Worked Problems on Transformers

EEEN20090 – Electrical Energy Systems

Problem 1

The tests for a 45 kVA 6000/230 V 50 Hz single phase transformer have given the following results:

Open-circuit: 2,200 W, 230 V, 45 A, all measures are taken on the low voltage winding.

Short-circuit: 2,750 W, 130 V, 52 A, all measures are taken on the high voltage winding.

a. Determine the parameters of the approximated equivalent circuit of the transformer, with all parameters referred to primary.

b. Assume that aload is connected to the secondary. The load consumes 22.5 kW with power factor. 0.85 lagging. The voltage on the secondary winding is 230 V. Determine the voltage on the primary winding and the efficiency of the transformer in such operating conditions.

c. Assume to have three transformers with same parameters as above and to connect these transformers in YY0. The primary is fed by a 10 kV three-phase grid and a three-phase load is connected to the secondary. The load is connected in triangle and has an impedance $1.5 + j2.7\ \Omega$ per branch. Determine the secondary voltage and the efficiency of the transformer in such operating conditions.

Problem 2

A single phase transformer has the following nominal data: 22 kVA, 400/200 V, $\epsilon_{sc} = 10\%$. The primary is fed at 375 V and a 100 A, power factor 1 load is connected to the secondary. In these conditions, the voltage on the secondary is 180 V.

Determine the primary voltage that leads to have on the secondary winding a voltage equal to 160 V when the transformer feeds a load of 14 kW and power factor 0.8 lagging.

Problem 3

A single phase transformer has the following nominal data: 200 kVA, tap ratio 11,000/380 V, 50 Hz. The tests gave the following results:

Open-circuit: 380 V, 100 A, 4,000 W (all measures are taken on the low voltage winding).

Cortocircuito: 750 V, corriente nominal, 7,000 W (all measures are taken on the high voltage winding).

Determine:

a. Parameters of the equivalent circuit, referred to primary, of the transformer.

b. Relative shortcircuit voltages (ϵ_{sc}, ϵ_{Rcc} and ϵ_{Xcc}).

Problem 4

A single phase transformer has the following nominal data: $S_N = 60$ kVA, $N_1/N_2 = 1500/700$, $\bar{Z}_1 = 0.22 + j1.2\ \Omega$, $\bar{Z}_2 = 0.05 + j0.3\ \Omega$ and $X_\mu = 180\ \Omega$. Iron losses are negligible.

The primary winding id fed by a voltage of 1,200 V. Determine:

a. Open-circuit secondary voltage.

b. Secondary voltage when the transformer feeds a load that consumes 110 A with power factor 0.8 leading.

c. Per unit variation of the magnetic induction of the transformer from open-circuit to the on-load conditions of the previous point.

Note: Use the exact equivalent circuit of the transformer.

Solution of Problem 1

a. Parameters of the shunt branch referred to the secondary winding:

$$R''_{\mathrm{Fe}} = \frac{V_0^2}{P_0} = 24.05\ \Omega \tag{1}$$
$$X''_{\mu} = \frac{V_0}{I_{0i}} = 5.23\ \Omega \tag{2}$$

where:

$$I_{0r} = \frac{V_0}{R''_{\mathrm{Fe}}} = 9.57\ \mathrm{A} \tag{3}$$
$$I_{0i} = \sqrt{I_0^2 - I_{0r}^2} = 43.97\ \mathrm{A} \tag{4}$$

Parameters of the shunt branch referred to the primary winding:

$$R_{\mathrm{Fe}} = R''_{\mathrm{Fe}} k_T^2 = 16.36\ \mathrm{k}\Omega \tag{5}$$
$$X_{\mu} = X''_{\mu} k_T^2 = 3.56\ \mathrm{k}\Omega \tag{6}$$

where the tap ratio is:

$$k_T = \frac{V_1}{V_2} = 26.1 \tag{7}$$

Short-circuit impedance:

$$R'_{sc} = \frac{P_{sc}}{I_{sc}^2} = 1.02\ \Omega \tag{8}$$
$$X'_{sc} = \sqrt{V_{sc}^2/I_{sc}^2 - R_{sc}^2} = 2.28\ \Omega \tag{9}$$

b. The primary voltage is (V_2 is assumed to be the reference phase):

$$\bar{S}_2 = P_2 + jP_2 \tan\phi_2 = (22.50 + j13.94)\ \mathrm{kVA} \tag{10}$$
$$\bar{I}'_2 = \frac{\bar{S}_2^*}{V_2 k_T} = 3.75 - j2.32\ \mathrm{A} \tag{11}$$
$$\bar{V}_1 = k_T V_2 + (R'_{sc} + jX'_{sc})\bar{I}'_2 = 6009.1 + j6.2\ \mathrm{V} \tag{12}$$

The efficiency is:

$$\bar{Z}_{\mathrm{Fe}} = \frac{jR_{\mathrm{Fe}}X_{\mu}}{R_{\mathrm{Fe}} + jX_{\mu}} = 739.3 + j3398.8\ \Omega \tag{13}$$
$$\bar{I}_0 = \frac{\bar{V}_1}{\bar{Z}_{\mathrm{Fe}}} = 0.37 - j1.69\ \mathrm{A} \tag{14}$$
$$\bar{I}_1 = \bar{I}_0 + \bar{I}'_2 = 4.12 - j4.01\ \mathrm{A} \tag{15}$$
$$P_1 = \Re\{\bar{V}_1 \bar{I}_1^*\} = 24.7\ \mathrm{kW} \tag{16}$$
$$\eta = 100\frac{P_2}{P_1} = 91.00\% \tag{17}$$

c. The secondary line voltage is (V_1 is assumed ot be the phase reference):

$$\begin{aligned}
\bar{Z}'_c &= k_T^2(1.5 + j2.7)/3 = 340.2 + j612.5\ \Omega && (18)\\
\bar{Z}'_t &= R'_{sc} + jX'_{sc} + \bar{Z}'_c = 341.3 + j614.8\ \Omega && (19)\\
\bar{Z}'_{\text{eq}} &= \frac{\bar{Z}_{\text{Fe}}\bar{Z}'_t}{\bar{Z}_{\text{Fe}} + \bar{Z}'_t} = 260.2 + j527.8\ \Omega && (20)\\
\bar{I}_1 &= \frac{V_1}{\bar{Z}'_{\text{eq}}} = 4.34 - j8.80\ \text{A} && (21)\\
\bar{I}'_2 &= \frac{\bar{Z}'_{\text{Fe}}}{\bar{Z}'_{\text{Fe}} + \bar{Z}'_t}\bar{I}_1 = 3.99 - j7.18\ \text{A} && (22)\\
\bar{V}'_2 &= \bar{Z}'_c\bar{I}'_2 = 5753 - 1.8\ \text{V (fase)} && (23)\\
V_2 &= \sqrt{3}|\bar{V}'_2|\ k_T = 381.97\ \text{V (línea)} && (24)
\end{aligned}$$

The efficiency is:

$$\begin{aligned}
P_1 &= \Re\{\bar{Z}'_{\text{eq}}\}|\bar{I}_1|^2 && (25)\\
P_2 &= \Re\{\bar{Z}'_c\}|\bar{I}'_2|^2 && (26)\\
\eta &= 100\frac{P_1}{P_2} = 91.59\% && (27)
\end{aligned}$$

Solution of Problem 2

First, we obtain the short-circuit impedance of the transformer. We have:

$$\bar{I}_2(R''_{sc} + jX''_{sc}) = V''_1(\cos\delta_1 + j\sin\delta_1) - \bar{V}_2 \qquad (28)$$

where:

$$V''_1 = \frac{V_1}{k_T} = 187.5\ \text{V} \qquad (29)$$

Assuming that V_2 is the reference phase, one has:

$$\begin{aligned}
I_2R''_{sc} + V_2 &= V''_1\cos\delta_1 && (30)\\
I_2X''_{sc} &= V''_1\sin\delta_1 && (31)
\end{aligned}$$

Moreover, the following relation holds:

$$R''^{\,2}_{sc} + X''^{\,2}_{sc} = Z''^{\,2}_{sc} \qquad (32)$$

where:

$$Z''_{sc} = \epsilon_{sc}\frac{V_{2N}^2}{S_N} = 0.1818\ \Omega \qquad (33)$$

Summing the squares of (30) and (31), one obtains:

$$I_2^2Z_{sc}^2 + 2I_2V_2R''_{sc} + V_2^2 = V''^2_1 \qquad (34)$$

and finally R''_{sc} and X''_{sc}:

$$\begin{aligned} R''_{sc} &= 0.0674\,\Omega \quad (35) \\ X''_{sc} &= 0.1689\,\Omega \quad (36) \end{aligned}$$

The primary voltage V_1 for the 14 kW load is:

$$V_1 = k_T|V_2 + (R''_{sc} + jX''_{sc})\frac{\bar{S}_2^*}{V_2}| = 354.6 \text{ V} \quad (37)$$

where we have assumed $\bar{V}_2 = V_2\angle 0$.

Solution of Problem 3

a. Tap ratio:

$$m = \frac{V_{1n}}{V_{2n}} = \frac{15000}{380} = 39 \quad (38)$$

Open-circuit data:

$$\begin{aligned} V_0 &= m \cdot 380 = 11000 \quad \text{V} \quad (39) \\ I_0 &= 100/m = 3.45 \quad \text{A} \\ P_0 &= V_0 I_0 \cos(\phi_0) = 4000 \quad \text{W} \\ \cos(\phi_0) &= \frac{P_0}{V_0 I_0} = 0.10526 \end{aligned}$$

Parameters of the shunt branch referred to the secondary winding:

$$\begin{aligned} R_{Fe} &= \frac{V_0}{I_0 \cdot \cos(\phi_0)} = \frac{11000}{\cdot 3.45 \cdot 0.10526} = 30250 \quad \Omega \quad (40) \\ X_\mu &= \frac{V_0}{I_0 \cdot \sin(\phi_0)} = \frac{11000}{\cdot 3.45 \cdot 0.99444} = 3202 \quad \Omega \end{aligned}$$

Short-circuit parameters:

$$\begin{aligned} V_{sc} &= 750 \quad \text{V} \quad (41) \\ I_{sc} &= I_{1n} = \frac{S_n}{V_{1n}} = \frac{200000}{\cdot 11000} = 18.2 \quad \text{A} \\ P_{sc} &= V_{sc} I_{sc} \cos\phi_{sc} = 7000 \quad \text{W} \\ \cos(\phi_{sc}) &= \frac{P_{sc}}{V_{sc} I_{sc}} = 0.51333 \end{aligned}$$

Parameters of the series branch referred to the secondary winding:

$$\begin{aligned} R'_{sc} &= \frac{V_{sc}}{I_{sc}} \cos(\phi_{sc}) = \frac{750}{18.2} \cdot 0.51333 = 21.175 \quad \Omega \quad (42) \\ X'_{sc} &= \frac{V_{sc}}{I_{sc}} \sin(\phi_{sc}) = \frac{750}{18.2} \cdot 0.85819 = 35.4 \quad \Omega \end{aligned}$$

b. Per-unit shortcircuit voltages:

$$\begin{aligned}
\epsilon_{sc} &= \frac{V_{sc}}{V_{1n}} \cdot 100 = \frac{750}{11000} \cdot 100 = 6.82\% \\
\epsilon_{Rcc} &= R'_{sc}\frac{I_{sc}}{V_{1n}} \cdot 100 = 21.175 \cdot \frac{18.2}{11000} \cdot 100 = 3.5\% \\
\epsilon_{Xcc} &= X'_{sc}\frac{I_{sc}}{V_{1n}} \cdot 100 = 35.4 \cdot \frac{18.2}{11000} \cdot 100 = 5.85\%
\end{aligned} \tag{43}$$

Solution of Problem 4

a. The open-circuit voltage on the secondary winding (using the primary voltage as phase reference):

$$\begin{aligned}
\bar{V}'_{20} &= \frac{\bar{Z}_\mu}{\bar{Z}_\mu + \bar{Z}_1}\bar{V}_1 \\
\Rightarrow \quad V_{20} &= \frac{V'_{20}}{k_T} = 556.6 \quad \text{V}
\end{aligned} \tag{44}$$

b. The on-load voltage on the secondary winding (using the secondary voltage as phase reference):

$$\bar{V}_1 = \bar{Z}_1\bar{I}_1 + \bar{Z}'_2\bar{I}'_2 + \bar{V}'_2 \tag{45}$$

$$\bar{I}_1 = \bar{I}'_2 + \frac{\bar{V}'_2 + \bar{Z}'_2\bar{I}'_2}{\bar{Z}_\mu} \tag{46}$$

from where one obtains:

$$V_1(\cos\theta_1 + j\sin\theta_1) = \bar{a} + \bar{b}V'_2 \tag{47}$$

where:

$$\bar{a} = a_r + ja_i = (\frac{\bar{Z}'_2\bar{Z}_1}{\bar{Z}_\mu} + \bar{Z}_1 + \bar{Z}'_2)\bar{I}'_2 \tag{48}$$

$$\bar{b} = b_r + jb_i = \frac{\bar{Z}_1}{\bar{Z}_\mu} + 1 \tag{49}$$

From the quare of (47), one obtains:

$$\alpha {V'}_2^2 + \beta V'_2 + \gamma = 0 \tag{50}$$

where:

$$\alpha = b_r^2 + b_i^2 \tag{51}$$

$$\beta = 2(a_rb_r + a_ib_i) \tag{52}$$

$$\gamma = a_r^2 + a_i^2 - V_1^2 \tag{53}$$

and, finally:

$$V_2 = \frac{V'_2}{k_T} = 580.9 \quad \text{V} \tag{54}$$

c. The per unit variation of the magnetic induction of the transformer from open circuit to on-load conditions detrmined in the previous question is:

$$\Delta B = \frac{E_2 - V_{20}}{V_{20}} = 0.017 \tag{55}$$

where E_2 is the emf induced in the on-load secondary winding:

$$\bar{E}_2 = \bar{V}_2 + \bar{Z}_2 \bar{I}_2 \tag{56}$$