Three Phase Systems

ELECTRICAL ENERGY SYSTEMS (EEEN20090)

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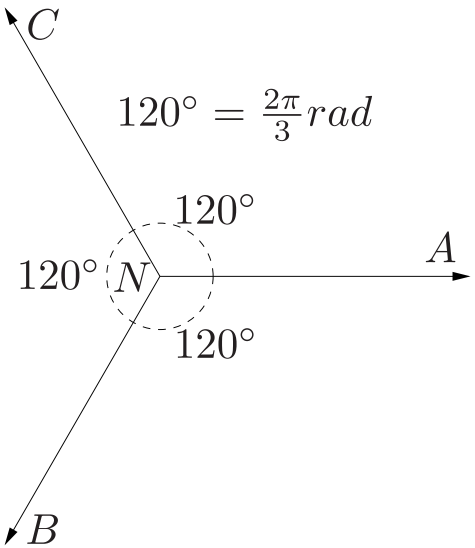
Phase Voltages

- 3 Phases: ABC (RST) (UVW)

$$e_A(t) = \sqrt{2}E\cos\omega t$$
$$e_B(t) = \sqrt{2}E\cos(\omega t - \tfrac{2\pi}{3})$$
$$e_C(t) = \sqrt{2}E\cos(\omega t + \tfrac{2\pi}{3})$$



- Phase Voltages or Phase-to-Neutral (N) Voltages: $\overline{E}_A, \overline{E}_B, \overline{E}_C$
- If the system is symmetrical: $|\overline{E}_A| = |\overline{E}_B| = |\overline{E}_C| = E$

$$\left.\begin{aligned} \overline{E}_A &= E\angle 0^\circ \\ \overline{E}_B &= E\angle -120^\circ \\ \overline{E}_C &= E\angle +120^\circ \end{aligned}\right\} \Rightarrow \overline{E}_A + \overline{E}_B + \overline{E}_C = 0$$

Phase-to-Phase Voltages

- It is possible to define a set of phase-to-phase voltages (also called ”line voltages”):

$$\overline{V}_{AB} = \overline{E}_A - \overline{E}_B$$
$$\overline{V}_{BC} = \overline{E}_B - \overline{E}_C$$
$$\overline{V}_{CA} = \overline{E}_C - \overline{E}_A$$

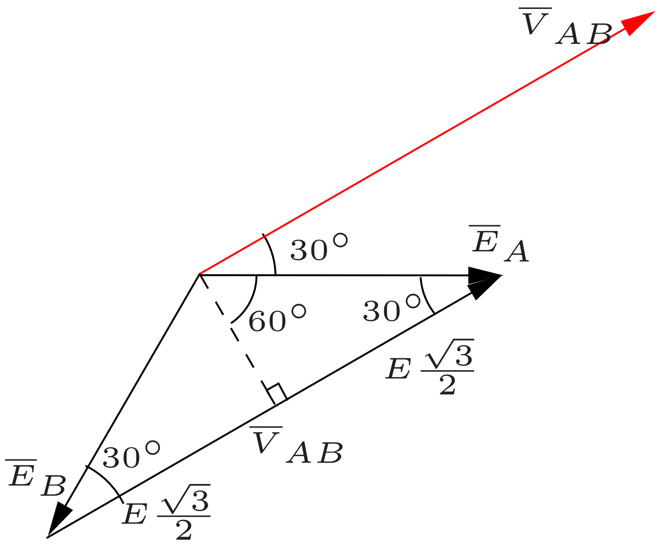
  - We have:

$$\begin{aligned}\overline{V}_{AB} = \overline{E}_A - \overline{E}_B = E - E\angle - 120^\circ = E(1 - \angle - 120^\circ) \\ = E(1 - (\cos 120^\circ - j\sin 120^\circ)) = E(\tfrac{3}{2} + j\tfrac{\sqrt{3}}{2}) = \sqrt{3}E(\tfrac{\sqrt{3}}{2} + j\tfrac{1}{2}) \\ = \sqrt{3}E\angle 30^\circ)\end{aligned}$$

