



Three Phase Systems

ELECTRICAL ENERGY SYSTEMS (EEEN20090)

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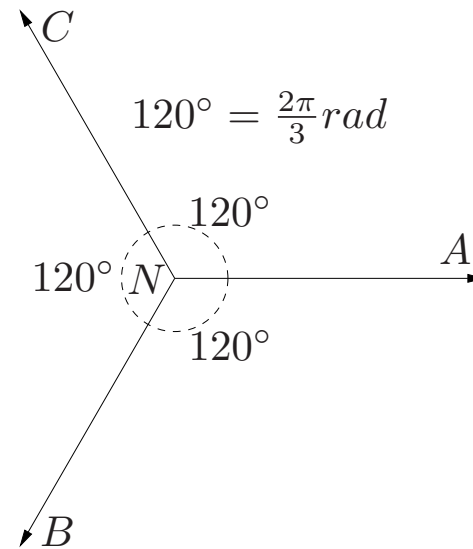
Phase Voltages

- 3 Phases: ABC (RST) (UVW)

$$e_A(t) = \sqrt{2}E \cos \omega t$$

$$e_B(t) = \sqrt{2}E \cos\left(\omega t - \frac{2\pi}{3}\right)$$

$$e_C(t) = \sqrt{2}E \cos\left(\omega t + \frac{2\pi}{3}\right)$$



- Phase Voltages or Phase-to-Neutral (N) Voltages: $\bar{E}_A, \bar{E}_B, \bar{E}_C$
- If the system is symmetrical: $|\bar{E}_A| = |\bar{E}_B| = |\bar{E}_C| = E$

$$\left. \begin{array}{l} \bar{E}_A = E \angle 0^\circ \\ \bar{E}_B = E \angle -120^\circ \\ \bar{E}_C = E \angle +120^\circ \end{array} \right\} \Rightarrow \bar{E}_A + \bar{E}_B + \bar{E}_C = 0$$

Phase-to-Phase Voltages

- It is possible to define a set of phase-to-phase voltages (also called "line voltages"):

$$\bar{V}_{AB} = \bar{E}_A - \bar{E}_B$$

$$\bar{V}_{BC} = \bar{E}_B - \bar{E}_C$$

$$\bar{V}_{CA} = \bar{E}_C - \bar{E}_A$$

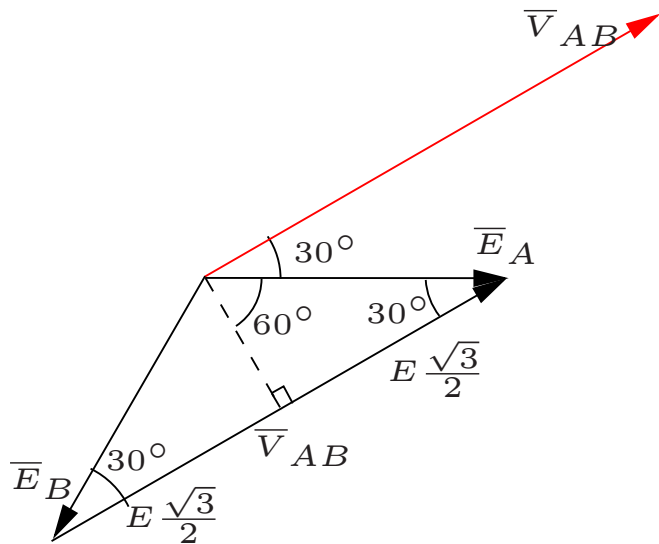
- We have:

$$\begin{aligned}\bar{V}_{AB} &= \bar{E}_A - \bar{E}_B = E - E\angle -120^\circ = E(1 - \angle -120^\circ) \\ &= E(1 - (\cos 120^\circ - j \sin 120^\circ)) = E\left(\frac{3}{2} + j\frac{\sqrt{3}}{2}\right) = \sqrt{3}E\left(\frac{\sqrt{3}}{2} + j\frac{1}{2}\right) \\ &= \sqrt{3}E\angle 30^\circ\end{aligned}$$

- Similarly, we can obtain \bar{V}_{BC} and \bar{V}_{CA} .

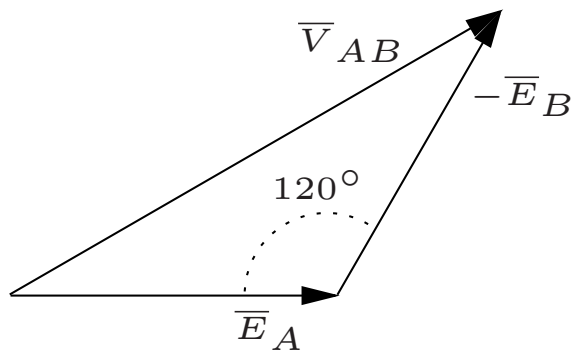
Graphical Method

- We can also use a graphical method:



$$\Rightarrow |\bar{V}_{AB}| = \sqrt{3}E$$

$$\angle \bar{V}_{AB} = 30^\circ$$



$$\Rightarrow \bar{V}_{AB} = \bar{E}_A - \bar{E}_B$$

Star and Delta Connections

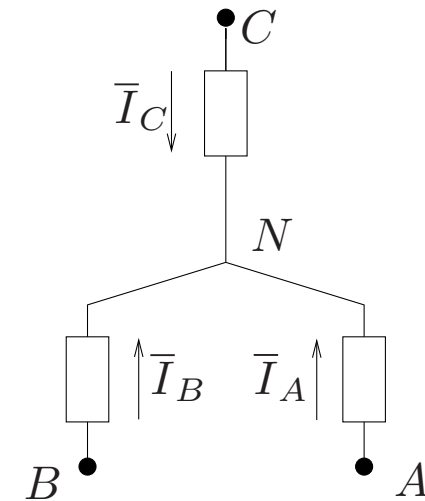
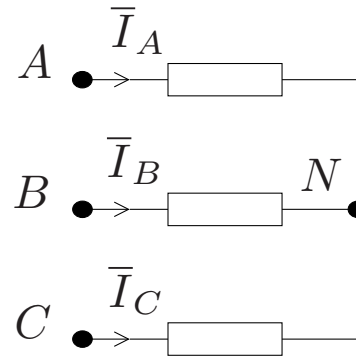
- We can also define 3-phase currents. There are two possible connections:

- Star Connection (Y)**

Line Currents:

$$\bar{I}_A, \bar{I}_B, \bar{I}_C$$

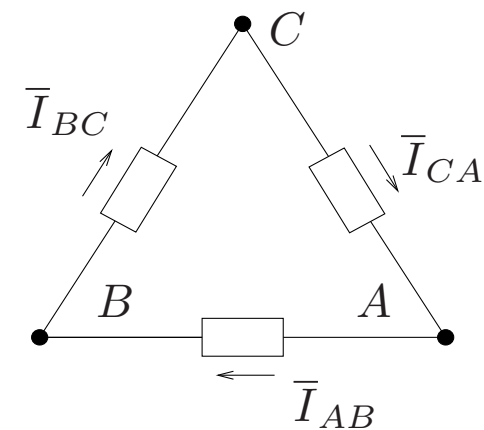
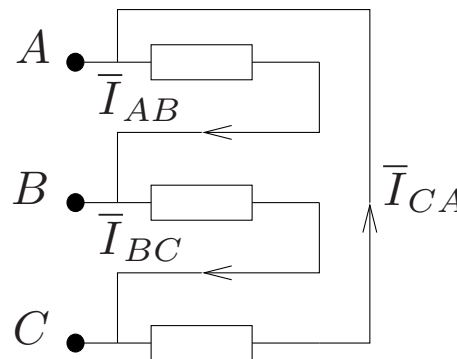
$$\bar{I}_A + \bar{I}_B + \bar{I}_C = 0$$



- Delta or Triangle Connection (Δ)**

Phase Currents:

$$\bar{I}_{AB}, \bar{I}_{BC}, \bar{I}_{CA}$$



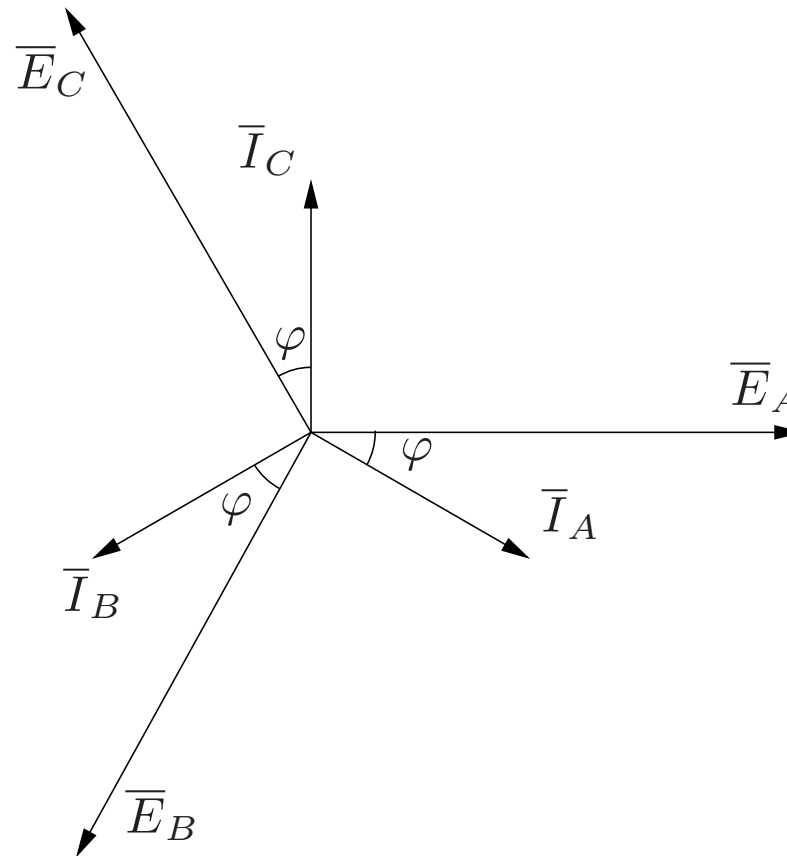
Line and Phase Currents

- If the impedances of each phase are equal and the voltages are symmetrical, the currents are balanced:

$$\bar{I}_A = I_L \angle -\varphi$$

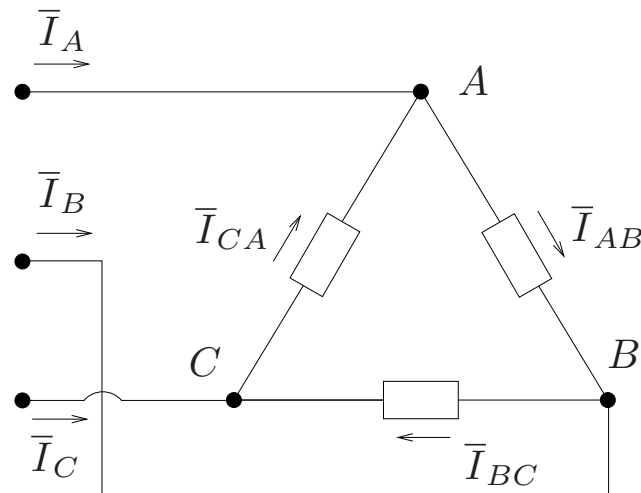
$$\bar{I}_B = I_L \angle -\varphi - 120^\circ$$

$$\bar{I}_C = I_L \angle -\varphi + 120^\circ$$



Line and Phase Currents

- Relationship between line and phase currents:



$$\left. \begin{aligned} \bar{I}_A &= \bar{I}_{AB} - \bar{I}_{CA} \\ \bar{I}_B &= \bar{I}_{BC} - \bar{I}_{AB} \\ \bar{I}_C &= \bar{I}_{CA} - \bar{I}_{BC} \end{aligned} \right\} \Rightarrow \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} \bar{I}_{AB} \\ \bar{I}_{BC} \\ \bar{I}_{CA} \end{bmatrix} = \begin{bmatrix} \bar{I}_A \\ \bar{I}_B \\ \bar{I}_C \end{bmatrix}$$

$$\Rightarrow |\bar{I}_{AB}| = \frac{1}{\sqrt{3}} |\bar{I}_A| = \frac{1}{\sqrt{3}} I_L = I_f$$

Equivalent $Y - \Delta$ Connection

- Equivalent connection means that the 3-phase power is the same:

$$\begin{aligned}\bar{S} &= 3\bar{z}_Y I_L^2 \\ &= 3\bar{z}_\Delta I_f^2 \\ &= 3\bar{z}_\Delta \frac{I_L^2}{3}\end{aligned}$$

- Thus:

$$\bar{z}_\Delta = 3\bar{z}_Y$$

or

$$\bar{z}_Y = \frac{1}{3}\bar{z}_\Delta$$

Power in Single Phase Systems

- For a single phase system, one has:

$$p = vi = V_M I_M \sin(\omega t) \sin(\omega t - \varphi)$$

$$= \frac{1}{2} V_M I_M (\cos \varphi - \cos(2\omega t - \varphi))$$

$$= \underbrace{VI \cos \varphi}_{\text{constant}} - \underbrace{VI \cos(2\omega t - \varphi)}_{\text{zero-average term}}$$

Power in Three Phase Systems

- For a three phase system, one has:

$$\begin{aligned} p &= e_A i_A + e_B i_B + e_C i_C \\ &= E_M I_M \sin(\omega t) \sin(\omega t - \varphi) + \\ &\quad E_M I_M \sin(\omega t - 120^\circ) \sin(\omega t - \varphi - 120^\circ) + \\ &\quad E_M I_M \sin(\omega t + 120^\circ) \sin(\omega t - \varphi + 120^\circ) \\ &= \underbrace{3 \frac{E_M I_M}{2} \cos \varphi}_{\text{only constant term}} \end{aligned}$$

- In fact:

$$\cos(2\omega t - \varphi) + \cos(2\omega t - \varphi - 120^\circ) + \cos(2\omega t - \varphi + 120^\circ) = 0$$

Three-Phase Power

- The relation above applies to:
 - phase-to-neutral voltages and line currents.
 - phase-to-phase voltages and phase currents.
- There are various ways to calculate the active power:

$$P = 3EI_L \cos \varphi = 3 \frac{V}{\sqrt{3}} I_L \cos \varphi = \sqrt{3} V I_L \cos \varphi = 3V I_f \cos \varphi$$



Nominal Quantities in Three Phase Systems

- Nominal quantities are **always** defined as follows:
 - S_N : three phase power (MVA)
 - V_N : phase to phase voltage (kV)
 - I_N : line current (kA)

- Hence,

$$S_N = \sqrt{3}V_N I_N$$

- **Tip:** To solve circuits, use phase-to-ground voltage and line current quantities:

$$\bar{S} = 3\bar{E} \bar{I}_L^*$$

Comparison of three-phase and single-phase systems

- Three phase systems are better than single phase ones for both technical and economical reasons.

- A relevant issue is related to losses: $P_{\text{losses}} \propto RI^2$

- We cannot reduce R , mainly for economical reasons:

$$R = \rho \frac{\ell}{A}$$

- ρ (resistivity): Depends on the material. Only copper and aluminum are economically viable. Hence ρ is basically assigned.
 - ℓ : Depends on the distances-nothing to do.
 - A : Cross section. It can be increased to reduce R , but the bigger A is, the higher the cost of transmission lines, the weight etc. Thus, increasing A is not viable.
- Then, the only way is to decrease I by increasing V . However, V cannot be high in distribution systems (for safety reasons).

Comparison of three-phase and single-phase systems

- Assume that the two systems feed the same load $P_L, \cos \varphi_L$.
- V is also the same for both systems.

1-phase AC system	3-phase AC system
$I_s = \frac{P_L}{V \cos \varphi_L}$	$I_t = \frac{P_L}{\sqrt{3}V \cos \varphi_L}$
$P_{\text{losses}} = 2R_s I_s^2 = 2\rho \frac{l}{A_s} \left(\frac{P_L}{V \cos \varphi_L} \right)^2$	$P_{\text{losses}} = 3R_t I_t^2 = 3\rho \frac{l}{A_t} \left(\frac{P_L}{\sqrt{3}V \cos \varphi_L} \right)^2$

- If we impose that the two systems have same losses, we obtain: $A_s = 2A_t$
- In the single-phase, we need 2 conductors: $A_{s,\text{tot}} = 2A_s$
- In the three-phase, we need 3 conductors: $A_{t,\text{tot}} = 3A_t$
- Hence, $A_{s,\text{tot}} = 2A_s = 4A_t = \frac{4}{3}A_{t,\text{tot}}$ which means that the three-phase system requires 25% less material to feed the same load and have the same losses with the single-phase system.

Comparison between DC and three-phase AC systems

- Let's assume again same load P_L and same V for both systems.

DC system	3-phase AC system
$I_{DC} = \frac{P_L}{V}$	$I_t = \frac{P_L}{\sqrt{3}V \cos \varphi_L}$
$P_{losses} = 2R_{DC}I_{DC}^2 = 2\rho \frac{l}{A_{DC}} \left(\frac{P_L}{V}\right)^2$	$P_{losses} = 3R_t I_t^2 = 3\rho \frac{l}{S_t} \left(\frac{P_L}{\sqrt{3}V \cos \varphi_L}\right)^2$

- If we impose that the two systems have same losses, we obtain: $\frac{2}{A_{DC}} = \frac{1}{A_t \cos^2 \varphi_L}$

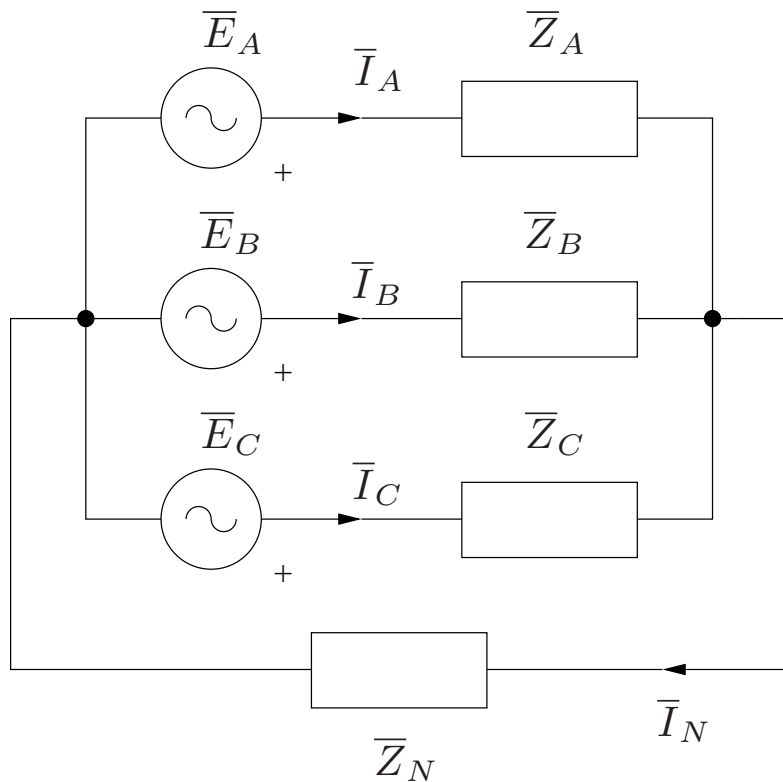
$$\left. \begin{array}{l} A_{DC,tot} = 2A_{DC} \\ A_{t,tot} = 3A_t \end{array} \right\} \Rightarrow A_{DC,tot} = 2S_{DC} = 4A_t \cos^2 \varphi_L = \frac{4}{3} \cos^2 \varphi_L A_{t,tot}$$

- Hence, $A_{DC,tot} < A_{t,tot}$ only if $\cos \varphi_L \leq \frac{\sqrt{3}}{2} = 0.866$
- Thus, in AC systems $\cos \varphi_L$ is bounded. Typically, loads must satisfy the condition:

$$1 \geq \cos \varphi_L \geq 0.9 \text{ (lagging)}$$

Single-phase equivalent of a three-phase system

- It is possible to study symmetrical and balanced 3-phase systems using an equivalent single-phase circuit.
- Let's consider the following circuit:



$$\bar{E}_A = \bar{Z}_A \bar{I}_A + \bar{Z}_N \bar{I}_N$$

$$\bar{E}_B = \bar{Z}_B \bar{I}_B + \bar{Z}_N \bar{I}_N$$

$$\bar{E}_C = \bar{Z}_C \bar{I}_C + \bar{Z}_N \bar{I}_N$$

Single-phase equivalent of a three-phase system

- If the voltages are symmetrical:

$$\bar{E}_A + \bar{E}_B + \bar{E}_C = 0 \Rightarrow \bar{Z}_A \bar{I}_A + \bar{Z}_B \bar{I}_B + \bar{Z}_C \bar{I}_C + 3\bar{Z}_N \bar{I}_N = 0$$

- If the system is also balanced, (i.e. $\bar{Z}_A = \bar{Z}_B = \bar{Z}_C = \bar{Z}$):

$$\bar{Z}(\bar{I}_A + \bar{I}_B + \bar{I}_C) = -3\bar{Z}_N \bar{I}_N$$

$$\bar{I}_A + \bar{I}_B + \bar{I}_C = 0 \Rightarrow \bar{I}_N = 0$$

- If the system is not balanced, (i.e. $\bar{Z}_A \neq \bar{Z}_B \neq \bar{Z}_C$):

$$\bar{I}_A = \frac{\bar{E}_A - \bar{V}_N}{\bar{Z}_A}, \bar{I}_B = \frac{\bar{E}_B - \bar{V}_N}{\bar{Z}_B}, \bar{I}_C = \frac{\bar{E}_C - \bar{V}_N}{\bar{Z}_C}$$

where: $\bar{V}_N = \bar{Z}_N \bar{I}_N$

Single-phase equivalent of a three-phase system

- Let:

$$\bar{Y}_A = \frac{1}{\bar{Z}_A}, \quad \bar{Y}_B = \frac{1}{\bar{Z}_B}, \quad \bar{Y}_C = \frac{1}{\bar{Z}_C}, \quad \bar{Y}_N = \frac{1}{\bar{Z}_N}$$

- The following condition holds (LKC):

$$\begin{aligned} \bar{I}_A + \bar{I}_B + \bar{I}_C &= \bar{I}_N \\ \Rightarrow \bar{Y}_A(\bar{E}_A - \bar{V}_N) + \bar{Y}_B(\bar{E}_B - \bar{V}_N) + \bar{Y}_C(\bar{E}_C - \bar{V}_N) &= \bar{Y}_N \bar{V}_N \\ \Rightarrow \bar{V}_N &= \frac{\bar{Y}_A \bar{E}_A + \bar{Y}_B \bar{E}_B + \bar{Y}_C \bar{E}_C}{\bar{Y}_A + \bar{Y}_B + \bar{Y}_C + \bar{Y}_N} \end{aligned}$$

- If $\bar{Z}_N = 0$ (i.e., neutral connection with negligible impedance):
 - $\bar{Y}_N \rightarrow \infty$
 - $\bar{V}_N = 0$

Single-phase equivalent of a three-phase system

- If $\bar{Z}_N \rightarrow \infty$ (i.e., no neutral connection):

- $\bar{Y}_N = 0$

- $\bar{V}_N = \frac{\bar{Y}_A \bar{E}_A + \bar{Y}_B \bar{E}_B + \bar{Y}_C \bar{E}_C}{\bar{Y}_A + \bar{Y}_B + \bar{Y}_C}$

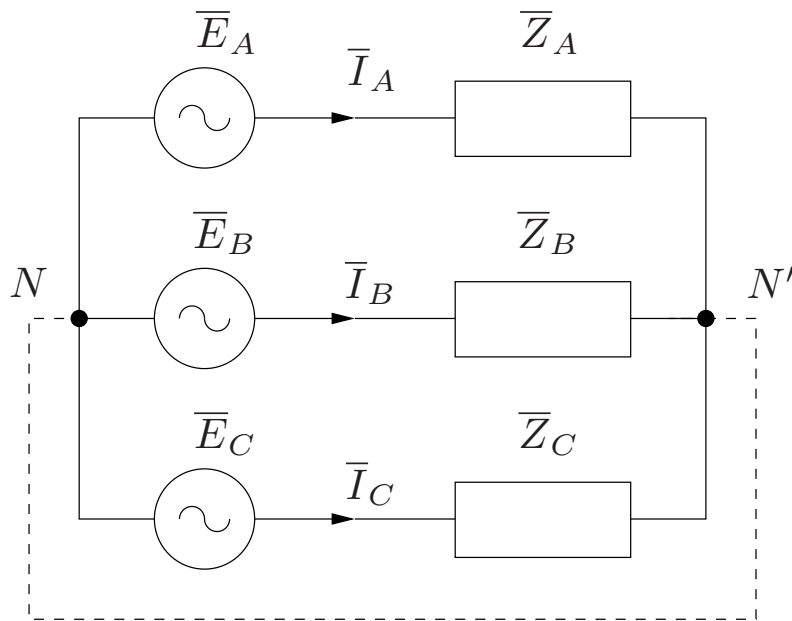
- If the system is balanced ($\bar{Y}_A = \bar{Y}_B = \bar{Y}_C = \bar{Y}$), we obtain again:

$$\bar{V}_N = \frac{\bar{Y}(\bar{E}_A + \bar{E}_B + \bar{E}_C)}{3\bar{Y} + \bar{Y}_N} = 0$$

- Hence, for a symmetrical and balanced system, **all neutral points have same potential.**

Single-phase equivalent of a three-phase system

- Let's define as single-phase equivalent of a three-phase system the following circuit:

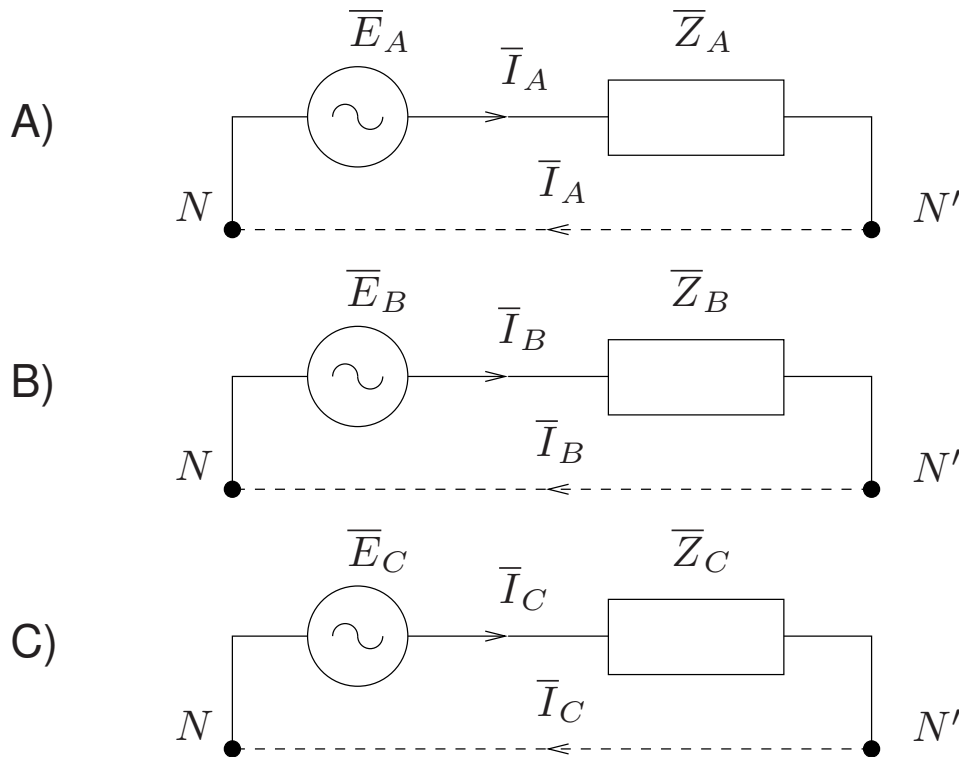


$$\bar{E}_A + \bar{E}_B + \bar{E}_C = 0$$

$$P = \sqrt{3}VI \cos \varphi = 3EI \cos \varphi$$

$$\bar{I}_A + \bar{I}_B + \bar{I}_C = 0$$

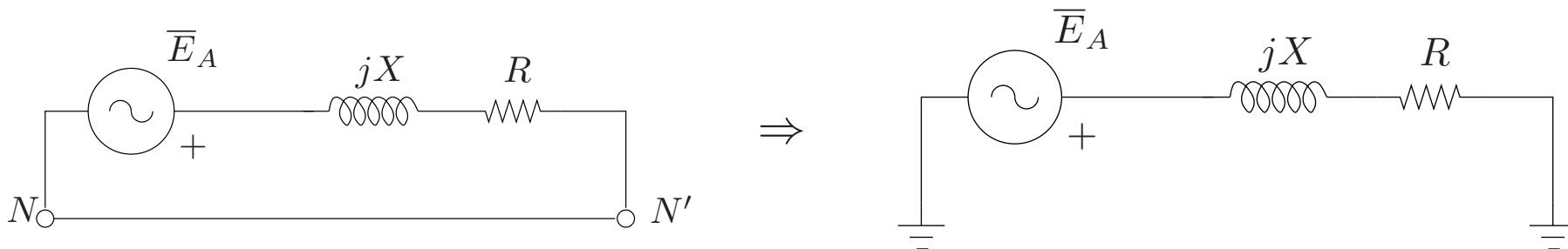
Single-phase equivalent of a three-phase system



- 3 independent circuits. We can study only one, e.g. circuit A.
- Note that: $P_A = P_B = P_C = EI \cos \varphi \Rightarrow P = 3P_A$

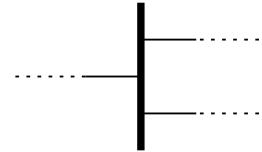
Single line diagram

- We can represent single-phase equivalents using a schematic, simplified representation.
- Since all the neutral points have the same potential, we can omit drawing the neutral connection and represent neutral points with a symbol.
- For example:

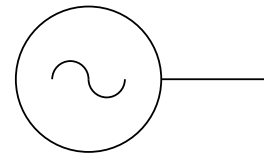


Some Symbols

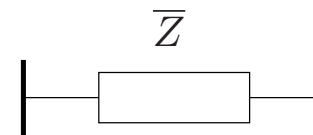
- Bus (connection among elements of the system)



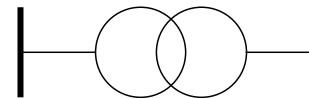
- Generator



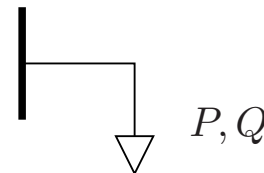
- Impedance (transmission line or cable)



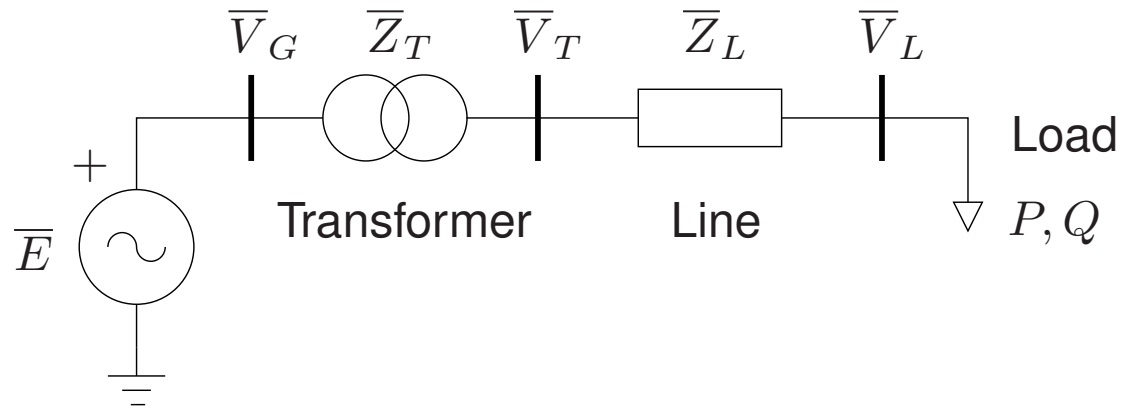
- Transformer



- Load



Example



- Equivalent single phase circuit:

