
$$Q_{L2} = 3X_{L2}I_2^2 = 0.49 \text{ MVAr}$$

where:

$$I_1 = \frac{P_1}{\sqrt{3}V_N \cos\phi_1} = 1.08 \text{ kA}$$
$$I_2 = \frac{P_2}{\sqrt{3}V_N \cos\phi_2} = 1.28 \text{ kA}$$

Finally:

$$
\begin{aligned}
P_G &= 35 \text{ MW} \\
Q_G &= Q_A + Q_B = 11.6 + 10.5 = 22.1 \text{ MVAr} \\
\cos\phi_G &= \frac{P_G}{\sqrt{P_G^2 + Q_G^2}} = 0.846 \text{ lagging}
\end{aligned}
$$

Solution of Problem 2

$$
\begin{aligned}
P_B &= P_1 + P_2 = 400 \text{ kW} \\
Q_B &= Q_1 + Q_2 = P_1 \tan\phi_1 + P_2 \tan\phi_2 \\
&= 92.96 + 121.08 = 214.02 \text{ kVAr} \\
\cos\phi_B &= \frac{P_B}{\sqrt{P_B^2 + Q_B^2}} = 0.88 \text{ (lagging)} \\
I_{AB} &= \frac{P_B}{\sqrt{3} V_B \cos\phi_B} = 87.48 \text{ A}
\end{aligned}
$$

Let's assume that $\bar{E}_B$ is the phase reference:

$$
\bar{E}_B = \frac{V_B}{\sqrt{3}\angle 0} = \sqrt{3}\angle 0 \text{ kV}
$$

then:

$$
I_{AB} = \frac{P_B - jQ_B}{3E_B} = 87.48 e^{-j28.3^\circ}
$$

then:

$$
\begin{aligned}
\bar{E}_A &= \bar{E}_B + (R + jX)\bar{I}_{AB} = 1.963 e^{j2.8^\circ} \\
V_A &= \sqrt{3} E_A = \sqrt{3} \cdot 1.963 \text{ kV} \\
P_A &= P_B + 3RI_{AB}^2 = 400 + 41.32 = 441.32 \text{ kW} \\
Q_A &= Q_B + 3XI_{AB}^2 = 214.02 + 50.51 = 264.55 \text{ kVAr} \\
\cos\phi_A &= \frac{P_A}{\sqrt{P_A^2 + Q_A^2}} = \cos(\tan^{-1}(\frac{Q_A}{P_A})) = 0.857
\end{aligned}
$$

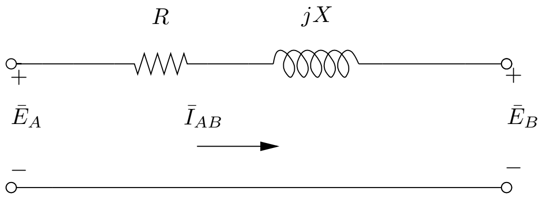

Figure 4

Solution of Problem 3

Let $\bar{E}_A = E\angle 0$ (phase reference). Then:

$$E = \frac{V_N}{\sqrt{3}} = 220 \text{ V}$$
$$\bar{V}_{NO} = \frac{\bar{E}_A\bar{Y}_A + \bar{E}_B\bar{Y}_B + \bar{E}_C\bar{Y}_C}{\bar{Y}_A + \bar{Y}_B + \bar{Y}_C} = 69.6e^{-j48^\circ}$$
$$\bar{I}_A' = \frac{\bar{E}_A - \bar{V}_{No}}{\bar{Z}_A} = 18.1e^{j16.8^\circ} \text{ A}$$
$$\bar{I}_B' = \frac{\bar{E}_B - \bar{V}_{No}}{\bar{Z}_B} = 29.5e^{j93.4^\circ} \text{ A}$$
$$\bar{I}_C' = \frac{\bar{E}_C - \bar{V}_{No}}{\bar{Z}_C} = 28.8e^{j122.7^\circ} \text{ A}$$

Single-phase load:

$$P_M = V_M I_M \cos(\phi_M)$$
$$\Rightarrow I_M = \frac{P_M}{V_M \cos(\phi_M)} = 8.77 \text{ A}$$

Note that:

$$\cos\phi_M = 0.8 \qquad \Rightarrow \phi_M = 36.8^\circ \text{ (lagging)}$$

Let's now determine the phase of $\bar{I}_M$ (see Figure).

The voltage is $\bar{V}_M = \bar{V}_{BC}$:

$$\bar{V}_{BC} = \sqrt{3}\, E\angle -90^\circ$$
$$\Rightarrow \bar{I}_M = 8.77e^{-j(90^\circ+36.8^\circ)} = 8.77e^{-j126.8^\circ} \text{ A}$$
$$\bar{I}_B = \bar{I}_B' + \bar{I}_M = 37.1e^{-j100^\circ} \text{ A}$$
$$\bar{I}_C = \bar{I}_C' + (-\bar{I}_M) = 32.9e^{j108^\circ} \text{ A}$$

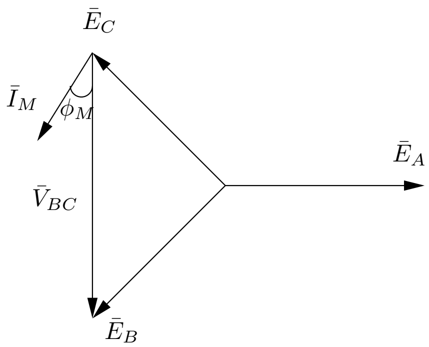

Figure 5

Total power:

$$\begin{aligned}
P_G &= P_A + P_B + P_C \\
&= E_A I_A \cos\phi_A + E_B I_B \cos\phi_B + E_C I_C \cos\phi_C \\
&= 3.816 + 7.669 + 7.079 \\
&= 18.564 \text{ W}
\end{aligned}$$

$$\begin{aligned}
Q_G &= Q_A + Q_B + Q_C \\
&= E_A I_A \sin\phi_A + E_B I_B \sin\phi_B + E_C I_C \sin\phi_C \\
&= -1137 - 2791 + 1504 \\
&= -2424 \text{ VAr}
\end{aligned}$$

Finally, the power factor is:

$$\cos\phi_G = \frac{P_G}{\sqrt{P_G^2 + Q_G^2}} = 0.991 \text{ (leading)}$$