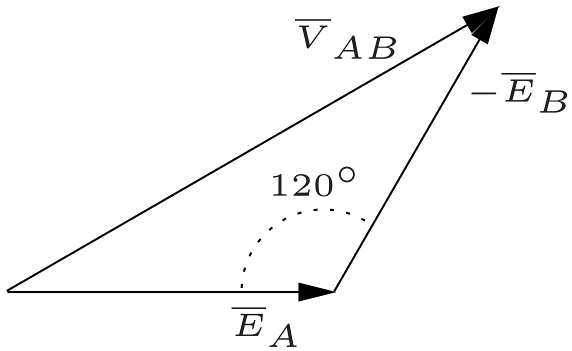
  - Similarly, we can obtain $\overline{V}_{BC}$ and $\overline{V}_{CA}$.

Graphical Method

- We can also use a graphical method:



$$\Rightarrow \quad |\overline{V}_{AB}| = \sqrt{3}E$$
$$\angle\overline{V}_{AB} = 30^\circ$$



$$\Rightarrow \quad \overline{V}_{AB} = \overline{E}_A - \overline{E}_B$$

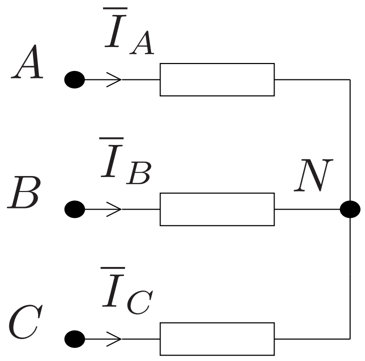
Star and Delta Connections

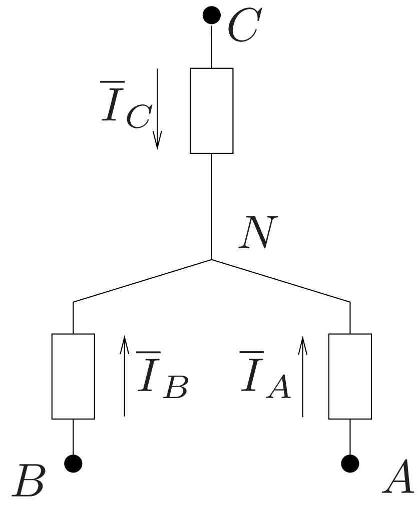
- We can also define 3-phase currents. There are two possible connections:
  - **Star Connection** (Y)

    Line Currents:

    $\overline{I}_A, \overline{I}_B, \overline{I}_C$

    $\overline{I}_A + \overline{I}_B + \overline{I}_C = 0$

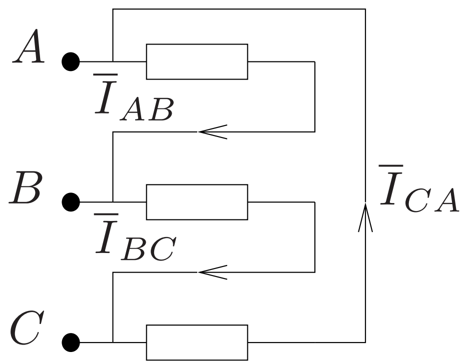


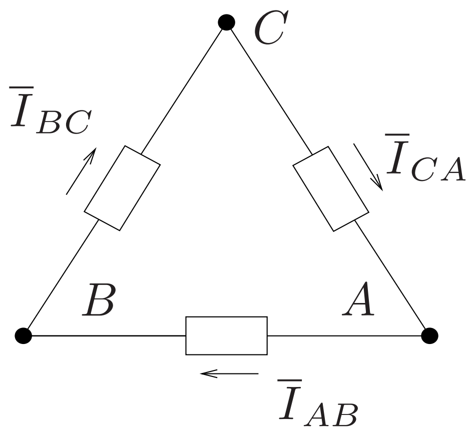


  - **Delta or Triangle Connection** (Δ)

    Phase Currents:

    $\overline{I}_{AB}, \overline{I}_{BC}, \overline{I}_{CA}$





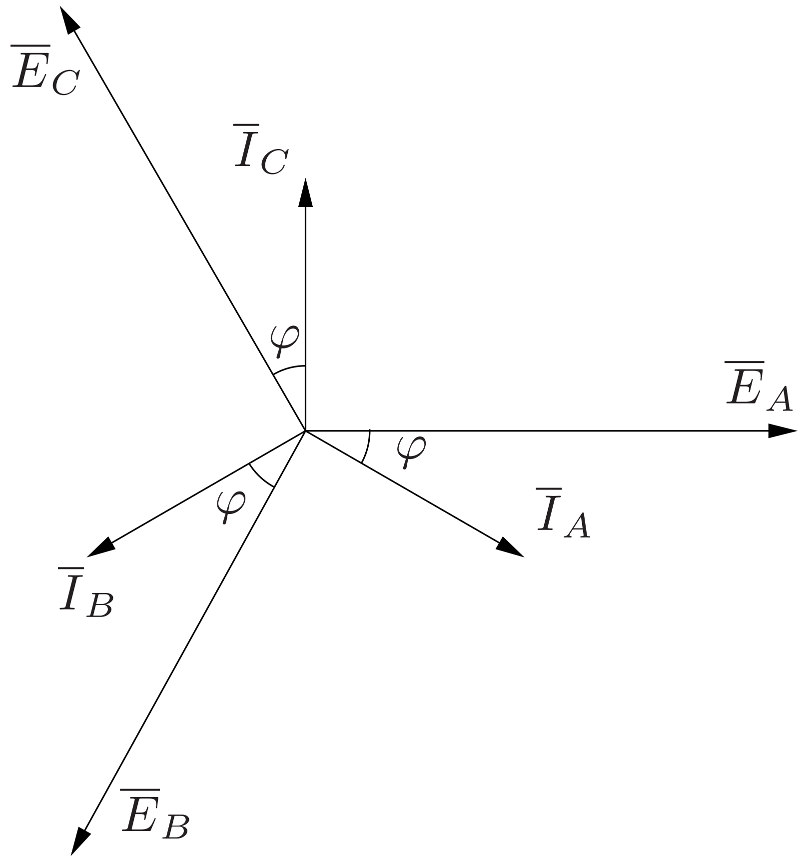
Line and Phase Currents

- If the impedances of each phase are equal and the voltages are symmetrical, the currents are balanced:

$$\overline{I}_A = I_L \angle -\varphi$$

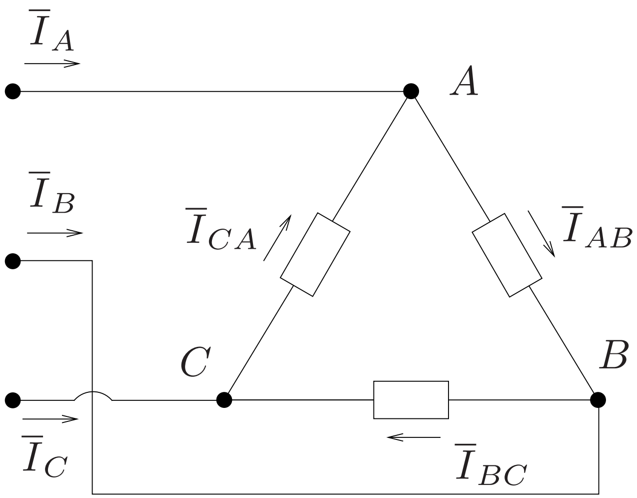
$$\overline{I}_B = I_L \angle -\varphi - 120^\circ$$

$$\overline{I}_C = I_L \angle -\varphi + 120^\circ$$



Line and Phase Currents

- Relationship between line and phase currents:



$$\left.\begin{array}{l}\overline{I}_A = \overline{I}_{AB} - \overline{I}_{CA}\\ \overline{I}_B = \overline{I}_{BC} - \overline{I}_{AB}\\ \overline{I}_C = \overline{I}_{CA} - \overline{I}_{BC}\end{array}\right\} \Rightarrow \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} \overline{I}_{AB} \\ \overline{I}_{BC} \\ \overline{I}_{CA} \end{bmatrix} = \begin{bmatrix} \overline{I}_A \\ \overline{I}_B \\ \overline{I}_C \end{bmatrix}$$

$$\Rightarrow |\overline{I}_{AB}| = \frac{1}{\sqrt{3}}|\overline{I}_A| = \frac{1}{\sqrt{3}} I_L = I_f$$

Equivalent $Y - \Delta$ Connection

- Equivalent connection means that the 3-phase power is the same:

$$\begin{aligned}\overline{S} &= 3\overline{z}_Y I_L^2 \\ &= 3\overline{z}_\Delta I_f^2 \\ &= 3\overline{z}_\Delta \frac{I_L^2}{3}\end{aligned}$$

- Thus:

$$\overline{z}_\Delta = 3\overline{z}_Y$$

or

$$\overline{z}_Y = \tfrac{1}{3}\overline{z}_\Delta$$

Power in Single Phase Systems

- For a single phase system, one has:

$$p = vi = V_M I_M \sin(\omega t) \sin(\omega t - \varphi)$$

$$= \frac{1}{2} V_M I_M (\cos\varphi - \cos(2\omega t - \varphi))$$

$$= \underbrace{VI\cos\varphi}_{\text{constant}} - \underbrace{VI\cos(2\omega t - \varphi)}_{\text{zero-average term}}$$

Power in Three Phase Systems

- For a three phase system, one has:

$$
\begin{aligned}
p &= e_A i_A + e_B i_B + e_C i_C \\
&= E_M I_M \sin(\omega t) \sin(\omega t - \varphi) + \\
&\quad E_M I_M \sin(\omega t - 120^\circ) \sin(\omega t - \varphi - 120^\circ) + \\
&\quad E_M I_M \sin(\omega t + 120^\circ) \sin(\omega t - \varphi + 120^\circ) \\
&= \underbrace{3 \frac{E_M I_M}{2} \cos\varphi}_{\text{only constant term}}
\end{aligned}
$$

- In fact:

$$
\cos(2\omega t - \varphi) + \cos(2\omega t - \varphi - 120^\circ) + \cos(2\omega t - \varphi + 120^\circ) = 0
$$

Three-Phase Power

- The relation above applies to:
  - phase-to-neutral voltages and line currents.
  - phase-to-phase voltages and phase currents.
- There are various ways to calculate the active power:

$$P = 3EI_L \cos\varphi = 3\frac{V}{\sqrt{3}} I_L \cos\varphi = \sqrt{3} V I_L \cos\varphi = 3 V I_f \cos\varphi$$

Nominal Quantities in Three Phase Systems

- Nominal quantities are **always** defined as follows:
  - S_N : three phase power (MVA)
  - V_N : phase to phase voltage (kV)
  - I_N : line current (kA)
- Hence,

$$S_N = \sqrt{3} V_N I_N$$

- **Tip**: To solve circuits, use phase-to-ground voltage and line current quantities:

$$\overline{S} = 3\overline{E}\,\overline{I}_L^*$$

Comparison of three-phase and single-phase systems

- Three phase systems are better than single phase ones for both technical and economical reasons.
- A relevant issue is related to losses: $P_{\text{losses}} \propto RI^2$
- We cannot reduce R, mainly for economical reasons:

$$R = \rho\frac{\ell}{A}$$

  - ρ (resistivity): Depends on the material. Only copper and aluminum are economically viable. Hence ρ is basically assigned.
  - ℓ : Depends on the distances-nothing to do.
  - A : Cross section. It can be increased to reduce R, but the bigger A is, the higher the cost of transmission lines, the weight etc. Thus, increasing A is not viable.
- Then, the only way is to decrease I by increasing V. However, V cannot be high in distribution systems (for safety reasons).

Comparison of three-phase and single-phase systems

- Assume that the two systems feed the same load P_L, $\cos\varphi_L$.
- V is also the same for both systems.

1-phase AC system	**3-phase AC system**
$I_s = \frac{P_L}{V \cos\varphi_L}$	$I_t = \frac{P_L}{\sqrt{3}V \cos\varphi_L}$
$P_{\text{losses}} = 2R_s I_s^2 = 2\rho \frac{l}{A_s} (\frac{P_L}{V \cos\varphi_L})^2$	$P_{\text{losses}} = 3R_t I_t^2 = 3\rho \frac{l}{A_t} (\frac{P_L}{\sqrt{3}V \cos\varphi_L})^2$

- If we impose that the two systems have same losses, we obtain: $A_s = 2A_t$
- In the single-phase, we need 2 conductors: $A_{s,\text{tot}} = 2A_s$
- In the three-phase, we need 3 conductors: $A_{t,\text{tot}} = 3A_t$
- Hence, $A_{s,\text{tot}} = 2A_s = 4A_t = \frac{4}{3}A_{t,\text{tot}}$ which means that the three-phase system requires 25% less material to feed the same load and have the same losses with the single-phase system.

Comparison between DC and three-phase AC systems

- Let's assume again same load P_L and same V for both systems.

DC system	3-phase AC system
$I_{\mathrm{DC}} = \frac{P_L}{V}$	$I_t = \frac{P_L}{\sqrt{3}V\cos\varphi_L}$
$P_{\mathrm{losses}} = 2R_{\mathrm{DC}}I_{\mathrm{DC}}^2 = 2\rho\frac{l}{A_{\mathrm{DC}}}(\frac{P_L}{V})^2$	$P_{\mathrm{losses}} = 3R_tI_t^2 = 3\rho\frac{l}{S_t}(\frac{P_L}{\sqrt{3}V\cos\varphi_L})^2$

- If we impose that the two systems have same losses, we obtain: $\frac{2}{A_{\mathrm{DC}}} = \frac{1}{A_t\cos^2\varphi_L}$

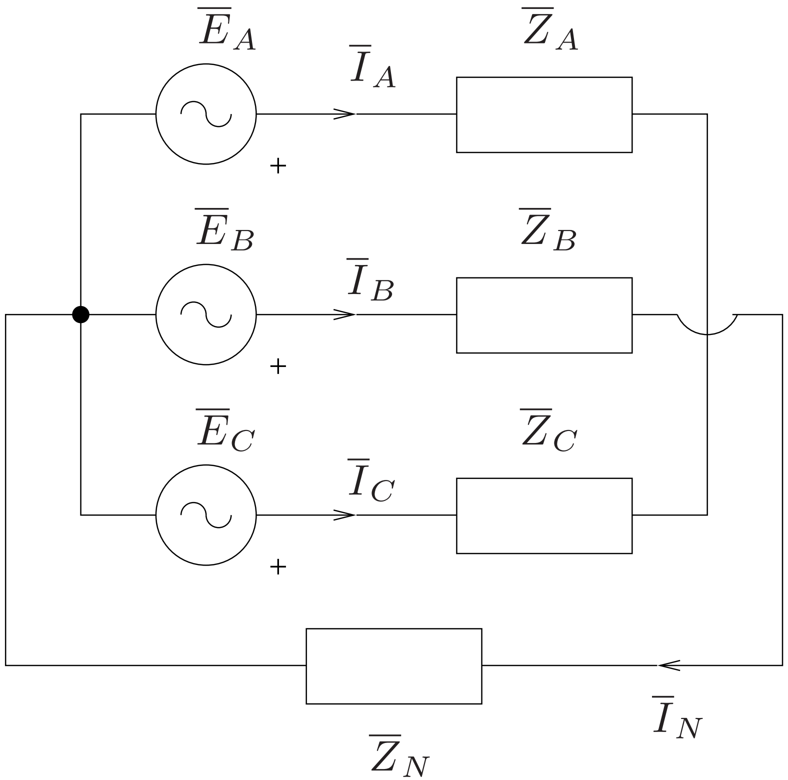
$$\left.\begin{array}{l} A_{\mathrm{DC,tot}} = 2A_{\mathrm{DC}} \\ A_{t,\mathrm{tot}} = 3A_t \end{array}\right\} \Rightarrow A_{\mathrm{DC,tot}} = 2S_{\mathrm{DC}} = 4A_t cos^2\varphi_L = \frac{4}{3}\cos^2\varphi_L A_{t,\mathrm{tot}}$$

- Hence, $A_{\mathrm{DC,tot}} < A_{t,\mathrm{tot}}$ only if $\cos\varphi_L \leq \frac{\sqrt{3}}{2} = 0.866$
- Thus, in AC systems $cos\varphi_L$ is bounded. Typically, loads must satisfy the condition:

$$1 \geq \cos\varphi_L \geq 0.9 \text{ (lagging)}$$

Single-phase equivalent of a three-phase system

- It is possible to study symmetrical and balanced 3-phase systems using an equivalent single-phase circuit.
- Let's consider the following circuit:



$$\overline{E}_A = \overline{Z}_A \overline{I}_A + \overline{Z}_N \overline{I}_N$$
$$\overline{E}_B = \overline{Z}_B \overline{I}_B + \overline{Z}_N \overline{I}_N$$
$$\overline{E}_C = \overline{Z}_C \overline{I}_C + \overline{Z}_N \overline{I}_N$$

Single-phase equivalent of a three-phase system

- If the voltages are symmetrical:

$$\overline{E}_A + \overline{E}_B + \overline{E}_C = 0 \Rightarrow \overline{Z}_A\overline{I}_A + \overline{Z}_B\overline{I}_B + \overline{Z}_C\overline{I}_C + 3\overline{Z}_N\overline{I}_N = 0$$

- If the system is also balanced, (i.e. $\overline{Z}_A = \overline{Z}_B = \overline{Z}_C = \overline{Z}$) :

$$\overline{Z}(\overline{I}_A + \overline{I}_B + \overline{I}_C) = -3\overline{Z}_N\overline{I}_N$$

$$\overline{I}_A + \overline{I}_B + \overline{I}_C = 0 \ \Rightarrow \ \overline{I}_N = 0$$

- If the system is not balanced, (i.e. $\overline{Z}_A \neq \overline{Z}_B \neq \overline{Z}_C$) :

$$\overline{I}_A = \frac{\overline{E}_A - \overline{V}_N}{\overline{Z}_A} \, , \, \overline{I}_B = \frac{\overline{E}_B - \overline{V}_N}{\overline{Z}_B} \, , \, \overline{I}_C = \frac{\overline{E}_C - \overline{V}_N}{\overline{Z}_C}$$

where: $\overline{V}_N = \overline{Z}_N\overline{I}_N$

Single-phase equivalent of a three-phase system

- Let:

$$\overline{Y}_A = \frac{1}{\overline{Z}_A}\,, \qquad \overline{Y}_B = \frac{1}{\overline{Z}_B}\,, \qquad \overline{Y}_C = \frac{1}{\overline{Z}_C}\,, \qquad \overline{Y}_N = \frac{1}{\overline{Z}_N}$$

- The following condition holds (LKC):

$$\begin{aligned} & \overline{I}_A + \overline{I}_B + \overline{I}_C = \overline{I}_N \\ \Rightarrow\quad & \overline{Y}_A(\overline{E}_A - \overline{V}_N) + \overline{Y}_B(\overline{E}_B - \overline{V}_N) + \overline{Y}_C(\overline{E}_C - \overline{V}_N) = \overline{Y}_N\overline{V}_N \\ \Rightarrow\quad & \overline{V}_N = \frac{\overline{Y}_A\overline{E}_A + \overline{Y}_B\overline{E}_B + \overline{Y}_C\overline{E}_C}{\overline{Y}_A + \overline{Y}_B + \overline{Y}_C + \overline{Y}_N} \end{aligned}$$

- If $\overline{Z}_N = 0$ (i.e., neutral connection with negligible impedance):
  - $\overline{Y}_N \to \infty$
  - $\overline{V}_N = 0$

Single-phase equivalent of a three-phase system

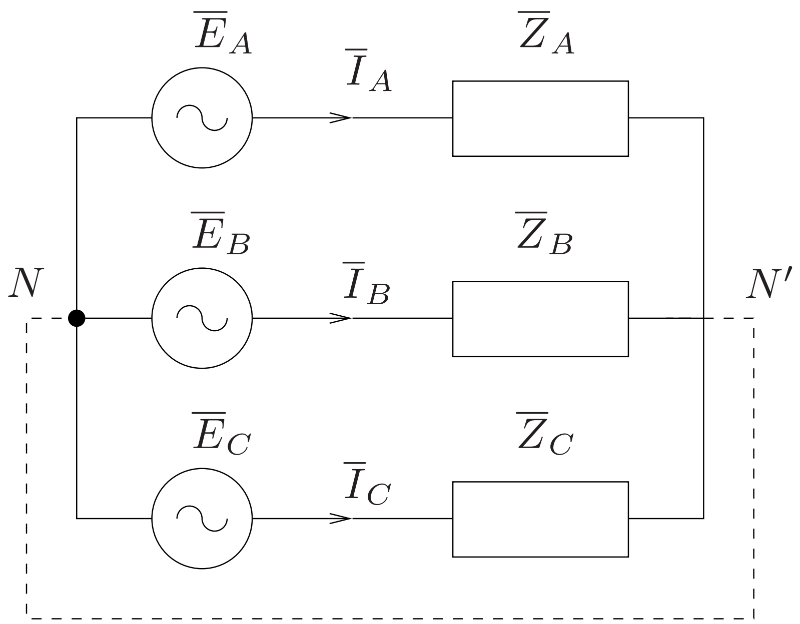
- If $\overline{Z}_N \to \infty$ (i.e., no neutral connection):
  - $\overline{Y}_N = 0$
  - $\overline{V}_N = \frac{\overline{Y}_A\overline{E}_A+\overline{Y}_B\overline{E}_B+\overline{Y}_C\overline{E}_C}{\overline{Y}_A+\overline{Y}_B+\overline{Y}_C}$
- If the system is balanced $(\overline{Y}_A = \overline{Y}_B = \overline{Y}_C = \overline{Y})$, we obtain again:

$$\overline{V}_N = \frac{\overline{Y}(\overline{E}_A + \overline{E}_B + \overline{E}_C)}{3\overline{Y} + \overline{Y}_N} = 0$$

- Hence, for a symmetrical and balanced system, **all neutral points have same potential**.

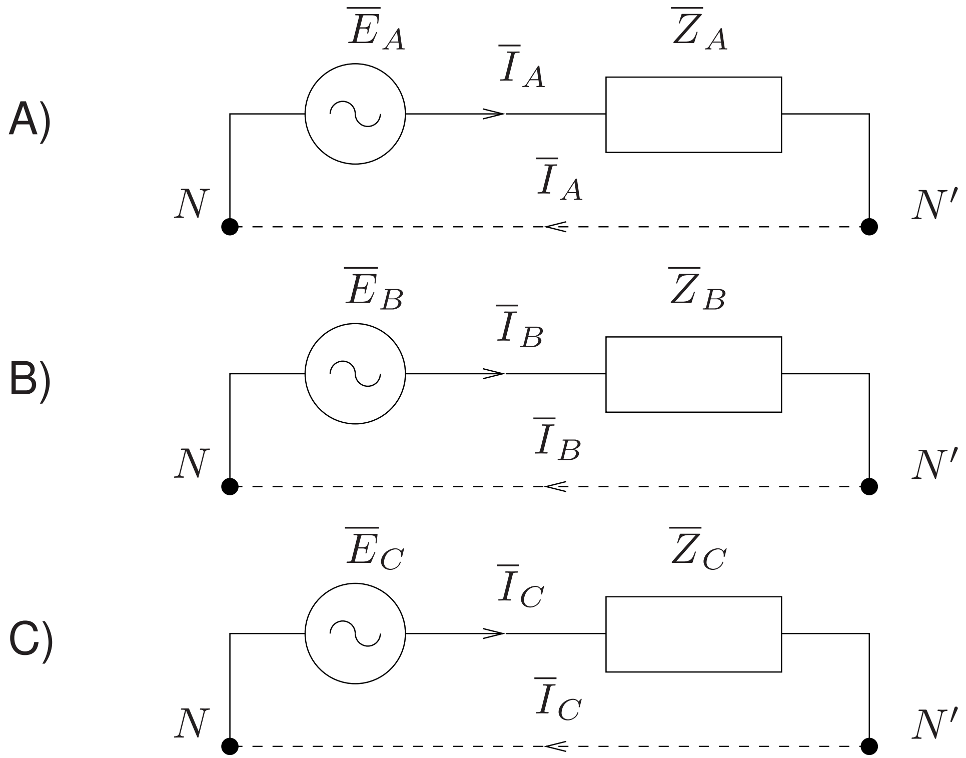
Single-phase equivalent of a three-phase system

- Let’s define as single-phase equivalent of a three-phase system the following circuit:



$$\overline{E}_A + \overline{E}_B + \overline{E}_C = 0$$
$$P = \sqrt{3} V I \cos\varphi = 3 E I \cos\varphi$$
$$\overline{I}_A + \overline{I}_B + \overline{I}_C = 0$$

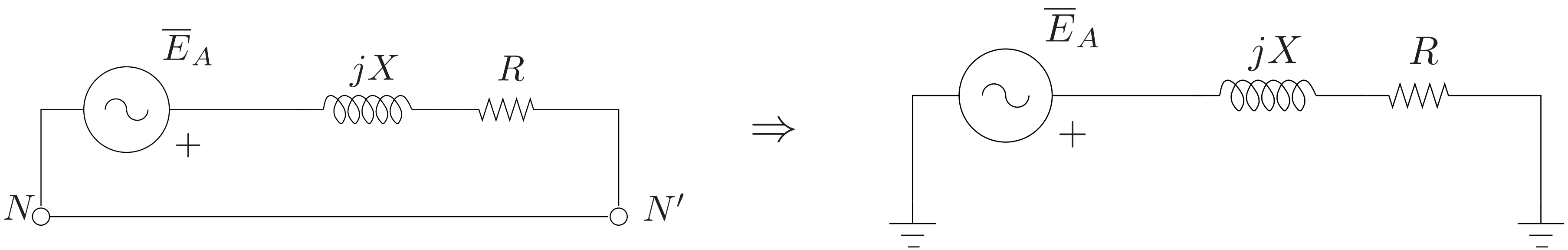
Single-phase equivalent of a three-phase system



- 3 independent circuits. We can study only one, e.g. circuit A.
- Note that: $P_A = P_B = P_C = EI\cos\varphi \;\Rightarrow\; P = 3P_A$

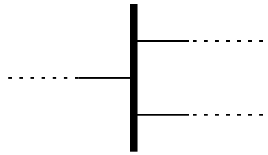
Single line diagram

- We can represent single-phase equivalents using a schematic, simplified representation.
- Since all the neutral points have the same potential, we can omit drawing the neutral connection and represent neutral points with a symbol.
- For example:

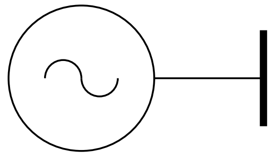


Some Symbols

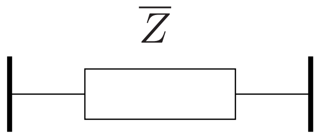
- Bus (connection among elements of the system)



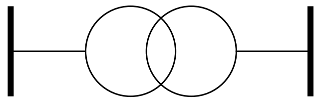
- Generator



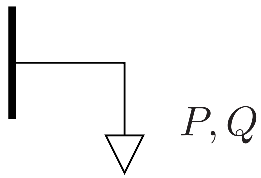
- Impedance (transmission line or cable)



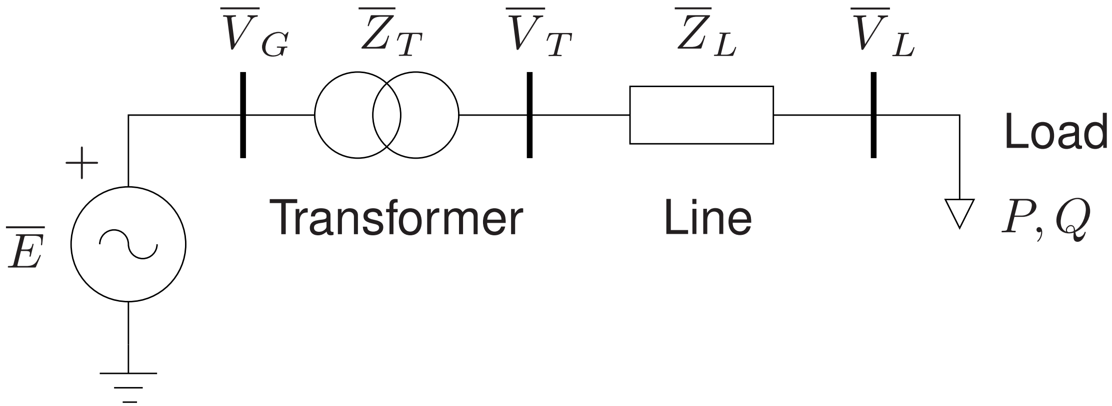
- Transformer



- Load



Example



- Equivalent single phase circuit: