Worked Problems on Three-Phase Systems

EEEN20090 – Electrical Energy Systems

Problem 1

A 3-phase inductive load is connected in Δ and in parallel with a capacitor bank. The load current is K and the phase-to-phase voltage is $\frac{1}{K}$. Determine the value of the line current of the capacitor bank such that the overall power factor is unity.

Problem 2

The voltages of the three-phase system shown in Figure are:

$$\bar{E}_a = 1 \text{ V}, \quad \bar{E}_b = -1 \text{ V}, \quad \bar{E}_c = j1 \text{ V}.$$

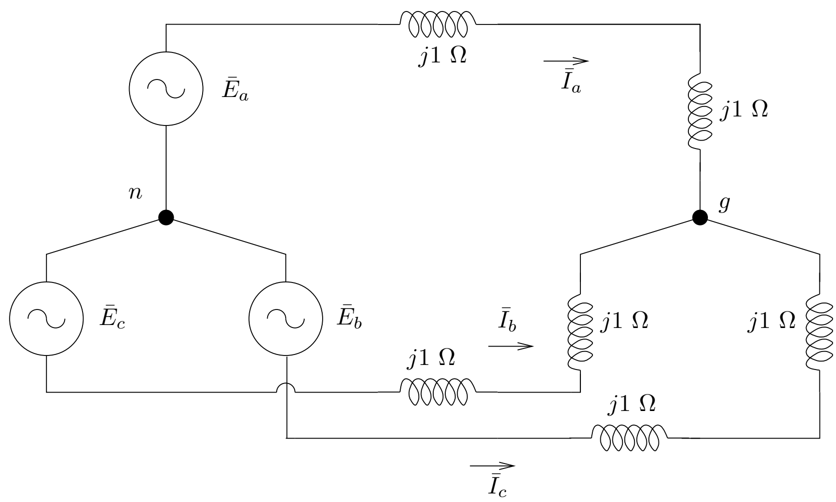

Figure 1

Determine:

a. The currents $\bar{I}_a$, $\bar{I}_b$ and $\bar{I}_c$.

b. The voltage $\bar{V}_{ng}$.

Problem 3

A 3-phase cable A feeds a 1000 kW, 6.6 kV load with power factor $\cos\varphi = 0.8$ lagging. Another cable B is connected in parallel with cable A. The impedance of cable B is $(3 + j4)\Omega$ per phase. When connected in parallel with cable B, cable A delivers 600 kW and its current is 68 A. Determine the impedance of cable A.

Problem 4

Consider the symmetrical, balanced 3-phase system shown in Figure .

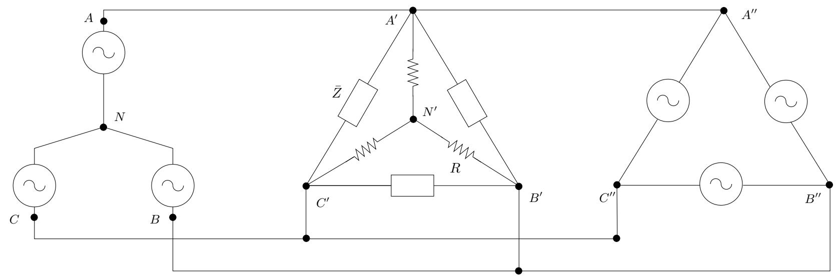

Figure 2

$$V = 220V,\ R = 100\Omega,\ \bar{V}_{C"A"} = \sqrt{3}V\angle 30^\circ,\ \bar{V}_{CN} = V\angle 0^\circ$$

The impedance $\bar{Z}$ is capacitive and, at nominal voltage (400 V) consumes 2400 VAr.

Determine:

a. The total load power consumed by the parallel of R and $\bar{Z}$.

b. The magnitude of current $I_{B'A'}$.

Solution of Problem 1

Let $\bar{S}_L$ be the power of the load and $\bar{S}_C$ the power of the capacitor bank. Then:

$$\bar{S}_L = \sqrt{3}K\frac{1}{K}\angle\varphi$$
$$\bar{S}_C = -\frac{\sqrt{3}}{K}I_C\angle 90°$$

The total power consumed by the load and the capacitors is:

$$\bar{S}_T = \sqrt{3}\cos\varphi + j\sqrt{3}\sin\varphi - j\frac{\sqrt{3}I_C}{K}$$

If we impose that $\cos\varphi_T = 1$, then the imaginary part of $\bar{S}_T$ is zero:

$$\mathrm{Im}\{\bar{S}_T\} = \sqrt{3}\sin\varphi - \frac{\sqrt{3}I_C}{K}\mathrm{Im}\bar{S}_T = 0 \Rightarrow I_C = K\sin\varphi$$

Solution of Problem 2

a. KVL gives:

$$\begin{cases} \bar{E}_a - jX\bar{I}_a + jX\bar{I}_b - \bar{E}_b = 0 \\ \bar{E}_a - jX\bar{I}_a + jX\bar{I}_c - \bar{E}_c = 0 \end{cases}$$

where $X = 2\ \Omega$. Then, from KCL: $\bar{I}_a + \bar{I}_b + \bar{I}_c = 0$. Hence:

$$\begin{bmatrix} -jX & jX & 0 \\ -jX & 0 & jX \\ 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} \bar{I}_a \\ \bar{I}_b \\ \bar{I}_c \end{bmatrix} = \begin{bmatrix} \bar{E}_b - \bar{E}_a \\ \bar{E}_c - \bar{E}_a \\ 0 \end{bmatrix}$$

From where we can obtain:

$$\begin{cases} \bar{I}_a = -0.1667 - j0.5 \text{ A} \\ \bar{I}_b = -0.1667 + j0.5 \text{ A} \\ \bar{I}_c = 0.3333 \text{ A} \end{cases}$$

b. The voltage $\bar{V}_{ng}$ is obtained as:

$$\bar{V}_{ng} = jX\bar{I}_a - \bar{E}_a = jX(-0.1667 - j0.5) - 1 = -j0.3333 \text{ V}$$

Note: An alternative solution is based on the Milligan's theorem, as follows:

$$\begin{aligned} \bar{V}_{ng} = -\bar{V}_{gn} &= -\frac{\bar{Y}_a\bar{E}_a + \bar{Y}_b\bar{E}_b + \bar{Y}_c\bar{E}_c}{\bar{Y}_a + \bar{Y}_b + \bar{Y}_c + \bar{Y}_{ng}} \\ &= -\frac{\bar{Y}\cdot(\bar{E}_a + \bar{E}_b + \bar{E}_c)}{3\bar{Y}} \\ &= -\frac{1}{3}(\bar{E}_a + \bar{E}_b + \bar{E}_c) = -j0.3333 \text{ V} , \end{aligned}$$

where it has been imposed that $\bar{Y}_{ng} = 0$ and $\bar{Y} = \bar{Y}_a = \bar{Y}_b = \bar{Y}_c$. Then currents can be deduced from the following expressions:

$$
\begin{aligned}
0 &= \bar{E}_a - jX\bar{I}_a + \bar{V}_{gn} \\
0 &= \bar{E}_b - jX\bar{I}_b + \bar{V}_{gn} \\
0 &= \bar{E}_c - jX\bar{I}_c + \bar{V}_{gn} \ .
\end{aligned}
$$

Solution of Problem 3

The data of the problem lead to the following equations:

$$I_{AR} + jI_{AI} + I_{BR} + jI_{BI} = \frac{P - jQ}{\frac{3}{\sqrt{3}}V} \tag{1}$$

$$\bar{z}_A(I_{AR} + jI_{AI}) = \bar{z}_B(I_{BR} + jI_{BI}) \tag{2}$$

$$\sqrt{(I_{AR}^2 + I_{AI}^2)} = 68 \text{ A} \tag{3}$$

$$I_{AR} = \frac{600}{\frac{3}{\sqrt{3}}V} \text{ A} \tag{4}$$

where

$$
\begin{aligned}
\bar{I}_A &= I_{AR} + jI_{AI} \\
\bar{I}_B &= I_{BR} + jI_{BI} \\
V &= 6.6 \text{ kV} \\
P + jQ &= (1000 + j750) \text{ kVA}
\end{aligned}
$$

Equation (1) imposes that the two cables when connected in parallel feed the load $P + jQ$. Equation (2) imposes that the voltage drop on the two cables when connected in parallel is the same. Equation (3) imposes the magnitude of the current in cable A. Equation (4) imposes the active power provided by cable (A) when connected in parallel with cable B. Equations (1)-(4) provide 6 conditions for 6 unknowns (I_{AR}, I_{AI}, I_{BR}, I_{BI}, $\bar{z}_A = R_A + jX_A$), hence the problem is well-posed and can be solved.

The resulting values are:

$$
\begin{aligned}
I_{AR} &= 52.5 \text{ A} \\
I_{AI} &= -43.2 \text{ A} \\
I_{BR} &= 35.0 \text{ A} \\
I_{BI} &= -22.4 \text{ A} \\
\bar{Z}_A &= (1.53 + j2.65)\ \Omega
\end{aligned}
$$

Solution of Problem 4

The equivalent single-phase circuit is shown in Figure 3, where:

$$\bar{Z} = -j\frac{V_N^2}{\frac{Q_N}{3}} = -j3\frac{400^2}{2400} = -j200\Omega$$

$$\bar{V}_{AN} = V(\angle\bar{V}_{CN} - \angle 120°)$$

$$= 220\angle{-120°})V$$

$$\bar{V}_{A''N''} = V(\angle\bar{V}_{C''A''} - \angle{-150°})$$

$$= V(\angle 30° - \angle{-150°})$$

$$= 220\angle{-120°})V$$

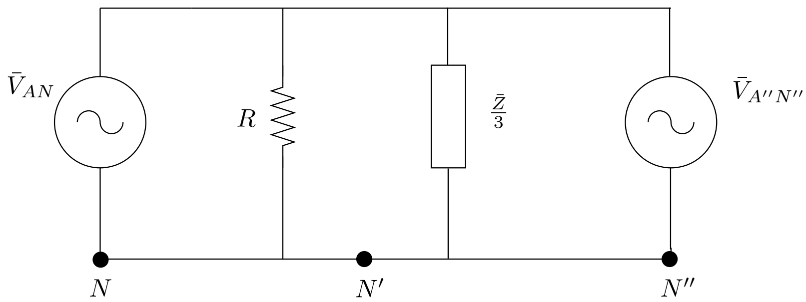

Figure 3

a. The total power of the parallel of $\bar{z}$ and R is:

$$\bar{S} = 3\frac{V^2}{R} - j3\frac{V^2}{\frac{|z|}{3}}$$

$$= 3\frac{220^2}{100} - j3\frac{220^2}{\frac{200}{3}}$$

$$= (1452 - j2178)VA$$

b. The magnitude of current $I_{B'A'}$ is:

$$\bar{I}_{B'A'} = \left[\frac{1}{\sqrt{3}}\frac{V\angle{-120°}}{\frac{z}{3}\angle{-90°}}\angle 30°\angle 180°\right]$$

$$= \frac{3\cdot 220}{\sqrt{3}\cdot 200}(\angle{-120°} + 90° + 30° + 180°)$$

$$= 1.9\angle 180°)A$$